

# Using Poems to Teach Exponential and Logarithmic Functions

By

Janine P. Bundy

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APPROVED FOR THE MATHEMATICS PROGRAM

I. Grzegorzczuk 7/30/10

Ivona Grzegorzczuk, Advisor Date

Cynthia F. Wyels 8/4/10

Cynthia Wyels Date

APPROVED FOR THE UNIVERSITY

G. A. Berg 9-1-10

Dr. Gary A. Berg Date

To my dad and mom, John and Marilyn Bundy, in gratitude for their  
love, encouragement, support and example.

Numbers 6:24-26, 2 Timothy 1:3

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# Abstract

Algebra students have a difficult time learning abstract concepts such as exponents and logarithms. The purpose of this thesis is to investigate the success of teaching exponential and logarithmic concepts to algebra students using specially designed poems compared to the traditional textbook methods. Thirty-six Intermediate Algebra students participated in this study. The study was conducted at California Lutheran University Spring Semester, 2010. Three topics were presented in this study: exponential functions, introduction to logarithms, and exponential and logarithmic equations. Two methods were used in presenting the material of the study: the traditional method as written in most college intermediate algebra textbooks and a special method employing poems. Using the special method technique the students' exponential and logarithmic conceptual knowledge improved. This thesis shows evidence of increased student understanding of exponential and logarithmic concepts worthy of further investigation. The results suggest that less emphasis on drill and manipulation and increased focus on the meaning of abstract concepts, in this case via poems, might increase student performance.

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# Chapter 1

## Introduction

Ask any high school or college intermediate algebra instructor what topics students have a difficult time understanding. Frequently they will say exponents and logarithms. According to Hammack & Lyons (1995), “Many students have difficulties mastering the concept even more so than with other functions” (p. 374). Students have limited intuition of the concepts. They are capable of doing calculations and manipulations yet are void of understanding fundamental concepts (Wood, 2005). Students are unable to explain why exponential and logarithmic properties are true (Weber, 2002b). Students who successfully complete course work have a limited understanding of its nature (Confrey, & Smith, 1995).

In mathematics, as abstract concepts increase -- as in exponential and logarithmic functions, student understanding decreases. Students mechanically memorize and apply rules without understanding them, seemingly dodging the exponential and logarithmic bullet, only to find out they may meet these exponents and logarithms at a more sophisticated level in future course work where comprehension of these concepts is crucial.

In addition to understanding exponential and logarithmic functions for future mathematics courses such as pre-calculus, calculus, differential equations,

and advanced topics, these are important mathematical concepts that are useful for other subjects and numerous applications.

Physics uses Newton's Law of Cooling and radioactive decay. Finance makes use of compound interest, present value, and future value growth. Chemistry and biology employ growth and decay models and solution pH. Physiology uses the Nernst Equation. Engineering makes use of magnitude of earthquakes and measuring decibels of sound. Without a strong understanding of exponential and logarithmic concepts, students are less likely to succeed in those subjects.

A study in foundational biochemistry and biological undergraduate courses by D. Watters and J. Watters (2006) found the "lack of basic understanding of logarithms and the inability of students to perform a simple calculation on logs without a calculator severely hampered their understanding of pH" (p. 278) and consequently limited their understanding of basic applications used in the course.

Mathematics educators have used many methods to teach exponents and logarithms to students. Some of these techniques include slide rules (Jackson, 1903), computers (Ronan, 1971), history (Toumasis, 1993), calculators (Rahn, & Berndes, 1994), and applications (Wood, 2005). An extensive literature search failed to identify any assessments as to the efficacy of these five specific techniques. Additionally, this extensive literature search resulted in only one study (Weber, 2002b), a pilot study, that focused on teaching exponential and logarithmic concepts and testing the effectiveness of the method.

If students are taught poems about exponents and logarithms, would they be better able to conceptualize exponents and logarithms? Would they understand the abstract representations? This thesis will describe a special instructional technique employing poems designed to teach exponents and logarithms. It will also report the results of a pilot study examining the efficacy of this technique.

*Statement of problem:* Algebra students have difficulties learning the abstract concepts pertaining to exponents and logarithms.

*Purpose:* The purpose of this study is to investigate the success of teaching exponential and logarithmic concepts to algebra students using specially designed poems.

*Significance of the study:* This study's significance consists of the development and implementation of a carefully designed experiment to determine whether a special teaching technique that incorporates poems leads to improvements in students' understanding of exponential and logarithmic functions.

## 1.1 Common mistakes made with logarithms

Students are more successful with carrying out processes of logarithms than with understanding the concepts of logarithms (Weber, 2002). Lack of understanding these concepts can in turn inhibit the students' performance using logarithmic properties. Students commonly have errors in their thinking process

as they manipulate logarithmic expressions. Many common mistakes students make while manipulating logarithms are summarized in Table 1<sup>1</sup>.

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<sup>1</sup> The author created this list of “common mistakes” then verified through a literature search that this represents the common view of common mistakes.

Correct property	Common Mistakes	Explanation
<p><b>Product Property of Logarithms</b></p> $\log_b(xy) = \log_b x + \log_b y$ <p>The sum of the logs is the log product.</p>	$\log_b(x + y) \neq \log_b x + \log_b y$  $\log_b(xy) \neq (\log_b x)(\log_b y)$	<ul style="list-style-type: none"> <li>- The log of a sum is not sum of the logs.</li> <li>- The logarithm of a sum can not be simplified (<math>\log_b(x + y)</math> can not be re-written).</li> <li>- The log of a product is not the product of logs.</li> </ul>
<p><b>Quotient Property of Logarithms</b></p> $\log_b \frac{x}{y} = \log_b x - \log_b y$ <p>The difference of the logs is the log quotient.</p>	$\log_b(x - y) \neq \log_b x - \log_b y$  $\log_b \frac{x}{y} \neq \frac{\log_b x}{\log_b y}$ <p style="text-align: center;">or</p> $\log_b x - \log_b y \neq \frac{\log_b x}{\log_b y}$	<ul style="list-style-type: none"> <li>- The log of a difference is not the difference of logs.</li> <li>- The logarithm of a difference can not be simplified.</li> <li>- The log of a quotient is not a quotient of logs.</li> <li>- The quotient of the logs is used in the change of base formula.</li> <li>- It is also not <math>x \div y</math> then take the log.</li> </ul>
<p><b>Power Product of Logarithms</b></p> $\log_b x^r = r \log_b x$ <p>When the argument is raised to a power the exponent can be turned into a coefficient.</p>	$(\log_b x)^r \neq r \log_b x$	<ul style="list-style-type: none"> <li>- An exponent on the log is not the coefficient of the log.</li> </ul>
<p><math>\log x</math> is a number</p>	<p><math>\log</math> times <math>x</math></p>	<ul style="list-style-type: none"> <li>- <math>\log x</math> is not a multiplication operation (<math>\log</math> times <math>x</math>)</li> </ul>

Table 1: Common mistakes made with logarithms.

# Chapter 2

## Description of Research

### 2.1 Location of Study

The study took place at California Lutheran University (CLU), a 225 acre campus located midway between Los Angeles and Santa Barbara, in the city of Thousand Oaks. CLU offers undergraduate, graduate and continuing education programs through its College of Arts and Sciences, School of Business and School of Education. The University offers 37 majors and 31 minors, in addition to professional preparation programs in specified fields of study. CLU is accredited by the Accrediting Commission for Senior Colleges and Universities of the Western Association of Schools and Colleges.

The 3,714 student enrollment at CLU is comprised of 2,352 undergraduate and 1,362 graduate students. The University maintains its residential emphasis with 61 percent of traditional undergraduate students living on campus. Average class size is 15 students. CLU's student body originates from 40 states and 36 counties<sup>2</sup>.

### 2.2 Participants of Study

The students who participated in this study were enrolled in the Spring 2010 Math 110 Intermediate Algebra courses at CLU. Math 110 is a four unit class at CLU. There were two sections of this class available, section 01 or 02, and both sections participated in the study. Each section met for sixty-five minutes three times each week on Monday,

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<sup>2</sup> All data in Sections 2.1 – 2.3 are taken from the California Lutheran University website, [www.clunet.edu](http://www.clunet.edu).

Wednesday and Friday. Section 01 met from 12:15 until 1:20 pm and section 02 met from 1:30 until 2:35 pm. There were eighteen students in each section for a total of thirty-six students.

Math 110 is designed to prepare students for their required college math class. According to the CLU catalog, “This course covers equations and inequalities, polynomials, rational and radical expressions, exponents, graphing linear equations and inequalities, linear systems, quadratic functions, exponential and logarithmic functions, and places extensive emphasis on word problems. This course is appropriate for students with Math SAT 500 or below.”

CLU Admissions requires incoming students have at least three years of high school mathematics, including Algebra II, although four years are preferred.

## 2.3 Duration of Study

The CLU Intermediate Algebra classes covered the logarithms sections discussed in this thesis during the last two weeks of spring 2010 semester, Monday, April 26 through Wednesday, May 5.

## 2.4 Outline of Study Method

A total of three sections of logarithms were presented:

- 1) Exponential Functions
- 2) Introduction to Logarithms
- 3) Exponential and Logarithmic Equations

These specific topics were chosen because they are required topics for the class. The topics of each section were presented to the students twice. The first time the topics were presented, they were presented in a traditional method using the textbook

*Intermediate Algebra with Applications*. The second time the topics were presented, they were presented in a special method using poems from the textbook *Al Zebra Explains Algebra*. The second presentation (special method) of the topics occurred following the traditional method during the next class meeting. The next class meeting occurred at least two days after the previous class meeting.

At the beginning of each lecture, whether traditional or special method was used, the students received a hard copy of the lecture notes. All lecture notes are in the Appendices. At the bottom of the last page of each topic of the traditional method notes, a homework assignment is listed for the student textbook *Intermediate Algebra with Applications*. Homework is collected each Wednesday and graded. It is unknown when each student in the study completed the homework (immediately following the lecture, in between the traditional method lecture and the special method lecture, the night before the homework was due, etc.). However, the homework questions were computational in nature and did not offer practice with concepts. The special method lecture did not offer homework questions. However, the students had the opportunity to memorize two of the special method poems for extra credit.

At the conclusion of each lecture (both traditional and special method), the students were told they would have a quiz from the lecture's material the following class meeting. The study used six quizzes total, Quiz 10 A & B, Quiz 11 A & B and Quiz 12 A & B, although there were only three different quizzes. Quiz 10 A & B were identical (except for the labels A & B), Quiz 11 A & B were identical (except for the labels A & B) and Quiz 12 A & B were identical (except for the labels A & B). The students were not told they would be receiving the same quiz twice. Students were not given any

answers to the quizzes until after the conclusion of the study. Prior to this study, students received one quiz every Friday throughout the semester up until the time of the study (with the exception of three Fridays during the semester when exams were given). Once the study began, students received at least one quiz, sometimes two, every class meeting for five consecutive class meetings. Prior to the study, students received only one lecture for each topic. The study differed from the previous lectures given during the semester in that the topics studied were presented twice. A topic was presented with the traditional method lecture and then it was presented the following class meeting a second time using the special method lecture.

## 2.5 Detailed description of the experiment

### Monday, April 26, 2010

Lecture notes for Exponential Functions traditional method were handed out to the students (Appendix A) at the beginning of class. The Exponential Functions traditional method lecture was given (approximately 60 minutes). At the conclusion of the lecture, students are told at the beginning of the next class meeting they will have a quiz covering the day's lecture.

### Wednesday, April 28, 2010

Students were given ten minutes to take the quiz on Exponential Functions traditional method, Quiz 10 A (Appendix G). Quizzes were collected. Following the quiz, lecture notes for Exponential Functions special method were handed out to the students (Appendix D). The Exponential Functions special method lecture was given (approximately 5 minutes). Students were told they would have a quiz from the material

at the beginning of the next class meeting. Then lecture notes for Introduction to Logarithms traditional method were handed out to the students (Appendix B). Introduction to Logarithms traditional lecture was given (approximately 45 minutes). At the conclusion of the lecture students were told they would have a quiz from the material at the beginning of the next class meeting, following the first quiz.

#### Friday, April 30, 2010

Students were given ten minutes to take the quiz on Exponential Functions special method, Quiz 10 B (Appendix H). Quizzes were collected. Then students were given ten minutes to take Introduction to Logarithms traditional method, Quiz 11 A (Appendix I). Quizzes were collected. Following the second quiz given, lecture notes for Introduction to Logarithms special method were handed out to the students (Appendix E). The Introduction to Logarithms special method lecture was given (approximately 5 minutes). Students were told they would have a quiz from the material at the beginning of the next class meeting.

Students were given the opportunity to memorize the poem from the Introduction to Logarithms special method lecture and write it out at the next class meeting for ten points of extra credit. The next class meeting was Monday, May 3, 2010. (This allowed students three days to memorize the poem prior to the next class meeting.) The incentive due to the ten points of extra credit was significant. Ten points is equivalent to one week's worth of homework and two-thirds of a fifteen point weekly quiz. At the end of the semester when this experiment took place, most students were eager to earn bonus points for their overall class grade.

Then lecture notes for Exponential and Logarithmic Equations traditional method were handed out to the students (Appendix C). Exponential and Logarithmic Equations traditional lecture was given (approximately 40 minutes). At the conclusion of the lecture students were told they would have a quiz from the material at the beginning of the next class meeting, following the first quiz.

Monday, May 3, 2010

Students were given fifteen minutes to take the quiz on Introduction to Logarithms special method, Quiz 11 B (Appendix J). (They were allotted an extra five minutes to write the poem on the back of the quiz for the ten extra credit points.) After the fifteen minutes expired, quizzes were collected and students were given ten minutes to take the quiz on Exponential and Logarithmic Equations traditional method, Quiz 12 A (Appendix K). After the second quizzes were collected, lecture notes for Exponential and Logarithmic Equations special method were handed out to the students (Appendix F). The Exponential and Logarithmic Equations special method lecture was given (approximately 5 minutes). Students were told they would have a quiz from the material at the beginning of the next class meeting. Students were given the opportunity to memorize the poem from the Exponential and Logarithmic Equations special method lecture for ten points of extra credit. The next class meeting was Wednesday, May 5, 2010. (This allowed students two days to memorize the poem prior to the next class meeting.) Then a topic of quadratic equations was presented. The quadratic equations material was not part of the study.

Wednesday, May 5, 2010

Students were allowed fifteen minutes to take the quiz on Exponential and Logarithmic Equations special method Quiz 12 B (Appendix L). (Once again they were allotted an extra five minutes to write the poem on the back of the quiz for 10 extra points.) After the fifteen minutes expired, prior to collecting the quizzes, the students answered two questions on the back of the quiz.

Question #1 You have taken six quizzes during the last four lectures. How many of the quizzes were different?

Question #2 Did learning the poems better help you to understand the logarithms concepts?

# Chapter 3

## Methods

The three topics presented for this study were Exponential Functions, Introduction to Logarithms, and Exponential and Logarithmic Equations. Two methods in this study were used to teach these concepts, the traditional method as written in most college intermediate algebra textbooks and a special method, the method employing poems. The methods used will now be described.

### 3.1 Traditional Method

The traditional method is a mechanical approach using rules and formulas that allow students to solve exponent and logarithm questions. The traditional method instruction included three topics, Exponential Functions, Introduction to Logarithms, and Exponential and Logarithmic Equations delivered during three different class meetings. A different topic was presented each day. The corresponding notes (student handouts) for each lecture can be found in the appendices. The notes were written to accompany the class textbook *Intermediate Algebra with Applications*. Examples used in the notes are from the textbook *Intermediate Algebra with Applications*.

#### 3.1.1 Exponential Functions – delivered day one of the study

In class just before the lecture began, the students received a copy of notes titled Exponential Functions Section 10.2. (Appendix A).

The instructor began the lecture telling the students this is Section 10.1 in the text, Exponential Functions. We have two objectives today in our lesson. The first objective is to evaluate exponential functions and the second is to graph exponential functions. Let's proceed.

The instructor said a definition of an exponential function is given in your notes. The instructor duplicated the information on the board:

***Definition of an Exponential Function***

The **exponential function** with base  $b$  is defined as

$$f(x) = b^x$$

Where  $b > 0, b \neq 1$ , and  $x$  is any real number.

The instructor then asked the students why  $b \neq 1$ . One student correctly offered up the explanation if  $b = 1$ , then the function will always be 1 regardless of the  $x$  value.

The instructor proceeded to do a few examples from the student handout evaluating exponential functions with specific  $x$  values. The examples used bases with  $b > 0$  and with  $0 < b < 1$ .

The instructor said a definition of a natural exponential function is given in your notes. The instructor duplicated the information on the board:

***Natural Exponential Function***

The function defined by

$$f(x) = e^x$$

is called the **natural exponential function**.

A quick explanation of  $e$  was given followed by the example evaluating a natural logarithm function listed in the student handout.

Then the lecture proceeded to address the second objective graphing exponential functions. Two examples were given. The first was an example of a polynomial function  $g(x) = x^2$ , the second was an example of an exponential function  $f(x) = 2^x$ . After the two graphs were drawn using the  $x, y$  chart technique, the instructor explained that the polynomial function has a variable for the base and a constant for the exponent, but the exponential function has a constant for the base and a variable for the exponent. The latter is the focus of our lecture.

Pointing to the graph of  $f(x) = 2^x$  the instructor asked the students, if a vertical line intersects the graph at only one point what can we conclude? Students correctly answered that we would conclude the graph is a graph of a function since it passes the vertical line test.

The instructor asked the students, did you notice on this same graph a horizontal line intersects the graph at only one point? This means this graph passed the horizontal line test. A graph that passed both the vertical and horizontal line test is said to be a one-to-one function. The exponential functions are one-to-one functions. The instructor

directed the students' attention to the next page of the notes that summarized these concepts.

While keeping the first two graphs on the board,  $g(x) = x^2$  and  $f(x) = 2^x$ , the instructor proceeded to graph a third function  $g(x) = \left(\frac{1}{2}\right)^x$  and then a fourth function

$f(x) = 2^{-x}$  using the  $x, y$  chart method. All four graphs remained on the board for comparison. The students clearly acknowledged the polynomial graph did not fit the criteria for the exponent graph based on its base as well as its graph. Students also recognized the third and fourth graphs were identical and were able to reason

$2^{-x} = \left(\frac{1}{2}\right)^x$ . Using the second and third graphs,  $f(x) = 2^x$  and  $g(x) = \left(\frac{1}{2}\right)^x$ , the

instructor showed the students if the base of the exponential function was greater than 1, the graph would be increasing and if the base was greater than 0 but less than 1, the graph would be decreasing.

The students then had an opportunity to graph four more exponential functions during the class lecture.

The students were told they would have a quiz from this lecture at the beginning of the next class.

### 3.1.2 Introduction to Logarithms – delivered day two of the study

In class just before the lecture began, the students received a copy of the one page of notes titled Introduction to Logarithms Section 10.2. (Appendix B)

The instructor began the lecture telling the students this is Section 10.2 in the text, Introduction to Logarithms. We have two objectives today in our lesson. The first objective is to write equivalent exponential and logarithmic equations and the second is to understand properties of logarithms.

The instructor said a definition of a logarithm is given in your notes. The instructor duplicated the information on the board:

<p style="text-align: center;"><b><i>Definition of a Logarithm</i></b></p> <p style="text-align: center;">If <math>x &gt; 0</math> and <math>b</math> is a positive constant not equal to 1, then</p> <p style="text-align: center;"><math>y = \log_b x</math></p> <p style="text-align: center;">is equivalent to</p> <p style="text-align: center;"><math>b^y = x</math></p>
--

A few examples were shown writing equivalent statements in logarithmic form for the given exponential form.

Then one example was given changing an exponential equation into a logarithmic equation. Then the students were asked to try a similar example on their own.

Then one example was given changing a logarithmic equation into an exponential equation. Then the students were asked to try a similar example on their own.

Next, the instructor said a definition of the Equality of Exponents Property is given in your notes. The instructor duplicated the information on the board:

***Equality of Exponents Property***

For  $b > 0, b \neq 1$ , if  $b^u = b^v$ ,

then  $u = v$

Two examples were shown evaluating logarithms using the Equality of Exponents Property. An additional example was shown solving a log using the same property.

Next, the instructor said a definition of antilogarithm is given in your notes. The instructor duplicated the information on the board:

***Definition of Antilogarithm***

If  $\log_b M = N$ , the antilogarithm base  $b$   
of  $N$  is  $M$ .

In exponential form,  $M = b^N$

The instructor explained antilogarithm is another way to state the equivalence of a logarithmic expression and an exponential expression.

Two examples were shown solving logarithms using the Antilogarithm Property.

The instructor gave a brief comparison of common logarithms versus natural logarithms followed by two examples of solving a logarithm with base ten and a logarithm with base  $e$ .

Next learning objective two was taught, the properties of logarithms. The instructor began this portion of the lecture writing on the board a comparison of exponential form and logarithmic form so students could visualize the similarities.

Exponential Form

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

Logarithmic Form

$$\log_2 1 = 0$$

$$\log_2 2 = 1$$

$$\log_2 4 = 2$$

The instructor proceeded to introduce the Product Property of Logarithms by showing  $\log_2 4 + \log_2 8 = \log_2 32$

$$\log_2 4 + \log_2 8 = 2 + 3 = 5$$

$$\log_2 32 = 5$$

$$\log_2 4 + \log_2 8 = \log_2 32$$

$$\log_2 32 = \log_2 (4 \times 8) = \log_2 4 + \log_2 8$$

Next, the instructor said a definition of the Product Property of Logarithms is given in your notes. The instructor duplicated the information on the board:

***Product Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b (xy) = \log_b x + \log_b y$$

Then, the instructor said a definition of the Quotient Property of Logarithms is given in your notes. The instructor duplicated the information on the board:

***Quotient Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

Then, the instructor said a definition of the Power Property of Logarithms is given in your notes. The instructor duplicated the information on the board:

***Power Property of Logarithms***

For any positive real numbers  $x$ , and  $b$ ,  
 $b \neq 1$ , for any real number  $r$ ,

$$\log_b x^r = r \log_b x$$

Four examples were given using these three properties writing logarithms in expanded form. Then four examples were given expressing logarithms as a single logarithm with a coefficient of 1.

Next, the instructor said three properties of logarithms are given in your notes. The instructor duplicated the information on the board:

***Properties of Logarithms***

**Logarithmic Property of One**

For any positive real number  $b, b \neq 1,$

$$\log_b 1 = 0$$

**Inverse Property of Logarithms**

For any positive real numbers  $x$  and  $b, b \neq 1,$

$$\log_b b^x = x$$

**1-1 Property of Logarithms**

For any positive real numbers  $x, y$  and  $b, b \neq 1,$

$$\text{if } \log_b x = \log_b y,$$

$$\text{then } x = y$$

Four examples were given using these three properties.

The lecture concluded with the change-of-base formula. The instructor said the change-of-base formula is given in your notes. The instructor duplicated the information on the board.

***Change-of-Base Formula***

$$\log_a N = \frac{\log_b N}{\log_b a}$$

The students were told they would have a quiz from this lecture at the beginning of the next class.

Due to the generous amount of material given in this lecture, students were told the quiz on this lecture would be regarding understanding the definitions of logarithms and changing logarithms into exponents.

### 3.1.3 Exponential and Logarithmic Equations – delivered day three of the study

In class just before the lecture began, the students received a copy of the notes titled Exponential and Logarithmic Equations Section 10.4. (Appendix C)

The instructor began the lecture telling the students this is Section 10.4 in the text, Exponential and Logarithmic Equations. We have two objectives today in our lesson. The first objective is to solve exponential equations and the second is to solve logarithmic equations.

The instructor reminded students an exponential equation is one in which a variable occurs in the exponent. Then the instructor said the Equality of Exponents Property is given in your notes. The instructor duplicated the information on the board:

***Equality of Exponents Property***

For  $b > 0, b \neq 1$ , if  $b^u = b^v$ ,

then  $u = v$

The instructor told the students if it is easy to express the both sides of the equation with like bases, try the Equality of Exponents Property. Two examples were given that were easy to express both sides of the equation with like bases.

The instructor proceeded. If it is NOT easy to express the sides of the equation with like basis, use logarithms to solve the exponential equations. Four examples were given using this approach.

Next learning objective two was taught, solving logarithmic equations. The instructor reminded students of the Product Property of Logarithms, Quotient Property of Logarithms, Power Property of Logarithms, and 1 – 1 Property of Logarithms from Section 10.2. The instructor said a definition of these properties was given in your Section 10.2 notes and it is in today's handout as well. The instructor duplicated the information on the board:

***Product Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b(xy) = \log_b x + \log_b y$$

***Quotient Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

***Power Property of Logarithms***

For any positive real numbers  $x$ , and  $b$ ,  
 $b \neq 1$ ,

for any real number  $r$ ,

$$\log_b x^r = r \log_b x$$

***1-1 Property of Logarithms***

For any positive real numbers  $x, y$  and  
 $b, b \neq 1$ , if  $\log_b x = \log_b y$ ,  
then  $x = y$

The instructor then solved three logarithmic equations using these properties. Specific information was given to the students describing how to approach each type of question.

Students were encouraged to simplify each side of the equation to one expression. The Product Property of Logarithms or the Quotient Property of Logarithms would be useful for this approach.

When one side of the equation has a constant and the other side of the equation has one log expression, the students were encouraged to change the logarithmic equation to an exponential equation and solve.

When both sides of the equation have just one log expression, use the 1-1 Property of Logarithms to solve the equation.

The students were told they would have a quiz from this lecture at the beginning of the next class. They were told the quiz would cover less complex questions than the last three equations we solved. The quiz questions would be basic simplification or expansion of logarithms using properties reviewed in this lecture, the Product Property of Logarithms, Quotient Property of Logarithms, and the Power Property of Logarithms. They were also told there would be one question from the previous section 10.2, Change of Base Property.

## 3.2 Special Method

The special method is an unconventional approach using poems to understand and solve exponential and logarithmic questions. Similar to the traditional method instruction, the special method instruction was delivered over three separate days. A different topic was presented each day. The same topics presented with the traditional method instruction were presented with the special method instruction; Exponential Functions, Introduction to Logarithms and Exponential and Logarithmic Equations. The corresponding notes (student handouts) for each lecture can be found in the appendices. The notes are from the textbook *Al Zebra Explains Algebra*.

### 3.2.1 Exponential Functions – delivered day two of the study

In class just before the lecture began, the students received a copy of the one page of notes titled Exponentials. (Appendix D)

The instructor told the class this is information on Exponential Functions.

Next the following poem from the lecture notes was read to the students by the instructor:

*Study rabbits – monitor their population size.  
It doubles each season. The doubling surprise!  
Two, four, eight, then sixteen... The graph grows so fast.  
The exponential growth function exposed at last!  
Bacteria can grow faster in a large Petri dish,  
From 10 to 100, to 1000... Trillions, if you wish.  
Multiplying by a factor that is bigger than one:  
Is about growth. While, decreasing can also be fun:  
Eat just half of your cake. Then just half of what's left.  
And one half again, and again. You will have to be deft  
As leftovers get smaller, almost none to display.  
And this graph is exposing exponential decay.*

Subsequently the poem was read again by the instructor. This time after the instructor read a portion of the poem, the instructor stated the represented graphic in the notes and pointed to the represented symbols or pictures mentioned in that portion of the poem.

The instructor read:

*Study rabbits – monitor their population size.  
It doubles each season. The doubling surprise!  
Two, four, eight, then sixteen... The graph grows so fast.  
The exponential growth function exposed at last!*

The instructor said this portion of the poem represents the graph of the rabbits.

The instructor then pointed to the graph of the rabbits.

The instructor continued reading:

*Bacteria can grow faster in a large Petri dish,  
From 10 to 100, to 1000... Trillions, if you wish.  
Multiplying by a factor that is bigger than one:  
Is about growth.*

The instructor said this portion of the poem represents the graphic  $a^x$  increasing for  $a > 1$ . The instructor then pointed to the graphic  $a^x$  increasing for  $a > 1$

The instructor continued reading:

*While, decreasing can also be fun:  
Eat just half of your cake. Then just half of what's left.  
And one half again, and again. You will have to be deft*

The instructor said this portion of the poem represents the graphic  $a^x$  decreasing for  $0 < a < 1$ . The instructor then pointed to the graphic  $a^x$  decreasing for  $0 < a < 1$ . The instructor continued reading:

*As leftovers get smaller, almost none to display.  
And this graph is exposing exponential decay.*

The instructor said this portion of the poem represents the graph of the cakes. The instructor then pointed to the graph of the cakes.

The students were told they would have a quiz from this lecture at the beginning of the next class.

### 3.2.2 Introduction to Logarithms – delivered day three of the study

In class just before the lecture began, the students received a copy of the one page of notes titled Logarithms. (Appendix E)

The instructor told the class this is information on an Introduction to Logarithms.

Next the following poem from the lecture notes was read to the students by the instructor:

*Logarithms, Logarithms, numbers full of magic!  
Always real, often long, sometimes even tragic.  
If you try and try again, this secret comes out next:  
Find the power  $y$  for the base  $a$  to make it equal  $x$ .  
Choose  $a$  and  $x$  both positive -- to avoid confusion,  
And keep away from the base one, its powers are an illusion.  
If  $x$  is one, to your surprise enchantment quickly fades  
As  $y$  is zero, always zero, no matter what's the base.  
When  $x$  and  $a$  are both the same, then  $y$  is simply one.  
That's a trivial riddle, a trivial case, no solving full of fun.*

Subsequently the poem was read again by the instructor. This time after the instructor read a portion of the poem, the instructor stated the represented graphic in the notes and pointed to the represented symbols or pictures mentioned in that portion of the poem.

The instructor read:

*Logarithms, Logarithms, numbers full of magic!  
Always real, often long, sometimes even tragic.  
If you try and try again, this secret comes out next:  
Find the power  $y$  for the base  $a$  to make it equal  $x$ .*

The instructor said this portion of the poem represents the equation  $\log_a x = y$  and  $a^y = x$ . The instructor then pointed to the equation  $\log_a x = y$  and  $a^y = x$ .

The instructor continued reading:

*Choose  $a$  and  $x$  both positive -- to avoid confusion,*

The instructor said this portion of the poem represents the condition  $a, x > 0$ .

The instructor then pointed to the condition  $a, x > 0$ .

The instructor continued reading:

*And keep away from the base one, its powers are an illusion.*

The instructor said this portion of the poem represents the condition  $a \neq 1$  as  $1^y = 1$  always. The instructor then pointed to the condition  $a \neq 1$  as  $1^y = 1$  always. The instructor continued reading:

*If  $x$  is one, to your surprise enchantment quickly fades  
As  $y$  is zero, always zero, no matter what's the base.*

The instructor said this portion of the poem represents the equation  $\log_a 1 = 0$   $a^0 = 1$ . The instructor then pointed to the equation  $\log_a 1 = 0$   $a^0 = 1$ . The instructor continued reading:

*When  $x$  and  $a$  are both the same, then  $y$  is simply one.  
That's a trivial riddle, a trivial case, no solving full of fun.*

The instructor said this portion of the poem represents the equation  $\log_a a = 1$   $a^1 = 1$ . The instructor then pointed to the equation  $\log_a a = 1$   $a^1 = 1$ .

The students were told they would have a quiz from this lecture at the beginning of the next class. They were also told they could memorize the poem from today's lecture for ten points of extra credit. They would have an opportunity to write it on their quiz during the next lecture.

### 3.2.3 Exponential and Logarithmic Equations – delivered day four of the study

In class just before the lecture began, the students received a copy of the one page of notes titled Logarithms. (Appendix F)

The instructor told the class this is information on Logarithms.

Next the following poem from the lecture notes was read to the students by the instructor:

*When different logs are in the game and all this makes you scared  
Just change the bases to what you like, and then you can compare.  
Two or ten are good for base. And then the number  $e$  some say.  
Now take the ratio in the chosen base:  $\log x$  over  $\log a$ .  
And now start adding them together, the  $x$ 's multiply,  
Divide the  $x$ 's when you subtract, and solve your problems now.  
When  $x$  is raised to power  $a^x$ , and you find it so confusing,  
Drop  $a^x$  in front of  $\log$  - or take it back up - your choosing.*

Subsequently the poem was read again by the instructor. This time after the instructor read a portion of the poem, the instructor stated the represented graphic in the notes and pointed to the represented symbols or pictures mentioned in that portion of the poem.

The instructor read:

*When different logs are in the game and all this makes you scared  
Just change the bases to what you like, and then you can compare.  
Two or ten are good for base. And then the number e some say.  
Now take the ratio in the chosen base: log x over log a.*

The instructor said this portion of the poem represents the equation  $\log_a x = \frac{\log_c x}{\log_c a}$ . The

instructor then pointed to the equation  $\log_a x = \frac{\log_c x}{\log_c a}$ .

The instructor continued reading:

*And now start adding them together, the x's multiply,*

The instructor said this portion of the poem represents the equation

$\log_a x_1 + \log_a x_2 = \log_a x_1 \cdot x_2$ . The instructor then pointed to the equation

$\log_a x_1 + \log_a x_2 = \log_a x_1 \cdot x_2$ .

The instructor continued reading:

*Divide the x's when you subtract, and solve your problems now.*

The instructor said this portion of the poem represents the equation

$\log_a x_1 - \log_a x_2 = \log_a \frac{x_1}{x_2}$ . The instructor then pointed to the equation

$\log_a x_1 - \log_a x_2 = \log_a \frac{x_1}{x_2}$ .

The instructor continued reading:

*When x is raised to power T, and you find it so confusing,  
Drop T in front of log - or take it back up - your choosing.*

The instructor said this portion of the poem represents the equation  $\log_a x^T = T \log_a x$ .

The instructor then pointed to the equation  $\log_a x^T = T \log_a x$ .

The students were told they would have a quiz from this lecture at the beginning of the next class. They were also told they could memorize the poem from today's lecture for ten points of extra credit. They would have an opportunity to write it on their quiz during the next lecture.

# Chapter 4

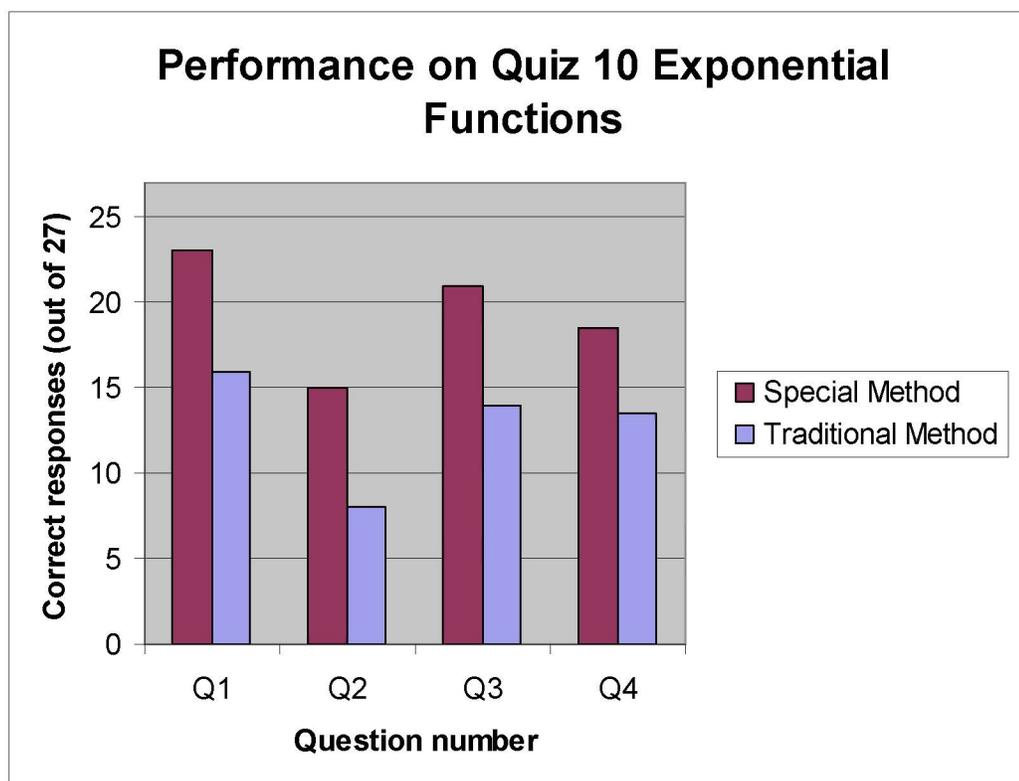
## Results

### 4.1 Results from Quiz 10 Exponential Functions

Quiz 10 examines students' conceptual understanding of the exponential growth and decay functions as well as of the definition of an exponential function.

Although there were a total of thirty-six students in the observed group, only twenty-seven student results are used for interpretation for Quiz 10. The data from students that were absent at least one of the two days the exponential function lectures were presented were not used in the results due to missing either Quiz A, Quiz 10 B or both.

The number of correct responses for each of the questions from Quiz 10 is displayed in Graph 1.



Graph 1: Performance on Quiz 10 Exponential Functions

Collectively the students scored higher on each of the four questions on the special method Quiz 10 B (Appendix H) compared to the traditional method Quiz 10 A (Appendix G).

The success rate of question 2 nearly doubled for the special method Quiz 10 B. Question 2 was a conceptual question asking students to choose a graph (from a selection of 8 graphs) that represented positive exponential growth function with base  $0 < b < 1$ . Less than 30% of the students answered that question correctly after the traditional lecture method. 56% of the students answered that question correctly after the special method lecture.

Data were analyzed for each question on Quiz 10 A and B. A question answered correctly scored 1 point. A question answered incorrectly scored 0 points. A comparison

of traditional method Quiz 10 A, and special method Quiz 10 B, was calculated for each question. Comparison results were tabulated for each of the four questions; 1 point if the student's score improved from traditional method Quiz 10 A, to special method Quiz 10 B, 0 points if there was no change in a student's score from traditional method Quiz 10 A, to special method Quiz 10 B, and -1 points if a student's score decreased from traditional method Quiz 10 A, to special method Quiz 10 B.

The number of students that experienced an increase in score was tabulated per quiz question. The number of students that experienced a decrease in score was tabulated per quiz question. Logistic regression was used to calculate P-values for each quiz question as well as for the overall Quiz 10 results. The results are shown in Table 2. A P-value less than 0.05 indicates our results are unlikely to be due to chance alone, suggesting there is evidence that the special method is improving students' success on the quiz.

	Question 1	Question 2	Question 3	Question 4	Quiz 10 Overall
Number of students with increased score	8	10	9	8	9
Number of students with decreased score	1	3	1	1	2
Total number of students with increased or decreased score	9	13	10	9	11
P-value	0.019531	0.046143	0.010742	0.019531	0.032715

Table 2: P-values for Quiz 10 Exponential Functions  
Note: Significant P-values ( $< .05$ ) are shaded

Question 1 on both Quiz 10 A and B was a conceptual question that asked students to choose a graph shape (from a selection of eight graphs) that represented positive exponential growth function with base  $> 1$ . The P-value result for question 1

shown in table 2 suggests there is evidence that the special method is improving students' success on question 1.

Question 2 on both Quiz 10 A and B was a conceptual question that asked students to choose a graph shape (from a selection of eight graphs) that represented positive exponential growth function with base  $0 < b < 1$ . The P-value result for question 2 shown in table 2 suggests there is evidence that the special method is improving students' success on question 2.

Question 3 on both Quiz 10 A and B was also a conceptual question regarding a person trying a new exercise plan. The exercise plan causes the person to lose weight. The first week the person loses 1 pound, the next week  $\frac{1}{2}$  pound, the next week  $\frac{1}{4}$  pound, the next week  $\frac{1}{8}$  inch pound, and the next week  $\frac{1}{16}$  pound. The weight loss continues with the same pattern. Students were asked to choose the graph shape (from a selection of eight graphs) that best represents the person's weight loss. The P-value result for question 3 shown in table 2 suggests there is evidence that the special method is improving students' success on question 3.

Question 4 on both Quiz 10 A and B was also a conceptual question asking in an exponential function, why do we say the base can not be 1? The P-value result for question 4 shown in table 2 suggests there is evidence that the special method is improving students' success on question 4.

Overall the students performed better on the special method Quiz 10 B than the traditional method Quiz 10 A.

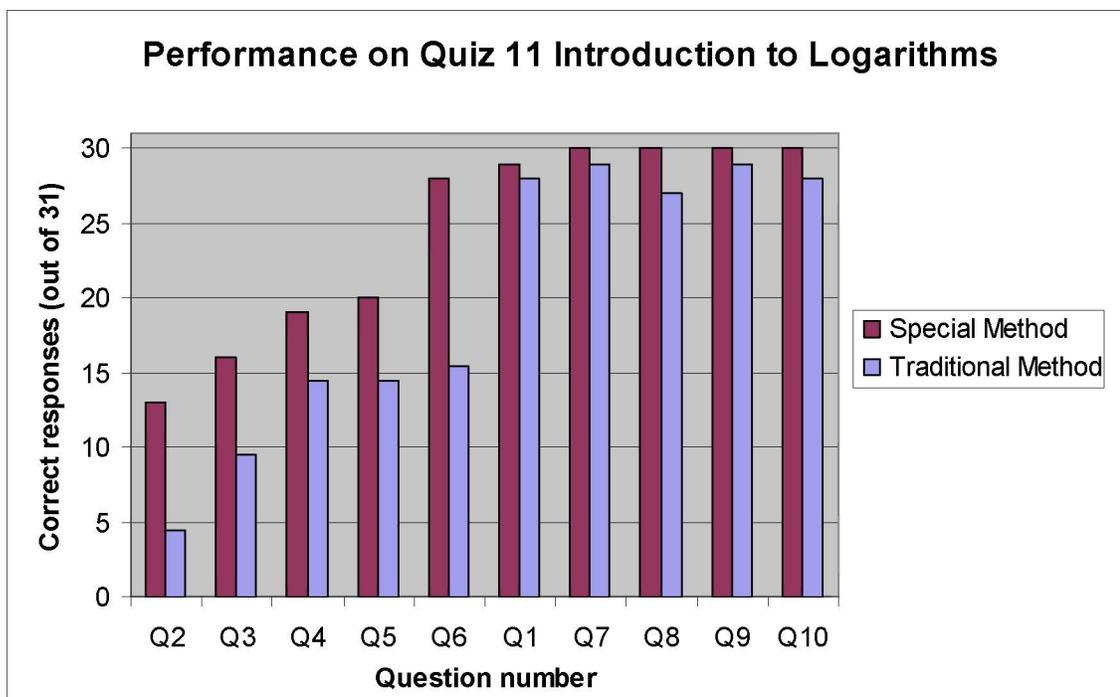
The P-values suggest students have improved understanding of concepts like exponential growth function with base  $> 1$  or  $0 < b < 1$ , and why the base can not equal 1 in an exponential function when the material is presented using the special method technique.

## 4.2 Results from Quiz 11 Introduction to Logarithms

Quiz 11 examines students' conceptual understanding of the definition of logarithms and the conversion of a logarithm to an exponent.

Although there were a total of thirty-six students in the observed group, thirty-one student results are used for interpretation for Quiz 11. The data from students that were absent at least one of the two days the exponential function lectures were presented were not used in the results due to missing either Quiz A, Quiz 11 B or both.

The number of correct responses for each of the questions from Quiz 11 is displayed in Graph 2.



Graph 2: Performance on Quiz 11 Introduction to Logarithms

Collectively the students scored higher on each of the ten questions on the special method Quiz 11 B (Appendix J) compared to the traditional method Quiz 11 A (Appendix I). Although both quizzes show the students generally performed well on questions 1, 7, 8, 9, and 10, the improvement is minor.

Question 1, 7, 8, 9, and 10 were all questions regarding a variation on writing a log in exponential form. Historically these are the types of questions students can easily manipulate. (Wood, 2005). They are grouped together on the right side of the graph. Questions 2, 3, 4, 5, and 6 are conceptual questions. The results for these conceptual questions are grouped together on the left side of the graph.

Question 2, regarding  $\log_b x = y$  describe the types of numbers  $b$  and  $x$  can be, the students scored nearly three times better on the special method Quiz 11 B (42% correct) compared to the traditional method Quiz 11 A (15% correct)

Question 3, regarding  $\log_b x = y$  describe the types of numbers  $b$  and  $x$  can be, the students' scores improved almost 68% on the special method Quiz 11 B (52% correct) compared to the traditional method Quiz 11 A (31% correct)

Question 4, regarding  $\log_b x = y$  describe what happens if  $x$  is one, the student score improvement was 31% on the special method Quiz 11 B (61% correct) compared to the traditional method Quiz 11 A (47% correct)

Concerning question 5, regarding  $\log_b x = y$  when is  $y$  zero, the overall student improvement was similar to question 4. An interesting answer from one of the students was she wrote when  $x = 1$  to your surprise enchantment quickly faded for  $y$  is 0 always 0 no matter what the base. This student quoted the poem when responding to the question.

Six students answered Question 6, of the special method Quiz 11 B quoting the poem. Question 6 was regarding  $\log_b x = y$  describe what happens when  $x$  and  $b$  are both the same? These six students quoted the poem by writing when  $b$  and  $x$  are both the same,  $y$  is simply 1, the exact quote from the poem (with the exception of replacing the "a" in the poem, the symbol for the base, for "b", the symbol used for the base in question 6). Many other students wrote when  $b$  and  $x$  are both the same,  $y$  is 1, capturing the essence of the poem.

In addition to the students remembering this particular concept, it is of interest to note students were able to replace a base with a symbol "a" given in the poem to a different base "b" that the question asked.

Question six had the highest number of improved scores when comparing the scores of special method Quiz 11 B (twenty-eight correct responses) compared to the traditional method Quiz 11 A (sixteen correct responses).

Data were analyzed for each question on Quiz 11 A and B. A question answered correctly scored 1 point. A question with only half of the answer received .5 point. A question answered incorrectly scored 0 points. A comparison of traditional method Quiz 11 A, and special method Quiz 11 B, was calculated for each question. Comparison results were tabulated for each of the four questions; 1 point if the student's score improved from traditional method Quiz 11 A, to special method Quiz 11 B, 0 points if there was no change in a student's score from traditional method Quiz 11 A, to special method Quiz 11 B, and -1 points if a student's score decreased from traditional method Quiz 11 A, to special method Quiz 11 B.

The number of students that experienced an increase in score was tabulated per quiz question. The number of students that experienced a decrease in score was tabulated per quiz question. Logistic regression was used to calculate P-values for each quiz question as well as for the overall Quiz 11 results. The results are shown in Table 3. A P-value less than 0.05 indicates our results are unlikely due to chance alone, suggesting there is evidence that the special method is improving students' success on some of the quiz questions, specifically the conceptual questions 2, 5, 6 and the quiz overall.

	Quest 2	Quest 5	Quest 6	Quiz 11 Overall	Quest 1	Quest 3	Quest 4	Quest 7	Quest 8	Quest 9	Quest 10
Number of students with increased score	17	7	15	19	3	13	8	1	4	1	2
Number of students with decreased score	1	1	1	7	2	6	5	0	1	0	0
Total number of students with increased or decreased score	18	8	16	26	5	19	13	1	5	1	2
P-value	7.2E-05	0.03516	0.00026	0.01448	0.5	0.08353	0.29053	0.5	0.1875	0.5	0.25

Table 3: P-values for Quiz 11 Introduction to Logarithms  
Note: Significant P-values (< .05) are shaded

Question 2 on both Quiz 11 A and B was a conceptual question that asked students to describe the types of numbers  $b$  and  $x$  can be in equation  $\log_b x = y$ . Seventeen students increased their score while only one student score was decreased. The P-value result for question 2 shown in table 3 suggests there is evidence that the special method is improving students' success on question 2.

Question 5 on both Quiz 11 A and B was a conceptual question that asked students to describe when  $y$  is zero in equation  $\log_b x = y$ . Seven students increased their scores while only one student score was decreased. The P-value result for question 5 shown in table 3 suggests there is evidence that the special method is improving students' success on question 5.

Question 6 on both Quiz 11 A and B was a conceptual question that asked students to describe happens when  $x$  and  $b$  are both the same in equation  $\log_b x = y$ . Fifteen students increased their score while only one student score was decreased. The P-value result for question 6 shown in table 3 suggests there is evidence that the special method is improving students' success on question 6.

A point of interest is that the results in Graph 2 look similar for questions 4 and 5, yet the P-values for each of these questions do not lead us to the same conclusion. Both questions 4 and 5 had fifteen of the thirty-one students correctly respond to traditional method Quiz 11A. Question 4 had nineteen of the thirty-one students correctly respond to special method Quiz 11B, while question 5 has twenty of the thirty-one students correctly respond to special method Quiz 11B. The overall scores were similar. However, question four responses improved for eight students while the responses decreased for five students. Question 5 responses improved for seven students while the

responses decreased for only one student. This explains why the quantity of responses for these questions is similar yet the P-values are significantly different.

Overall the students performed better on the special method Quiz 11 B than the traditional method Quiz 11 A. The P-values suggest students have improved understanding of  $\log_b x = y$  when the material is presented using the special method technique. The P-values suggest, due to the special method technique, the students have improved conceptual knowledge about the types of numbers  $b$  and  $x$  can be, what happens when  $y$  is zero and when  $x$  and  $b$  are both the same.

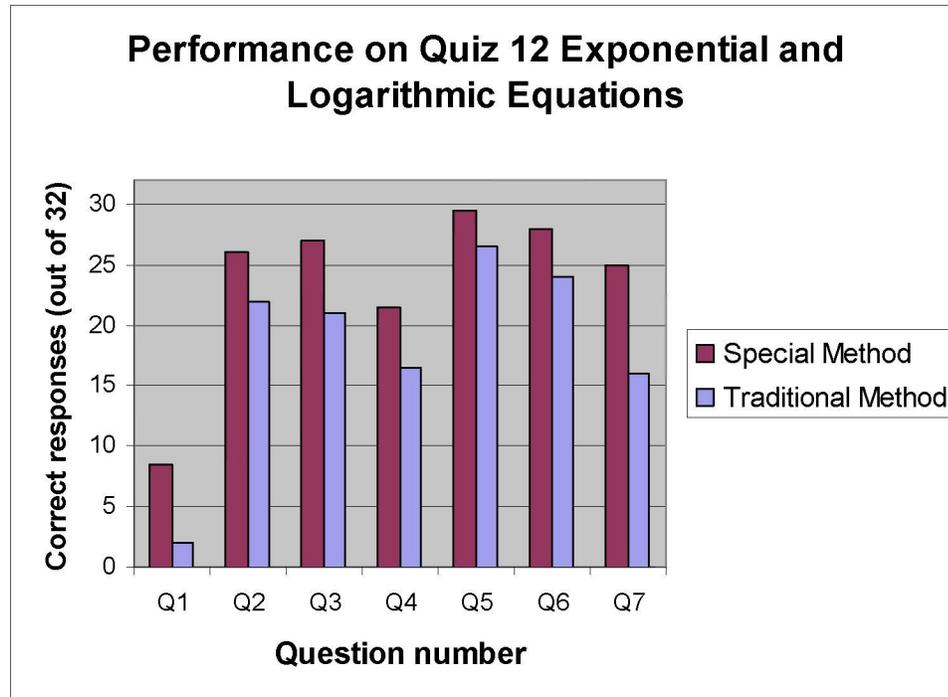
## 4.3 Results from Quiz 12 Exponential and Logarithmic

### Equations

Quiz 12 examines students' understanding of the Product Property of Logarithms, Quotient Property of Logarithms, and the Power Property of Logarithms and Change of Base Property. Students typically make the common mistakes with logarithms shown in Table 1 when addressing the types of computational questions given in Quiz 12.

Although there were a total of thirty students in the observed group, thirty-two student results are used for interpretation for Quiz 12. The data from students that were absent at least one of the two days the exponential function lectures were presented were not used in the results due to missing either Quiz A, Quiz 12 B or both.

The number of correct responses for each of the questions from Quiz 12 is displayed in Graph 3.



Graph 3: Performance on Quiz 12 Exponential and Logarithmic Equations

Collectively the students scored higher on each of the seven computational questions on the special method Quiz 12 B (Appendix L) compared to the traditional method Quiz 12 A (Appendix K).

Question 1 special method Quiz 12 B had over four times the correct responses as did the traditional method Quiz 12 A. This was a change of base question, change  $\log_4 16$  to base 16.

Question 7 special method Quiz 12 B had 50% improvement from the traditional method Quiz 12 A. This was a Product Property of Logarithms question, express  $\log_y 9^4$  as a product.

Although not shown on Graph 3, a point of interest is in the improvement of common mistakes made with logarithms (Table 1) from the responses of the traditional method Quiz 12 A to the special method Quiz 12 B. Collectively students had a total of thirteen responses (5.8% of the Quiz 12 A total responses) using one of the common logarithmic errors shown in Table 1 on the traditional method Quiz 12 A. However, collectively, the students had only one response (less than 1% of the Quiz 12 B total responses) with a common logarithmic error (Table 1) on the special method Quiz 12 B. The only response with a common logarithmic error (the student wrote the coefficient of the log as the exponent on the log) on the special method Quiz 12 B was for question 4. This is interesting because common logarithmic mistakes were not mentioned to the students in any of the lectures. It is suggested this improvement is a result of the special method technique.

Data were analyzed for each question on Quiz 12 A and B. A question answered correctly scored 1 point. A question with only half of the answer received .5 point. A

question answered incorrectly scored 0 points. A comparison of traditional method Quiz 12 A, and special method Quiz 12 B, was calculated for each question. Comparison results were tabulated for each of the four questions; 1 point if the student's score improved from traditional method Quiz 12 A, to special method Quiz 12 B, 0 points if there was no change in a student's score from traditional method Quiz 12 A, to special method Quiz 12 B, and -1 points if a student's score decreased from traditional method Quiz 12 A, to special method Quiz 12 B.

The number of students that experienced an increase in score was tabulated per quiz question. The number of students that experienced a decrease in score was tabulated per quiz question. Logistic regression was used to calculate P-values for each quiz question as well as for the overall Quiz 12 results. The results are shown in Table 4. A P-value less than 0.05 indicates our results are unlikely due to chance alone, suggesting there is evidence that the special method is improving students' success on some of the quiz questions, specifically the questions 1, 3, 7 and the quiz overall.

	Question 1	Question 3	Question 7	Quiz 12 Overall	Question 2	Question 4	Question 5	Question 6
Number of students with increased score	7	10	10	19	8	8	5	6
Number of students with decreased score	0	3	1	7	3	4	1	2
Total number of students with increased or decreased score	7	13	11	26	11	12	6	8
P-value	0.007812	0.046143	0.005859	0.01448	0.113281	0.193848	0.109375	0.144531

Table 4: P-values for Quiz 12 Exponential and Logarithmic Equations

Note: Significant P-values (< .05) are shaded

All questions on Quiz 12 were computational questions, the type of questions on which students typically make common mistakes. Question 1, the change of base question, change  $\log_4 16$  to base 16, had no student decrease their scores, yet seven

students improved their scores on the special method Quiz 12 B. The P-value result for question 1 shown in table 4 suggests there is evidence that the special method is improving students' success on question 1.

Out of all the questions on this quiz, question 3, express as a single logarithm and simplify if possible  $\log_2 16 - \log_2 8$ , had the largest quantity of common error answers on the traditional method Quiz 12 A. This changed on special method Quiz 12 B, as not one student made a common error answering this question. The P-value result for question 3 shown in table 4 suggests there is evidence that the special method is improving students' success on question 3.

Question 7, express  $\log_y 9^4$  as a product had the lowest P-value for this quiz. Only one student decreased a score on this question, while ten students increased their scores. The P-value result for question 7 shown in table 4 suggests there is evidence that the special method is improving students' success on question 7.

The P-values suggest students have improved understanding of logarithmic properties including the Quotient Property and the Power Property as well as Change of Base Property when the material is presented using the special method technique.

## 4.4 Comparison of means

Table 5 compares the means of each quiz and shows the total percent increase for each quiz.

	Quiz 10	Quiz 11	Quiz 12	Quiz 10, 11 & 12 Average
Traditional Method Average	50%	64%	57%	57%
Special Method Average	75%	79%	74%	76%
Percent Increase	50%	23%	30%	33%

Table 5: Comparison of quiz means and percent increase

## 4.5 Bias

In order to learn what correlations or bias might exist with student performance, regression analysis was performed with quiz scores and student data.

Prior to the experiment, data was collected on each student via a questionnaire (Appendix M). The one page questionnaire was distributed at the end of class a week before the study began. Students were offered one point of extra credit if the students brought back the questionnaire filled out by the next class session. Within two class sessions all of the questionnaires were returned.

Regression analysis was performed comparing each of the three quiz outcomes (the change from Quiz A to Quiz B, i.e., total improved, declined or stagnant) to the responses to these categories:

- Male or female
- Math classes taken in high school.
- Overall high school GPA
- SAT score
- Subjects most enjoyed in high school
- Type of learner
- Experience a good math teacher in elementary school
- Experience a good math teacher in middle school
- Experience a good math teacher in high school
- Repeating the algebra class previously taken at CLU
- Last time student took a math class prior to this class
- First member of your immediate family to attend college
- Student major
- Dream job

Specific regression analysis was performed on each individual category (relative to the quiz outcomes). Some categories had additional regression analysis performed by combining the subcategories of a category. For example, the “math classes taken in high school” category offered seventeen math classes from which to choose. Regression

analysis was performed on each individual subcategory within “math classes taken in high school.” Then regression analysis was performed a second time using the category as a set of grouped factors. For example, one group consisted of any type of algebra class, any type of geometry class formed another group, and analogously for statistics classes and calculus classes.

Similarly, “subjects most enjoyed in high school” offered eleven subjects from which to choose. Regression analysis was performed on each individual subcategory within “subjects most enjoyed in high school”. Then regression analysis was performed a second time using the category as a set of grouped factors. For example, one group consisted of any type of elective class that was science oriented, regular science or technology/computer science class.

In addition to regression analysis performed on each individual category of “do you think you had a good math teacher in elementary school, middle school and high school”, it was also performed a second time using all of the “had a good math teacher in elementary, middle or high school” categories.

Student majors were grouped into subcategories such as medical, entertainment, teaching, etc. and regression analysis was performed on each subcategory. Similarly, dream jobs were grouped into subcategories and regression analysis was performed.

Yet, with all of the regression analysis performed, individual categories as well as subcategory combination in a category, there were no more significant P-values than would be expected by chance alone.

One last type of regression analysis was performed with the two quiz outcomes from Quiz 11 and 12 (the delta from Quiz A to Quiz B, total improvement, decline or

stagnant) and student memorizing and receiving extra credit for writing the poem on the quiz. The P-value for Quiz 11 was 0.0401, for Quiz 12 it was 0.0746. The P-value result for Quiz 11, 0.0401, (a significant P-value  $< 0.05$ ) suggests there is evidence that the special method of memorizing a poem is improving students' success on Quiz 11. The P-value result for Quiz 12, 0.0746, while small, does not provide a significant P-value  $< 0.05$ . Then the regression analysis was performed with the two quiz outcomes from Quiz 11 and 12 (the delta from Quiz A to Quiz B, total improvement, decline or stagnant) and students attempting to receive extra credit for writing the poem on the quiz. Attempting to receive extra credit means the student began to write out the poem and it was incomplete, finished but incorrect or correctly written. The P-value for Quiz 11 was 0.0939, for Quiz 12 it was 0.6064, significantly different than the results of students actual memorizing the poem. The P-values suggest there is evidence that memorizing the Quiz 11 poem is improving students' success on Quiz 11.

Were there other factors that may have swayed the results? There may have been a selection bias. The classes were taught during mid day, so only mid day student results are collected. Additionally students taking algebra the second semester of the school year may have presented bias (are first semester students more or less motivated?). Additional studies conducted during different times of the day and during fall and spring semester could further investigate this possibility.

## 4.6 Student Feedback

After the last quiz was collected, the students offered up some verbal feedback on the process they experienced learning logarithms.

When asked, “Did you find the special method lectures, the second lectures of the same material helpful?”

The answers given:

- It was another way to understand the concepts.
- It was very helpful.
- The logarithms poem (second lecture) was the most helpful piece of the material.
- One student thought presenting material with two methods gave too much information and found it to be confusing.

When asked, “Did you recall any part of the poem to help you answer questions?”

The responses were enthusiastic yeses, although one student volunteered, “Due to the extra credit points, memorizing the poem was a priority for the extra credit, but thinking or understanding what the poem meant was not a priority for me.”

Students also wrote the responses to the question on the back of the quiz; Did learning the poems better help you to understand the logarithms concepts? Nineteen of the students said they did. One student wrote, Yea, didn’t think it would but it did. Another student wrote it made it easier to remember the rules. Another student wrote yes, greatly.

Out of curiosity, a question the author would like to know the answer to is “How many extra credit points would it take to entice you to memorize each poem?”

## 4.7 Summary of the data

Students performed better after the special method lecture on all exponential and logarithmic conceptual and computational questions. The P-values suggest students have improved understanding of exponents and logarithms when the material is presented using the special method technique using poems. Further, the P-values suggest

memorizing the poem from the Introduction to Logarithms special method lecture increased the students' success on Quiz 11.

Additionally, an unexpected result surfaced; common mistakes made with logarithms, as mentioned in Table 1, improved after the special method lecture. They were nearly non-existent.

# Chapter 5

## Conclusion and Recommendations

### 5.1 Validity of results

This thesis investigated a special method teaching technique using poems to help students better understand exponents and logarithms. The data shows the students' performance improved after the special method lecture using the poems. The results from the special method teaching technique are very encouraging. The results suggest that less emphasis on drill and manipulation and increased focus on the meaning of abstract concepts, in this case via poems, might improve student performance.

There are areas to acknowledge regarding the validity of the results. The most obvious: the author of this thesis served as the lecturer for the students in the study and the examiner of the results. This concern can be addressed by duplicating the research with a different instructor.

The order of the study allowed the traditional method to be taught first followed by the special method. This may have filled gaps of knowledge that were previously unclear to students and consequently aided the special methods technique results. Additionally the students were learning the same material twice which may have also aided the results. In order to address this concern, it would be interesting to duplicate this study in an intervention.

Quiz A and Quiz B were identical quizzes with the exception of the quiz name, either Quiz A or Quiz B. Students may have recalled the questions and investigated them prior to the next class overestimating the success of the special method teaching technique.

However, if students were to recall the questions, scores on Quiz B would have been much higher across all of the questions answered. An example of this is question 1 on Quiz 12. It is a change of base question, change  $\log_4 16$  to base 16. Being the first question on the quiz, students could have easily made a mental note that it was a change of base question for later investigation, even if the student was unable to recall the question asked to change to base 16. This question had an improvement of only seven student scores (out of thirty possible improvement scores). Further, students did not know they were going to be receiving the same quiz twice. Additionally, students were not given any answers to the quizzes until the last quiz of the study had been collected. Lastly at the conclusion of the last quiz, students wrote on the back of the quiz the number of different quizzes they believed they received out of six quizzes. Of the thirty-five students that were present that day, twelve students, more than one third of the class, wrote a number different than three. At least one third of the class did not realize Quiz A and B were the same.

The concern of identical quizzes can be partially addressed by giving different quizzes for each topic, instead of giving the same quiz twice. Additionally this concern can be eliminated in the future by providing an intervention study.

When the special method lecture was given following any Quiz A of the same material, some students may have thought about the quiz questions they did not understand and looked for those answers as the special method lecture material was being presented. This concern can be eliminated in the future by providing an intervention study.

Homework was assigned at the conclusion of the traditional method lecture while extra credit was offered (memorizing poems) after the special method lecture. Was it possible the timing of homework completion affected student performance in-between Quiz A and Quiz B? This is very unlikely. Regardless of when the assigned homework was completed (the day it was assigned, prior to Quiz A, after Quiz A was given but before Quiz B was given, or if it was not completed at all) it was computational in nature and would not have helped students to better understand the conceptual questions on the quizzes.

## 5.2 Additional investigation

This thesis has shown evidence worthy of further investigation. One suggestion is to engage in an experimental intervention. Two groups of students will be taught the material. Half of the students will be taught with the traditional method and the other half will be taught with the special method technique. In this manner, both groups can be tested using the same quizzes for comparison without enhancing the outcome due to receiving dual lectures on the same topic. Implementation with more students than were used in this study is suggested.. Additionally, either the poems will need additional computational supplementation, as the poem addresses the concepts of exponents and logarithms, without examples, or the all quizzes should be modified to reflect conceptual

questions only. If an experimental intervention is not possible, the next suggestion is to duplicate this study with a larger population of students. It is recommended instead of giving identical quizzes twice, give different quizzes for each topic.

It also might be interesting to teach the methods in reverse order, teaching the special method technique first followed by the traditional method. In this case, it is suggested that all quizzes should reflect conceptual questions only.

### 5.3 Conclusion and Recommendations

The special method technique of using poems may change how exponential and logarithmic functions are taught. Based on the evidence shown in this thesis, specifically the improvement of students' performance on every quiz question in this study, the improved understanding of exponential and logarithmic concepts and the common mistakes made with logarithms improvement by students, the author recommends using the special method technique. Until further research has been conducted, it is recommended as a supplement to any math course where students have a challenging time understanding abstract concepts including high school algebra classes and remedial college math classes.

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# Appendices

## Section 10.1 Exponential Functions

### Learning Objectives:

1. Evaluate exponential functions
2. Graph exponential functions

### Learning Objective 1 Evaluate exponential functions

#### *Definition of an Exponential Function*

The **exponential function** with base  $b$  is defined as

$$f(x) = b^x$$

Where  $b > 0, b \neq 1$ , and  $x$  is any real number.

### **Examples:** Evaluate

a)  $f(x) = 2^x$  at  $x = 3$  and  $x = -2$

b)  $f(x) = \left(\frac{2}{3}\right)^x$  at  $x = 3$  and  $x = -2$

c)  $f(x) = 2^{2x+1}$  at  $x = 3$  and  $x = -2$

d)  $f(x) = 2^{3x-1}$  at  $x = 1$  and  $x = -1$

e)  $f(x) = (\sqrt{5})^x$  at  $x = 4$ ,  $x = -2.1$  and  $x = \pi$

***Natural Exponential Function***

The function defined by

$$f(x) = e^x$$

is called the **natural exponential function**.

**Examples:** Evaluate

a)  $f(x) = e^x$  at  $x = 2$ ,  $x = -3$  and  $x = \pi$



***One-to-One Functions***

A graph of a function is the graph of a 1-1 function if it satisfies both the vertical line and horizontal line tests.

***Vertical Line Test***

If every vertical line intersects a graph at most once, then the graph is the graph of a function.

***Horizontal Line Test***

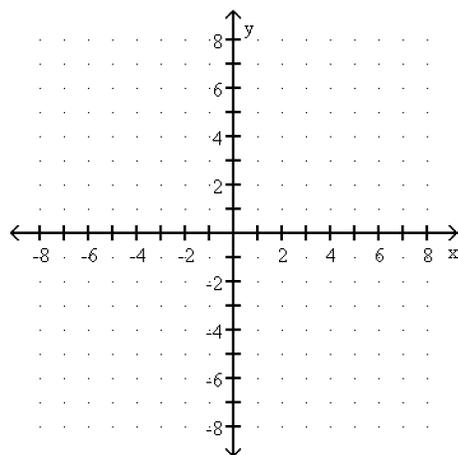
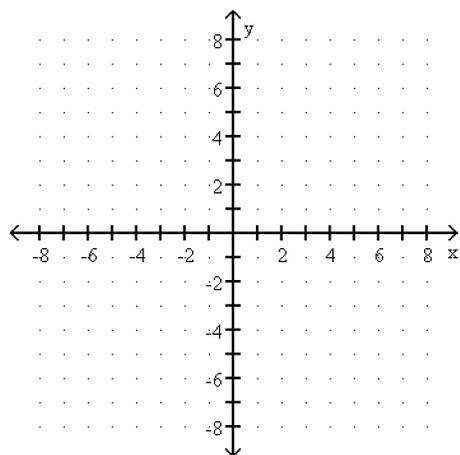
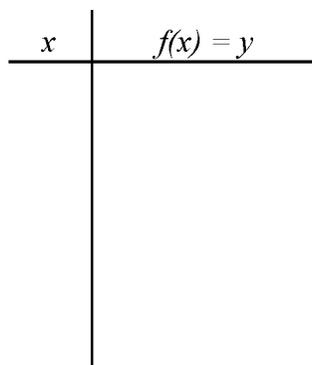
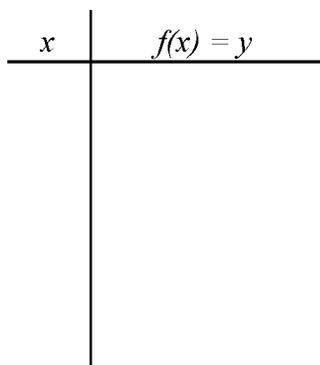
**A graph of a function** is the graph of a 1-1 function if any horizontal line intersects the graph at no more than one point.

***Exponential Functions are 1-1***

The exponential function defined by  $f(x) = b^x$ ,  $b > 0$ ,  $b \neq 1$ , is a 1-1 function.

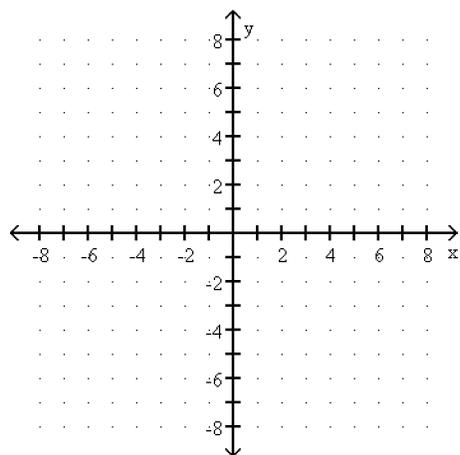
c)  $g(x) = \left(\frac{1}{2}\right)^x$

d)  $f(x) = 2^{-x}$



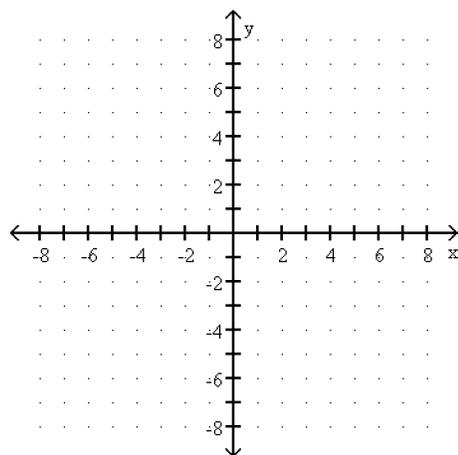
e)  $f(x) = 3^{x-1}$

$x$	$f(x) = y$
-----	------------



f)  $f(x) = \left(\frac{1}{3}\right)^x$

$x$	$f(x) = y$
-----	------------





## Section 10.2 Introduction to Logarithms

### Learning Objectives:

1. Write equivalent exponential and logarithmic equations
2. Properties of logarithms

### Learning Objective 1 Write equivalent exponential and logarithmic equations

#### *Definition of a Logarithm*

If  $x > 0$  and  
 $b$  is a positive constant not equal to 1, then

$$y = \log_b x$$

is equivalent to

$$b^y = x$$

### Examples:

Exponential Form

$$2^4 = 16$$

$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$10^{-1} = 0.1$$

$$b^a = x$$

Logarithmic Form

### Examples:

a) Write  $3^{-4} = \frac{1}{81}$  in logarithmic form.

b) Write  $4^5 = 1024$  in logarithmic form.

**Examples:**

a) Write  $\log_{10} 0.0001 = -4$  in exponential form.

b) Write  $\log_7 343 = 3$  in exponential form.

***Equality of Exponents Property***

For  $b > 0, b \neq 1$ , if  $b^u = b^v$ ,

then  $u = v$

**Examples:**

a) Evaluate:  $\log_4 64$

b) Evaluate:  $\log_3 \left( \frac{1}{9} \right)$

c) Solve  $\log_4 x = -2$  for  $x$ .

***Definition of Antilogarithm***

If  $\log_b M = N$ , the antilogarithm base  $b$   
of  $N$  is  $M$ .

In exponential form,  $M = b^N$

**Examples:**

a) Solve  $\log_2 x = -4$  for  $x$ .

b) Solve  $\log_6 x = 2$  for  $x$ .

**Common Logarithms**

Logarithms with base 10  
Omit the base  
 $\log_{10} x$   
 $\log x$

**Natural Logarithms**

Logarithms with base  $e$   
 $\ln x$   
We say “ $l$ ” “ $n$ ”  $x$

**Examples:**

a) Solve  $\log x = 1.5$  for  $x$ .

b) Solve  $\ln x = -1$  for  $x$ .

**Learning Objective 2 Properties of logarithms**

Exponential Form

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

Logarithmic Form

$$\log_2 1 = 0$$

$$\log_2 2 = 1$$

$$\log_2 4 = 2$$

$$\log_2 4 + \log_2 8 = 2 + 3 = 5$$

$$\log_2 32 = 5$$

$$\log_2 4 + \log_2 8 = \log_2 32$$

$$\log_2 32 = \log_2 (4 \times 8) = \log_2 4 + \log_2 8$$

***Product Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b (xy) = \log_b x + \log_b y$$

***Quotient Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

***Power Property of Logarithms***

For any positive real numbers  $x$ , and  $b$ ,  
 $b \neq 1$ , for any real number  $r$ ,

$$\log_b x^r = r \log_b x$$

**Examples:** Write the logarithm in expanded form

a)  $\log_b \frac{x^2}{y}$

b)  $\ln y^{\frac{1}{3}} z^3$

c)  $\log_8 \sqrt[3]{xy^2}$

d)  $\log_8 \sqrt{x^3 y}$

**Examples:** Express as a single logarithm with a coefficient of 1.

a)  $2\log_b x - 3\log_b y - \log_b z$

b)  $\frac{1}{3}(\log_4 x - 2\log_4 y + \log_4 z)$

c)  $\frac{1}{2}(2\ln x - 5\ln y)$

d)  $\frac{1}{2}(\log_3 x - 3\log_3 y + \log_3 z)$

***Properties of Logarithms***

**Logarithmic Property of One**

For any positive real number  $b, b \neq 1$ ,  $\log_b 1 = 0$

**Inverse Property of Logarithms**

For any positive real numbers  $x$  and  $b, b \neq 1$ ,

$$\log_b b^x = x$$

**1-1 Property of Logarithms**

For any positive real numbers  $x, y$  and  $b, b \neq 1$ , if

$$\log_b x = \log_b y,$$

then  $x = y$

**Examples:** Simplify

a)  $8\log_4 4$

b)  $\log_8 1$

c)  $\log_{16} 1$

d)  $12\log_3 3$

***Change-of-Base Formula***

$$\log_a N = \frac{\log_b N}{\log_b a}$$

**Examples:**

a) Evaluate  $\log_7 32$

b) Rewrite  $f(x) = -3\log_7(2x - 5)$   
in terms of natural logarithms.

## Section 10.4 Exponential and Logarithmic Equations

### Learning Objectives:

1. Solve exponential equations
2. Solve logarithmic equations

### Learning Objective 1 Solve exponential equations

An **exponential equation** is one in which a variable occurs in the exponent.

***Equality of Exponents Property***

For  $b > 0, b \neq 1$ , if  $b^u = b^v$ ,  
then  $u = v$

If it is easy to express the terms with like bases, try the technique shown in these examples.

**Examples:** Solve and check:

a)  $10^{3x+5} = 10^{x-3}$

b)  $9^{x+1} = 27^{x-1}$

If it is NOT easy to express the terms with like bases, try the technique shown in these examples using logarithms.

**Examples:** Solve for  $x$ :

a)  $4^{3x} = 25$

b)  $(1.06)^x = 1.5$

c)  $4^x = 7$

d)  $3^{2x} = 4$

**Learning Objective 2** Solve logarithmic equations***Product Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b(xy) = \log_b x + \log_b y$$

***Quotient Property of Logarithms***

For any positive real numbers  $x$ ,  $y$ , and  $b$ ,  
 $b \neq 1$ ,

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

***Power Property of Logarithms***

For any positive real numbers  $x$ , and  $b$ ,  
 $b \neq 1$ , for any real number  $r$ ,

$$\log_b x^r = r \log_b x$$

***1-1 Property of Logarithms***

For any positive real numbers  $x, y$  and  $b, b \neq 1$ ,  
if  $\log_b x = \log_b y$ ,  
then  $x = y$

**Examples:** Solve.

a)  $\log_9 x + \log_9(x - 8) = 1$

Get one expression on each side, use the Product Property

**Write the equation in exponential form.**  
**(Use this technique when one side of the equation has a log and the other side of the equation has a constant.)**

Solve for  $x$ .

b)  $\log_4(x^2 - 3x) = 1$

Get one expression on each side, done for us.

Write the equation in exponential form.  
(Use this technique when one side of the equation has a log and the other side of the equation has a constant.)

Solve for  $x$ .

c)  $\log_3 x + \log_3(x + 3) = \log_3 4$

Get one expression on each side, use the Product Property

**Use the 1-1 Property of Exponents.**  
**(Use this technique when BOTH sides of the equation have a log.)**

Solve for  $x$ .

d)  $\log_3(2x-1) = 2$

Get one expression on each side, done for us.

Write the equation in exponential form.  
(Notice one side of the equation has a log and the other side of the equation has a constant.)

Solve for  $x$ .

e)  $\log_2 x - \log_2(x-1) = \log_2 2$

Get one expression on each side, use the Quotient Property

Use the 1-1 Property of Logarithms.  
(Notice BOTH sides of the equation have a log.)

Solve for  $x$ .

## exponentials

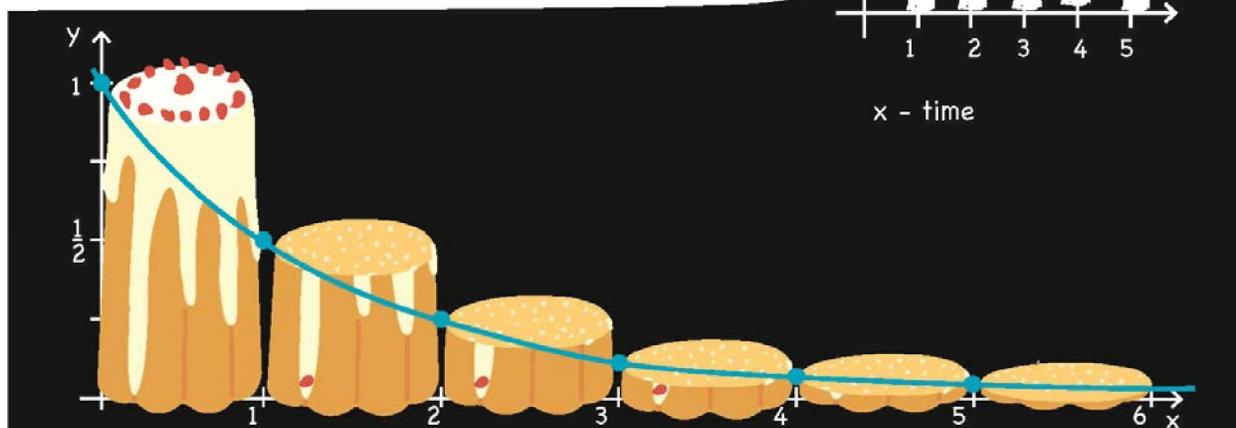
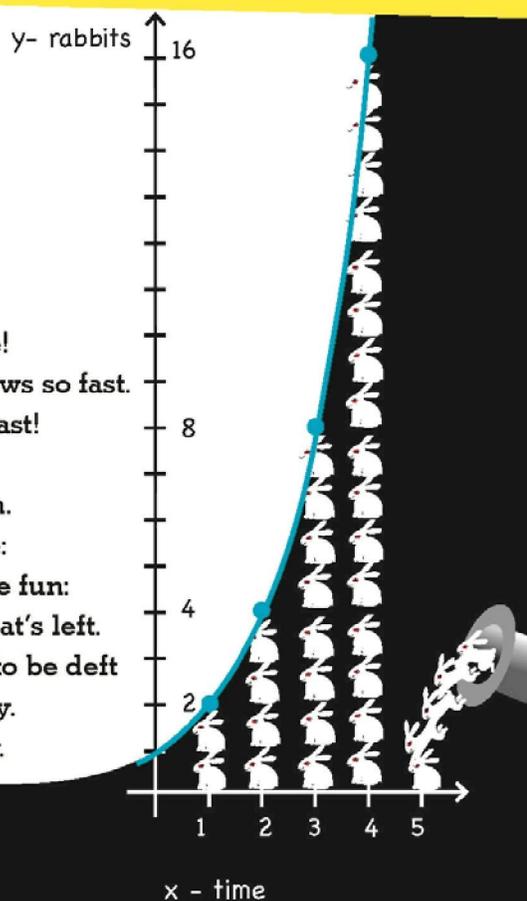
$$y = a^x \quad a > 0 \quad a = 1 \quad a^0 = 1 \quad a^1 = a$$

$$a^x > 0 \text{ always}$$

$$a^x \text{ increasing for } a > 1$$

$$a^x \text{ decreasing for } 0 < a < 1$$

Study rabbits – monitor their population size.  
 It doubles each season. The doubling surprise!  
 Two, four, eight, then sixteen... The graph grows so fast.  
 The exponential growth function exposed at last!  
 Bacteria can grow faster in a large Petri dish,  
 From 10 to 100, to 1000... Trillions, if you wish.  
 Multiplying by a factor that is bigger than one:  
 Is about growth. While, decreasing can also be fun:  
 Eat just half of your cake. Then just half of what's left.  
 And one half again, and again. You will have to be deft  
 As leftovers get smaller, almost none to display.  
 And this graph is exposing exponential decay.



### AI's PUZZLES

- Let's say that a sick person can spread his disease to two other people per day (on average). Estimate how long it will take to get 100 people sick? 1000? 1,000,000?
- Can you fold a newspaper in half 100 times?
- If you keep eating half of what is left of your a cake (say every hour), will you completely finish the cake? How long it will take?
- What is a better deal: to sell 10 rabbits for \$100 each, or to sell the first for \$1, the second for \$2, the third for \$4, ... and so on doubling the prize each time?

# Logarithms

$$\text{Log}_a x = y$$

$$\log_a x = y \quad a^y = x$$

$$a, x > 0$$

$$a \neq 1 \text{ as } 1^y = 1$$

$$\log_a 1 = 0 \quad a^0 = 1$$

$$\log_a a = 1 \quad a^1 = a$$

$$\log_a a^2 = 2 \quad a^2 = a^2$$

always

Logarithms, Logarithms, numbers full of magic!

Always real, often long, sometimes even tragic.

If you try and try again, this secret comes out next:

Find the power  $y$  for the base  $a$  to make it equal  $x$ .

Choose  $a$  and  $x$  both positive – to avoid confusion,

And keep away from the base one, its powers are an illusion.

If  $x$  is one, to your surprise enchantment quickly fades

As  $y$  is zero, always zero, no matter what's the base.

When  $x$  and  $a$  are both the same, then  $y$  is simply one.

That's a trivial riddle, a trivial case, no solving full of fun.

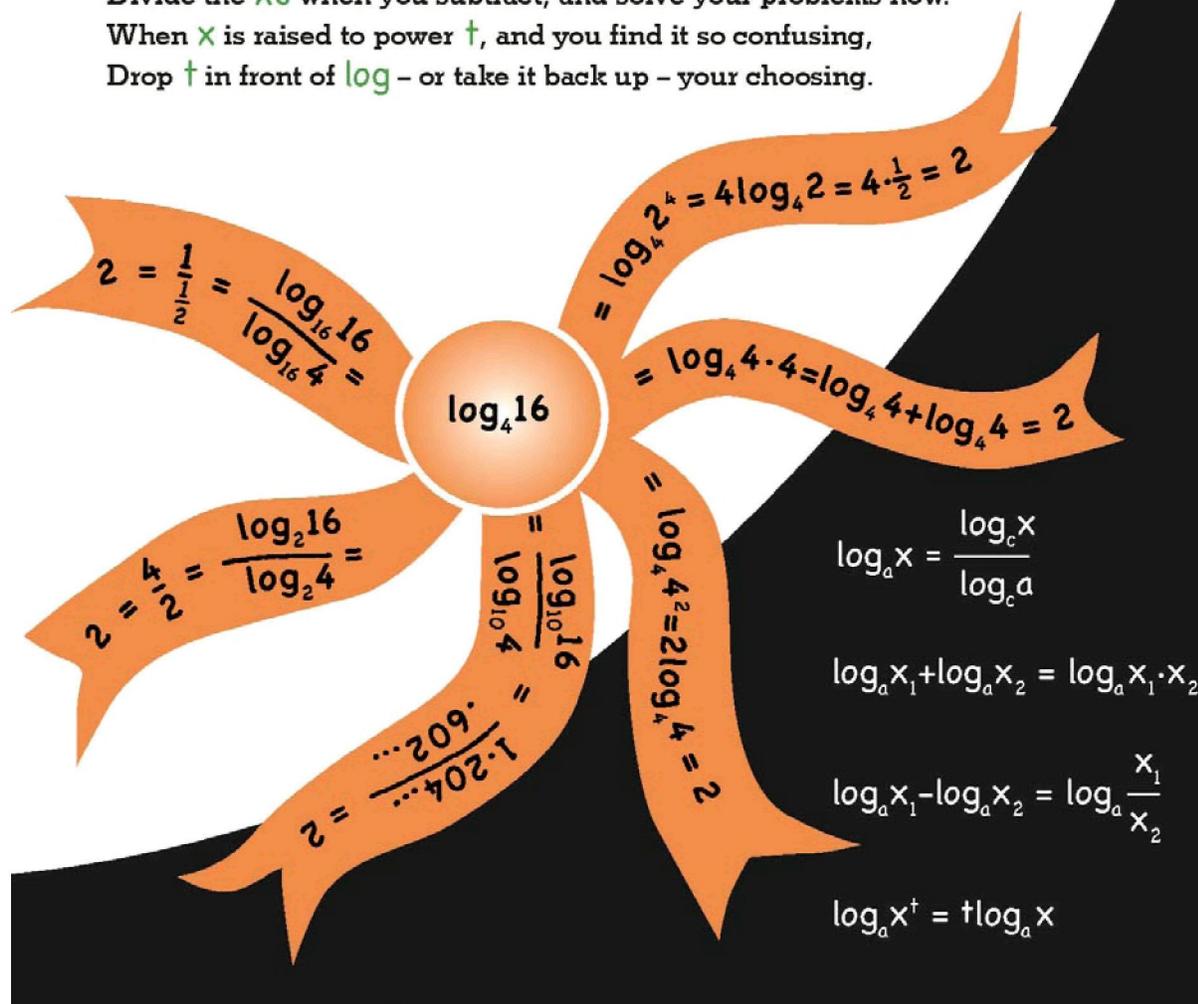


## AI's PUZZLES

- Write an equation describing your \$100 doubling in value every year. Approximately how many years will it take to get \$1000? \$100,000? \$1,000,000?
- The oil spill clean up crew gets 10% of the oil out of the beach. If they start today, how long do they need to clean, till only 10% of the oil spill is left on the beach? 1%? Can the beach be ever completely clean?

## logarithms

When different logs are in the game and all this makes you scared  
 Just change the bases to what you like, and then you can compare.  
 Two or ten are good for base. And then the number  $e$  some say.  
 Now take the ratio in the chosen base:  $\log x$  over  $\log a$ .  
 And now start adding them together, the  $x$ 's multiply,  
 Divide the  $x$ 's when you subtract, and solve your problems now.  
 When  $x$  is raised to power  $t$ , and you find it so confusing,  
 Drop  $t$  in front of  $\log$  - or take it back up - your choosing.

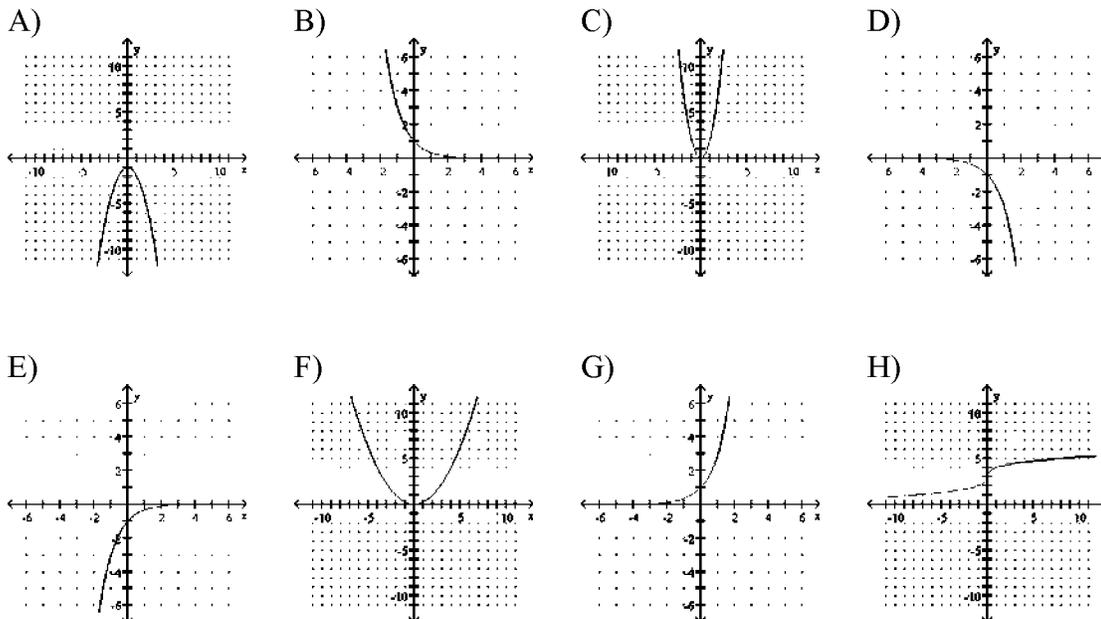


### AI's PUZZLES

- Approximate number 10 as a log with bases 10, 2, 5, 100,  $\frac{1}{10}$
- Can a negative number  $y$  be written as  $y = \log_a x$  for some numbers  $a$  and  $x$ ?
- Calculate  $\log_2 [\log_{10} (\log_2 2^{100})^2]^2$ .

## Quiz 10 A

Use the following graphs to answer questions 1, 2 and 3 below.



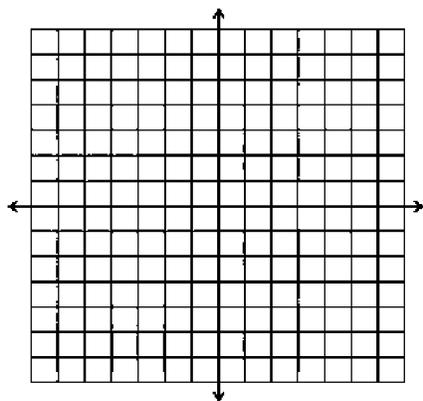
- 1) Choose the graph shape that represents positive exponential growth function with base  $> 1$ .
  
- 2) Choose the graph shape that represents positive exponential decay function with base  $0 < b < 1$ .
  
- 3) A person tries a new exercise plan. The exercise plan causes the person to lose weight. The first week the person loses 1 pound, the next week  $\frac{1}{2}$  pound, the next week  $\frac{1}{4}$  pound, the next week  $\frac{1}{8}$  inch pound, and the next week  $\frac{1}{16}$  pound. The weight loss continues with the same pattern. Choose the graph shape that best represents the person's weight loss.
  
- 4) In an exponential function, why do we say the base can not be 1?

Extra Credit Questions on the back  $\Rightarrow$

## Extra Credit

A person tries a new exercise plan. The exercise plan causes the person to lose weight. The first week the person loses 1 pound, the next week  $\frac{1}{2}$  pound, the next week  $\frac{1}{4}$  pound, the next week  $\frac{1}{8}$  inch pound, and the next week  $\frac{1}{16}$  pound. The weight loss continues with the same pattern.

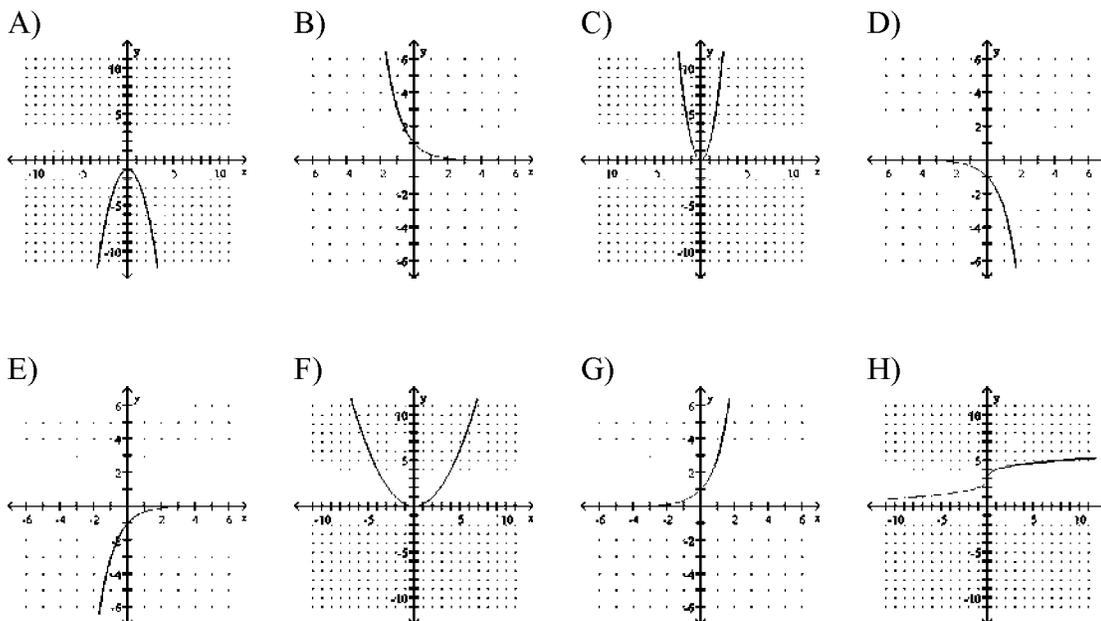
A) If the person weighed 200 pounds when the exercise plan began, graph the person's weight change.



B) Create a formula for the person's weight loss.

## Quiz 10 B

Use the following graphs to answer questions 1, 2 and 3 below.



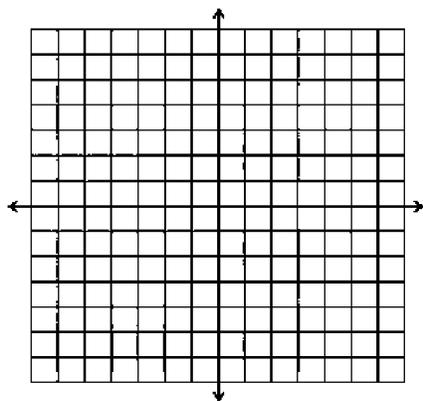
- 1) Choose the graph shape that represents positive exponential growth function with base  $> 1$ .
  
- 2) Choose the graph shape that represents positive exponential decay function with base  $0 < b < 1$ .
  
- 3) A person tries a new exercise plan. The exercise plan causes the person to lose weight. The first week the person loses 1 pound, the next week  $\frac{1}{2}$  pound, the next week  $\frac{1}{4}$  pound, the next week  $\frac{1}{8}$  inch pound, and the next week  $\frac{1}{16}$  pound. The weight loss continues with the same pattern. Choose the graph shape that best represents the person's weight loss.
  
- 4) In an exponential function, why do we say the base can not be 1?

Extra Credit Questions on the back  $\Rightarrow$

## Extra Credit

A person tries a new exercise plan. The exercise plan causes the person to lose weight. The first week the person loses 1 pound, the next week  $\frac{1}{2}$  pound, the next week  $\frac{1}{4}$  pound, the next week  $\frac{1}{8}$  inch pound, and the next week  $\frac{1}{16}$  pound. The weight loss continues with the same pattern.

A) If the person weighed 200 pounds when the exercise plan began, graph the person's weight change.



B) Create a formula for the person's weight loss.

## Quiz 11 A

- 1) Write  $\log_b x = y$  as an exponent.
- 2) Regarding  $\log_b x = y$  describe the types of numbers  $b$  and  $x$  can be.
- 3) Regarding  $\log_b x = y$  describe what  $b$  can not be and why it is not allowed.
- 4) Regarding  $\log_b x = y$  describe what happens if  $x$  is one.
- 5) Regarding  $\log_b x = y$  when is  $y$  zero?
- 6) Regarding  $\log_b x = y$  describe what happens when  $x$  and  $b$  are both the same?
- 7) Write  $\log_4 1 = 0$  as an exponent.
- 8) Write  $\log_5 5 = 1$  as an exponent.
- 9) Write  $\log_9 3^2 = 1$  as an exponent.
- 10) Write  $\log_3 3^2 = 2$  as an exponent.

## Quiz 11 B

- 1) Write  $\log_b x = y$  as an exponent.
- 2) Regarding  $\log_b x = y$  describe the types of numbers  $b$  and  $x$  can be.
- 3) Regarding  $\log_b x = y$  describe what  $b$  can not be and why it is not allowed.
- 4) Regarding  $\log_b x = y$  describe what happens if  $x$  is one.
- 5) Regarding  $\log_b x = y$  when is  $y$  zero?
- 6) Regarding  $\log_b x = y$  describe what happens when  $x$  and  $b$  are both the same?
- 7) Write  $\log_4 1 = 0$  as an exponent.
- 8) Write  $\log_5 5 = 1$  as an exponent.
- 9) Write  $\log_9 3^2 = 1$  as an exponent.
- 10) Write  $\log_3 3^2 = 2$  as an exponent.





Name \_\_\_\_\_

1) What math classes did you take in high school? *(Please check all that apply, and list the grade you earned next to the checked box.)*

- |  |   |
|--|---|
| Practical Mathematics <input type="checkbox"/> ____  | Statistics <input type="checkbox"/> ____                                |
| Beginning Algebra <input type="checkbox"/> ____  | Statistics Advanced Placement <input type="checkbox"/> ____             |
| Intermediate Algebra <input type="checkbox"/> ____   | Descriptive Statistics Advanced Placement <input type="checkbox"/> ____ |
| Algebra 1 <input type="checkbox"/> ____  | College Algebra <input type="checkbox"/> ____                           |
| Algebra 2 <input type="checkbox"/> ____  | Pre-Calculus <input type="checkbox"/> ____                              |
| Algebra 2 - Honors <input type="checkbox"/> ____   | Pre-Calculus Honors <input type="checkbox"/> ____                       |
| Geometry Plane/Solid <input type="checkbox"/> ____   | Calculus AB Advanced Placement <input type="checkbox"/> ____            |
| Geometry - Honors <input type="checkbox"/> ____  | Calculus BC - Advanced Placement <input type="checkbox"/> ____          |
| Others not listed above <i>(please list the class(es))</i> _____ <input type="checkbox"/> ____ |   |

2) What was your overall high school GPA? *(Grade Point Average out of 4)* \_\_\_\_\_

3) What was your score on the following tests? *(Please list all you have taken.)*

SAT \_\_\_\_\_ ACT \_\_\_\_\_ TOEFL \_\_\_\_\_

4) What subjects did you like most in high school? *(Please check all that apply.)*

- English  Mathematics  Language  Social Studies  History  Science   
 Business  Art  Technology/Computer Science  Physical Education   
 Electives  *(Please list all you most enjoyed.)* \_\_\_\_\_

5) What kind of learner are you? *(Please check all that apply.)*

- |  |   |
|--|---|
| Learning by hearing (auditory) <input type="checkbox"/>  | Learning by doing (tactile) <input type="checkbox"/>            |
| Learning by seeing (visual) <input type="checkbox"/>   | Learning by poems, jokes, or mnemonics <input type="checkbox"/> |
| Learning in groups <input type="checkbox"/>  | Learning using a computer <input type="checkbox"/>              |
| Learning independently (by yourself) <input type="checkbox"/>  | Learning surfing the Internet <input type="checkbox"/>          |
| Learning listening to lectures <input type="checkbox"/>  | Learning through games <input type="checkbox"/>                 |
| Learning through entertainment <input type="checkbox"/> <i>(Please list the type(s) of entertainment.)</i> |   |
- 

6) Do you think you had a good math teacher in elementary school? Y \_\_\_ N \_\_\_

7) Do you think you had a good math teacher in middle school? Y \_\_\_ N \_\_\_

8) Do you think you had a good math teacher in high school? Y \_\_\_ N \_\_\_

9) Have you taken this class previously at CLU? Y \_\_\_ N \_\_\_

10) When was the last time you took a math class prior to this class?

*(Please list the semester and year.)* \_\_\_\_\_

11) Are you the first member of your immediate family to attend college? Y \_\_\_ N \_\_\_

12) What is your major? *(If you do not know yet, please write undecided.)* \_\_\_\_\_

13) What is your dream job? \_\_\_\_\_