Art Projects to Further the Understanding of Algebra

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Lorrene Black

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APPROVED FOR THE MATHEMATICS PROGRAM

5/11/11

Dr. Ivona Grzegorczyk　　　　Date

C. Wyels　　　　5/18/11

Dr. Cynthia J. Wyels　　　　Date

APPROVED FOR THE UNIVERSITY

5/18/11

Dr. Gary A. Berg　　　　Date

ABSTRACT

Art Projects to Further the Understanding of Algebra

Many students arrive in college thinking of mathematics as a set of procedures to memorize and apply. This is in direct opposition to mathematicians' conviction that mathematical learning consists of developing an understanding of mathematical concepts and the ability of using them in various contexts. This thesis explores the efficacy of using art projects to supplement the standard lecture model in Intermediate Algebra courses.

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ACKNOWLEDGMENTS

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Chapter I

INTRODUCTION

Students entering the California State University system take a basic skills mathematics test to ensure that they are adequately prepared for college level courses that assume mastery of high school mathematics. This test, known as the ELM (Entry Level Mathematics) exam, has the student demonstrate knowledge of selected topics in the areas of Algebra I, Algebra II and Elementary Geometry (Entry Level Math (ELM) Exam, 1997). Students who do not do well on this exam are placed into California State University: Channel Island's Math 94 or Math 95 classes.

This research project compares three sections of Intermediate Algebra, Math 95 that had the same content, the same instructor, and the same basic teaching schedule. However, each class did a different number of art projects hence students had different learning experiences. The class that will be referred to as Section 1, was based on traditional way of teaching without any art projects; Section 2 worked on one art project; Section 3 had art projects for every chapter covered, seven in total. The analysis will show that the art and mathematics activities improved student's learning experience.

Chapter II

WHERE

The study was done at the California State University: Channel Islands (CSUCI). The university is a student-centered, four-year public university known for it's interdisciplinary, multicultural, and international perspectives and its emphasis on experiential and service learning. Channel Islands' strong academic programs focus on arts and humanities, sciences, business, teaching credentials. Students benefit from individual attention, up-to-date technology, and classroom instruction augmented by stellar faculty research.

WHO

Based on their scores on the Elementary Level Mathematics (ELM) test, students are put into three different groups:

1. Ready for college-level math courses

2. Requires instruction in Intermediate Algebra (Math 95)

3. Requires instruction in Basic and Intermediate Algebra (Math 94 and 95).

The students that were a part of this study were enrolled in three different sections of Math 95 Intermediate Algebra in Fall '09, Spring '10 and Fall '10. Math 95 is a credit/no credit course, which means that no letter grade is assigned as a final result. There were a total of 63 students in this study over the three semesters, Section 1 had 26 students in the Fall '10 semester, Section 2 had 22 in the Spring '10 semester, and Section 3 had 14 in the Fall '09 semester. All three sections were taught by the same instructor, hence there should be consistency and no bias resulting from the instructor's way of teaching.

The catalog description for the class is: "*Prerequisite: MATH 094 or an Entry Level Mathematics Score between 35 and 49.* A review of concepts of geometry and intermediate algebra with applications. Students who earn Credit in this course satisfy the Entry Level Mathematics (ELM) requirement. This course is offered Credit/No Credit only. Credit will not apply toward the baccalaureate degree but will apply as 5 units of University Credit."

WHEN

This data collection of activities was conducted between September 2009 and December 2010 in Fall 09, Spring 10, and Fall 10 semesters. Each section met three times a week for a total of four hours. An additional one hour computer based lab is required, where students solve problems on a self paced, independent basis under the supervision of a lab instructor. Data analysis was preformed between November 2010 and January 2011.

HOW

There are seven topics of Intermediate Algebra that Math 95 focuses on: functions and graphs, systems of linear equations and problem solving, inequalities and problem solving, exponents and radicals, quadratic functions and equations, exponential and logarithmic functions, and conic sections. Each of these topics was covered in all sections using a lecture supported by a PowerPoint presentation (Appendix 1A). The students were given partial lecture notes handouts (Appendix 1B) that they used as a basis for their lecture notes. At the beginning of each session students were given time to ask questions about either the previously covered content or homework assignments. During the class time, the students were given many opportunities to ask questions, or ask for additional examples.

A list of all homework assignments that followed each topic is in Appendix 1C. On most Fridays the students were given a short quiz. Even though the quizzes were not identical due to scheduling issues every semester, they covered the same topics and were similar in presentation.

The specific activities that were being implemented to see if they would improve student learning were art projects. They were given on selected Fridays. Students were given a prompt describing the specific mathematical requirements for each art project. To enforce content that students were learning during the class, they were allowed to collaborate and discuss their projects. (See Appendix 1D for prompts.)

The results from the Midterms 1 (Appendix 2A) and Midterms 2 (Appendix 2B) and Final exams (Appendix 2C) were used for statistical analysis. For data consistency tests were very similar each semester and each included graphing questions.

DESCRIPTION OF EXPERIMENT

Three section of Intermediate Algebra were used in this study. In two section students were involved in art projects in addition to their regular lectures and course activities. Data from all three sections was collected and analyzed for this study.

1. The class that will be referred to as Section 1 was held in the Fall of 2010, which took place after the study was conducted on the classes that will be known as Section 2 and Section 3. The students were given no art projects during the course of the semester, while other instruction related parameters (such as instructor, material, lectures, testing, etc.) were similar to the other sections. Therefore Section 1 is the control group for this study.

2. The class that will be referred to as Section 2 was held in the Fall of 2009. They worked on one art project. This art project was given at the end of the semester and would only affect the results of the final exam.

3. The class that will be referred to as Section 3 was held in the Spring of 2010. This class was given an art project at the completion of every chapter. Therefore, all of the test scores would be affected by the art projects.

Each class time was structured a certain way. The following is a specific description of the teaching method and data collecting methods.

1. Description of a typical class day

   - Students receive prepared partial notes for the content to be covered.

   - Time for questions and discussion related to previously covered content or assignments.

   - A short lecture is given, which included questions and discussion of the new material.

- Specific homework problems assigned at the end of each session

2. Sessions with art activities

   - Conducted similarly, but once the lecture was completed and homework assigned, the students were given an art activity.

   - The students would work alone or in teams on the art project for the remaining time. The instructor would answer any related questions.

   Note: Some sessions included short quizzes.

3. Midterms

   - The two class sessions before the Midterm were used for review for the midterm.

   - The students were given a review sheet for the midterm, which if completed and turned in on the day of the test would give a small amount of extra credit.

   - The students could take up to two hours to complete their test.

   - Midterm results and students' performances on the tests were used for statistical analysis.

4. Final

   - The entire week before the Final Exam was used for review of the material covered.

- The students were given a review sheet for the final, if completed and turned in on the day of the test were extra credit.

- The students had up to two hours to complete the final exam.

- Final exam results and students’ performance were used for statistical analysis.

Topics used for this study include: functions and graphs, systems of linear equations and problem solving, inequalities and graphing, exponents and radicals, quadratic functions and equations, exponential and logarithmic functions, and conic sections. The teaching methods evaluated in this study were both traditional and special in the following sense:

TRADITIONAL METHOD

The lecture and the textbook-based teaching is being called the traditional method. It involves lecturing and drills of the rules and formulas that are presented in the book for the students to use for problem solving. The traditional method that was presented in this study added PowerPoint presentations, pre-prepared lecture notes for the students and whiteboard use for examples. All the homework assignments used came directly from the textbook.

SPECIAL METHOD

In this work, the special method refers to the addition of art projects that required mastery of specific math content. The projects were first discussed, a quick example was usually provided, and students could work in groups or alone to complete them. Collaboration and discussions were encouraged.

To cover specific mathematics content, direction for each project included the use of examples of specific algebraic curves such as lines or parabolas to help students in their artistic creations (Prompts are in Appendix 1D). The instructor provided additional clarification and suggestions if necessary.

Before drawings were submitted, the students were asked to check that they have all of the mathematical requirements in their art. They were also asked to provide the equations for all of the required graphs.

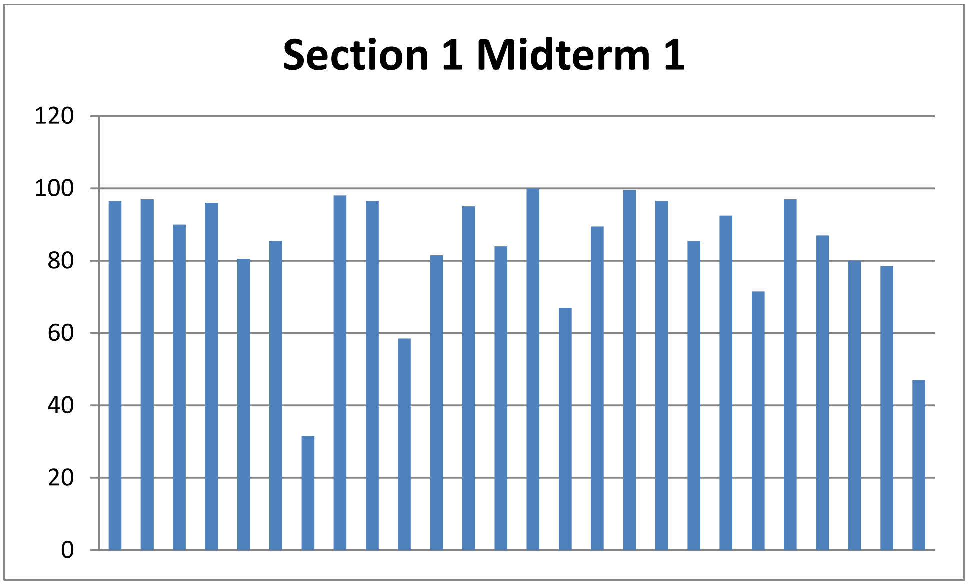
While figuring out their equations, the students were allowed to work with their teams and discuss the problems they might have been having with others. If students needed more time to complete their projects, they were able to finish their artistic activities at home.

Chapter IV

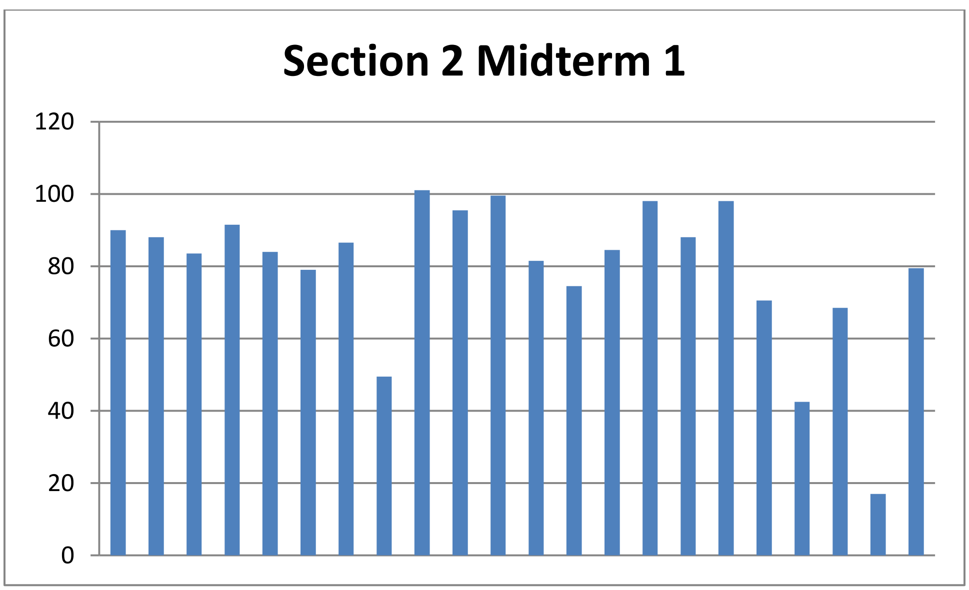
This thesis is using the null hypothesis that there will be no changes in the students' understanding of Algebra, with the alternate hypothesis that their performance will improve. The data analysis includes grades on the major tests in all three sections of the Math 95 classes. (The scores for all of the students and all of the tests are provided in Appendix 3A.) Section 1 was taught as the last course in this study and is used as the control group (26 subjects), as the student in this class did not work on art projects at all. Section 2 had one art project (22 subjects) and students in the last section in this sequence, Section 3, had 7 art projects (14 subjects).

ANALYSIS OF RESULTES OF MIDTERM 1

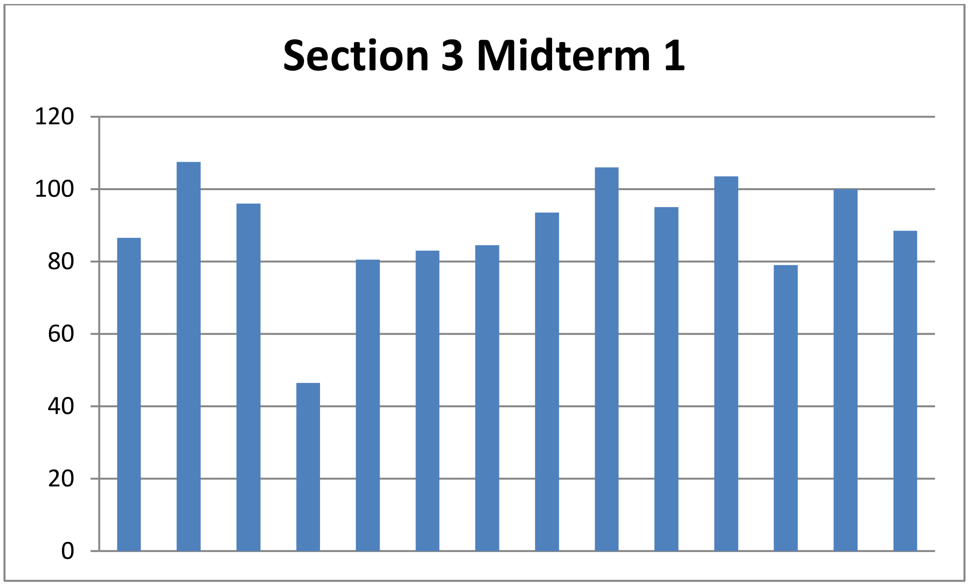
Midterm 1 was identical for all sections and tested the students on linear functions, and systems linear of equations and their graphs. Only Section 3 had a related art project at the time of the midterm. In all of the sections, the score that a student could have gotten was between zero and one hundred, with the passing score of seventy or 70%. However, students could score an additional ten points if they completed bonus problems. Each student's score is represented in the following graphs. Scores from each section are plotted for each student and the statistical parameters are calculated below each graph.



The students in Section 1 had a minimum score of 31.5 and the highest score achieved was 100. The mean value for this midterm was about 84, with the standard deviation at about 17.



The students in Section 2 had a minimum score of 17 and the highest score was 101. The mean value of the midterm was about 80, with a standard deviation of about 20.

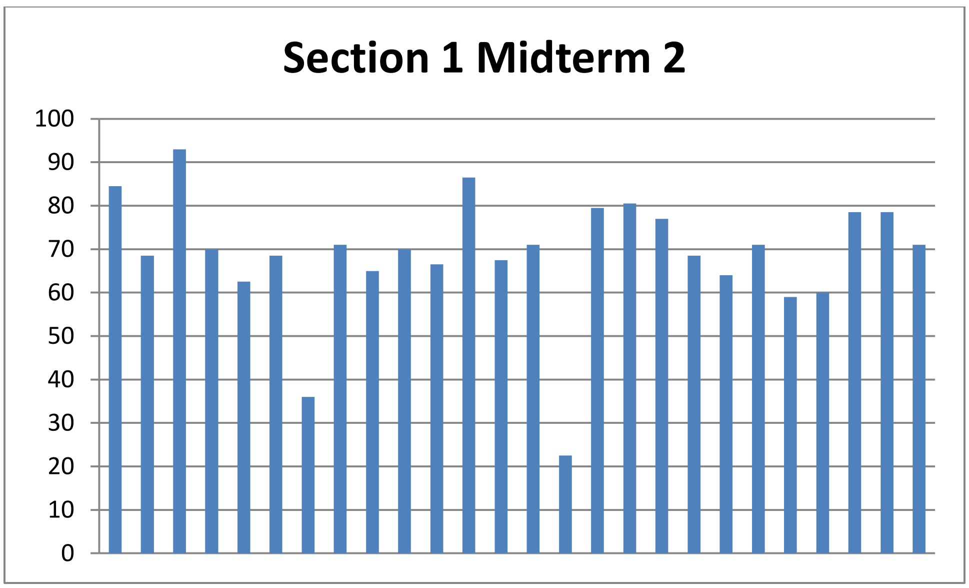


Lastly, the students in Section 3 had a minimum score of 46.5 and the highest score of 107.5. The mean value for this section was about an 89, and the standard deviation was about 15.

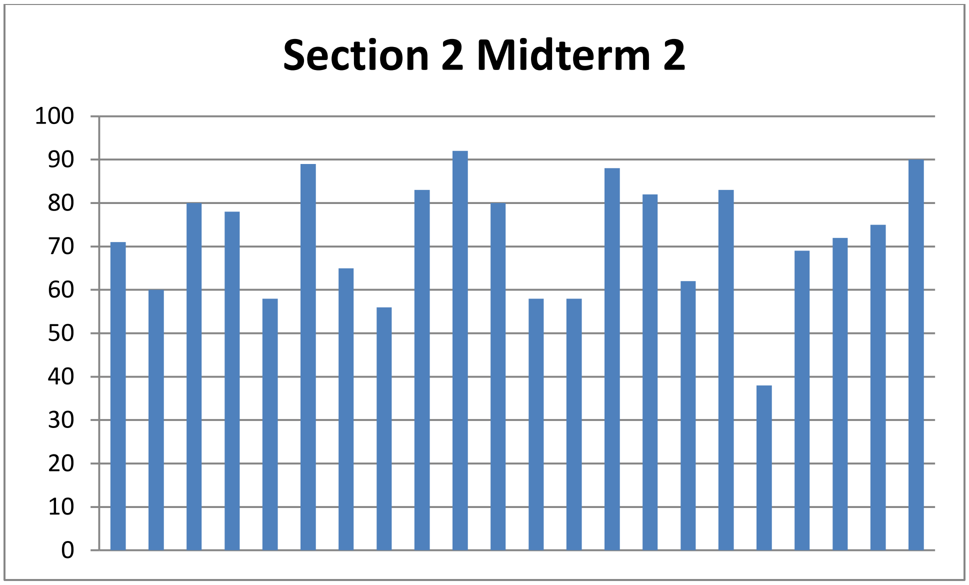
Note that the section with the three highest scores was Section 3, which had done art projects, and the sections with the three lowest scores were both Sections 1 and 2, who had not seen an art project. However at this time, the scores of all three sections are quite similar, as students in each class are comparable in terms of their abilities.

ANALYSIS OF RESULTS OF MIDTERM 2

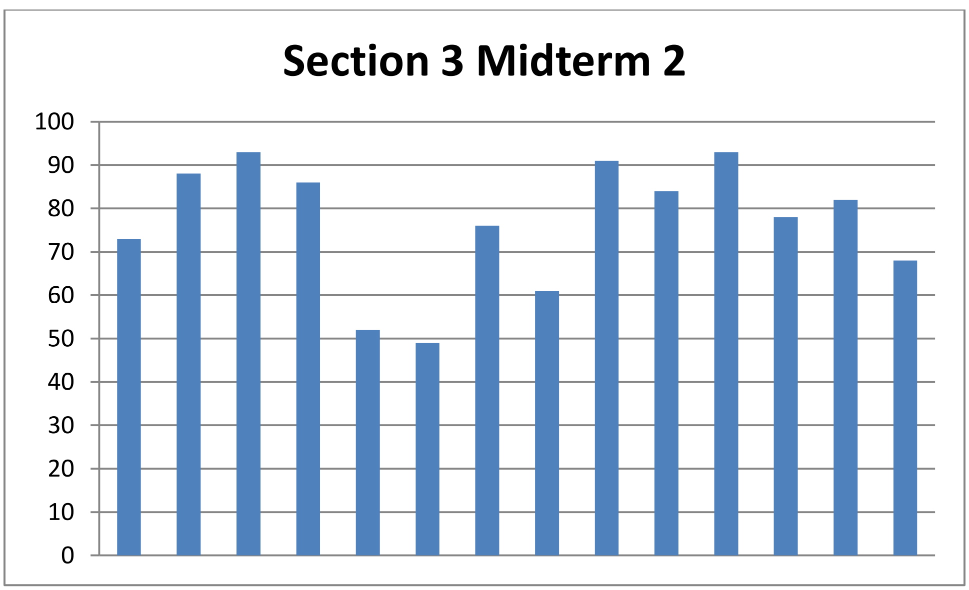
Midterm 2 tested the students on inequalities, exponents, and radicals. Again, Section 3 is the only class to have done an art project at this point. In all of the sections, the score that a student could have gotten was between zero and one hundred, with the passing score being seventy or 70%. For each section Midterm 2 scores are represented on the graphs below with statistical parameters calculated below each graph.



The students in Section 1 had a minimum score of 22.5 and the highest score achieved was 93. The mean value for this midterm was about 69, with the standard deviation at about 14.



The students in Section 2 had a minimum score of 38 and the highest score was 92. The mean value of the midterm was about 72, with a standard deviation of about 14.

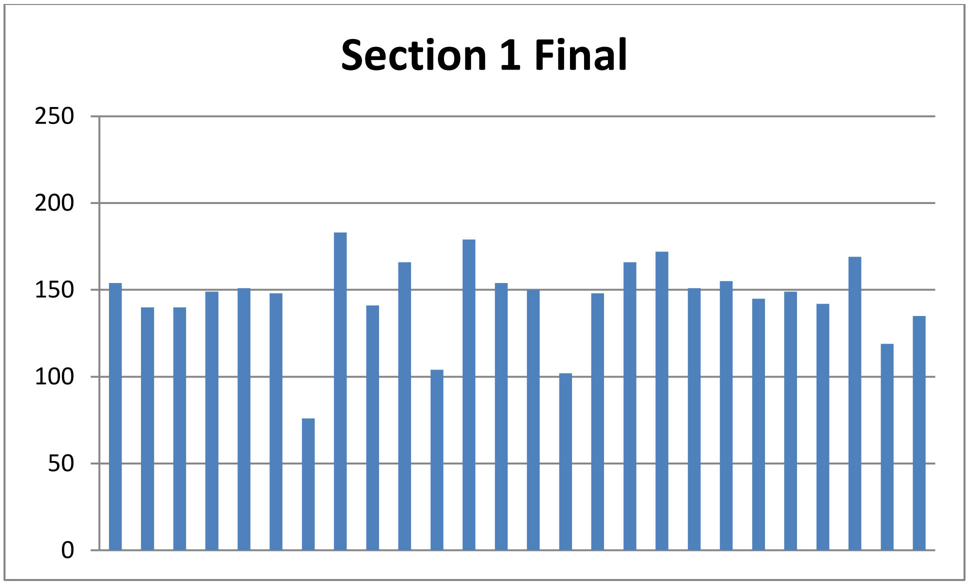


Lastly, the students in Section 3 had a minimum score of 49 and the highest score of 93. The mean value for this section was about a 78, and the standard deviation was about 15. Again, Section 3 out preformed Section 2.

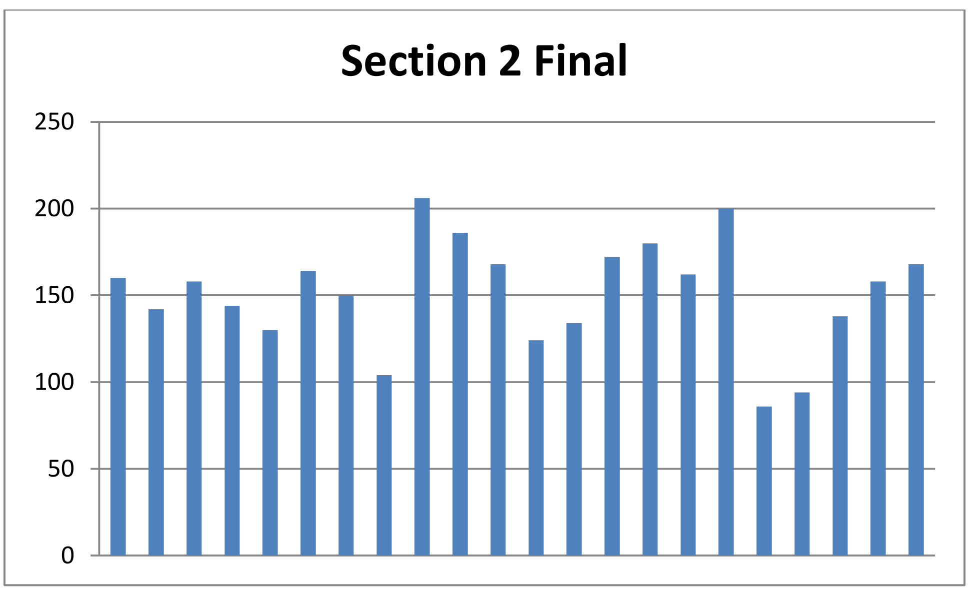
The highest scores seem to be distributed among the three different sections. However, the three lowest scores are again in Section 1 and 2; the sections that had not seen an art project. Even the weakest students in Section 3 scored above 50% on this midterm.

ANALYSIS OF RESULTS OF FINAL

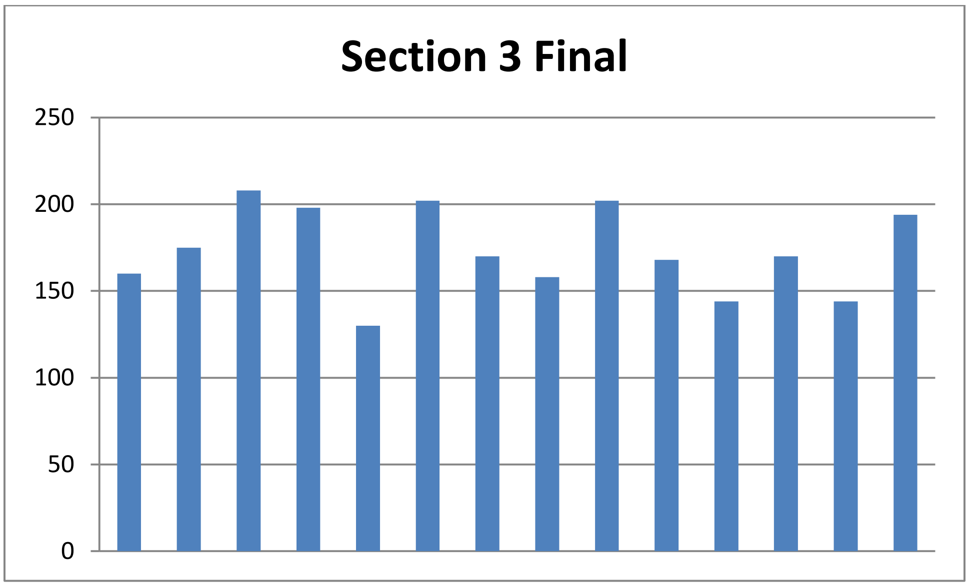
The comprehensive final exam covered the previous concepts along with quadratic functions and equations, exponential and logarithmic functions, parabolas, circles and ellipses. Section 2 had one art project, and Section 3 had all 7 art projects done by this time. So at this point, the only students not involved in art projects were in Section 1. In all of the sections, the score that a student could have gotten was between zero and two hundred, with the passing score being One hundred and forty or 70%. For each section the scores on the final exam are represented on the graphs on the following pages and the statistical parameters are calculated below each picture.



The students in Section 1 had a minimum score of 76 and the highest score achieved was 183. The mean value for this midterm was about 146, with the standard deviation at about 24.



The students in Section 2 had a minimum score of 86 and the highest score was 206. The mean value of the midterm was about 151, with a standard deviation of about 31.



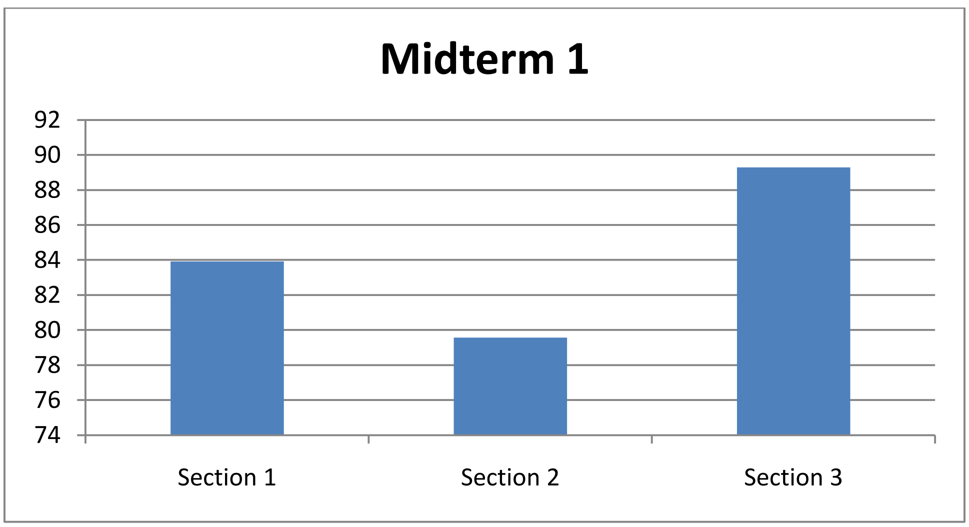
Lastly, the students in Section 3 had a minimum score of 130 and the highest score of 208. The mean value for this section was approximately 173, and the standard deviation was just below 25. Only one student did not achieve the passing score of 140 and 5 students were close to the perfect score (over 33% of the class).

It is interesting to note students with the eight highest scores were in Section 2, and Section 3 and both sections worked on an art project at this time. Section 1, however, (the section without any art projects) had the eight lowest scores. Hence some differences between sections can be seen already.

COMPARISONS

In order to see whether there was a significant change in the scores between sections, the mean values of the test scores were examined first (Appendix 3B). Since the samples are small, the t-test was used (Appendix 3C) with 95 % confidence level. In this context a p-score of less than 0.05 shows that the difference between sections achieves the significant level.

Before Midterm 1, only students in Section 3 had seen two art projects. The picture below represents mean scores of the first midterm.



It is interesting to notice that at this time the section with students who did some art projects (Section 3) has a higher mean than the other two sections. However, it is too early to draw any conclusions at this stage. The t-test comparison between the sections confirms this statement that statement – there is no significant statistical difference yet. See the tables below for specific calculations.

The table below states the t-test results for comparison of section 1 and 2.

Midterm 1		
	Section 1	*Section 2*
Mean	83.9231	79.5682
Variance	288.0538	415.1499
Observations	26	22
Hypothesized Mean Difference	0	
t Stat	0.7958	
P(T<=t) one-tail	0.2154	
t Critical one-tail	1.6829	

The fact that the p-score does not give any significant difference between Sections 1 and Section 2 at this time means that the sections are very similar as it should be since both sets of students had not seen an art project. The p-scores say that there is no significant difference between these sections at this point in the study.

Now to compare the t-test scores for the students in Section 1 and section 2 with the students in Section 3 (who had art projects already):

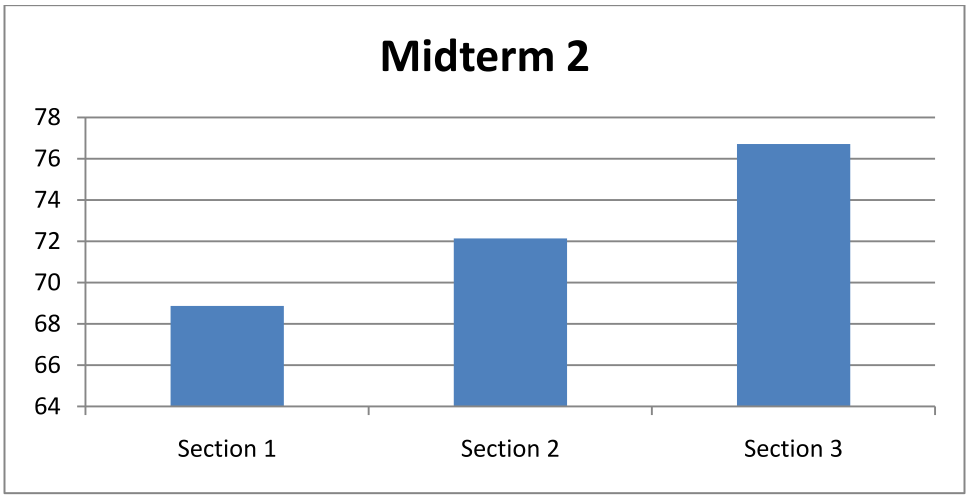
Midterm 1		
	Section 1	*Section 3*
Mean	83.92308	89.28571
Variance	288.0538	239.0659
Observations	26	14
Hypothesized Mean Difference	0	
t Stat	-1.010647	
P(T<=t) one-tail	0.160272	
t Critical one-tail	1.699127	

Midterm 1		
	Section 2	*Section 3*
Mean	79.56818	89.28571
Variance	415.1499	239.0659
Observations	22	14
Hypothesized Mean Difference	0	
t Stat	-1.620792	
P(T<=t) one-tail	0.05729	
t Critical one-tail	1.69236	

Again, the p-scores between the students in Section 1 and Section 3 do not show significant difference at this time. However the p-score for the comparison of

Section 2 and Section 3 is very close to being significant. The thing to remember is that only students in Section 3 had an art project at this time, and this section is performing better than the other two sections, even though the difference is not quite significant yet.

The following picture displays the mean scores for midterm 2.



Again, only students in Section 3 would have worked on art projects at this point. On this test the mean sores of students in section 3 are the highest and the section 2 scores improved as compare with the previous test, even though they did not do any art yet. The t-test comparing the performance of students from Section 1 and Section 2 is displayed in the table below.

Midterm 2		
	Section 1	*Section 2*
Mean	68.8654	72.1364
Variance	206.0712	194.5996
Observations	26	22
Hypothesized Mean Difference	0	
t Stat	-0.7987	
P(T<=t) one-tail	0.2143	
t Critical one-tail	1.6794	

Note that the scores from this test are extremely similar to that of the scores for the first midterm for the same sections. Since both groups of students had not seen an art project yet, the fact that there is no significant difference between the sections suggests that they are statistically similar.

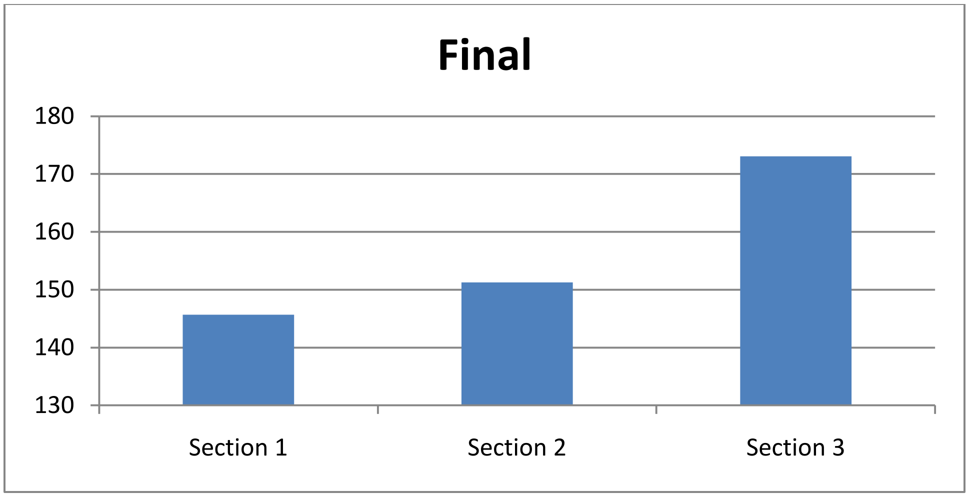
The tables below compare t-test scores and p-scores for the students in Section 1 and Section 3, and the students in Section 2 and Section 3.

Midterm 2		
	Section 1	*Section 3*
Mean	68.865385	76.71429
Variance	206.07115	211.2967
Observations	26	14
Hypothesized Mean Difference	0	
t Stat	-1.635954	
P(T<=t) one-tail	0.0569496	
t Critical one-tail	1.7056179	

Midterm 2		
	Section 2	*Section 3*
Mean	72.136364	76.71429
Variance	194.59957	211.2967
Observations	22	14
Hypothesized Mean Difference	0	
t Stat	-0.935673	
P(T<=t) one-tail	0.1788678	
t Critical one-tail	1.7032884	

The p-scores above show that the students in Section 3 performed better than the students in Section 1, and it is almost a significant difference. Comparison of students in Section 2 and Section 3 shows only slightly better performance in Section 3, still not really a significant difference.

The mean scores for the final exam are displayed on the following picture.



Much like Midterm 2, there is an increase in the mean score between Section 1 and Section 2, and then between Section 2 and Section 3. At this point of the course, Section 2 has done their art project and Section 3 has finished all of their art projects. Again as with the other two exams, the students who had art projects for all chapters had the higher mean score.

To analyze the significance of these results, the t-test and p-scores for the students in Section 1 and Section 2 are looked at and the results are summarized in the table below.

Final		
	Section 1	*Section 2*
Mean	145.6923	151.2727
Variance	573.0215	972.3983
Observations	26	22
Hypothesized Mean Difference	0	
t Stat	-0.6857	
P(T<=t) one-tail	0.2485	
t Critical one-tail	1.6849	

Even though this p-score does not show significant difference in performance it still shows better performance of the art based teaching even though doing only one art project does not significantly increase test scores.

Analysis of the p-score for the students in Section 1 compared to the students in Section 3 is shown in the table.

Final		
	Section 1	*Section 3*
Mean	145.6923	173.0714
Variance	573.0215	611.1484
Observations	26	14
Hypothesized Mean Difference	0	
t Stat	-3.378009	
P(T<=t) one-tail	0.001155	
t Critical one-tail	1.705618	

The p-score in this case shows significant difference of student learning in the art based section. It shows that there is a very significant increase in exam scores for students who worked on art projects.

The last p-scores are for the students in Section 2 compared to the students in Section 3 and are displayed in the table on the following page.

Final		
	Section 2	*Section 3*
Mean	151.2727	173.0714
Variance	972.3983	611.1484
Observations	22	14
Hypothesized Mean Difference	0	
t Stat	-2.325688	
P(T<=t) one-tail	0.013266	
t Critical one-tail	1.693889	

The p-score shows significantly better performance in the students who had the art projects for the full length of the course.

For the final exam, it can be said with 95% confidence that there is a significant increase in scores for students in Section 3 who were exposed to art activities in a significant way as compared to students in Section 1 or Section 2.

SUMMARY OF THE DATA

The students in Sections 2 and 3 performed better after they completed their art-based activities that were given to them. Statistical analysis shows that the students regularly doing art activities have significantly improved understanding for the material in Intermediate Algebra. The results show that the special art based method applied throughout the semester improves student's performance significantly, while sporadic use of the projects may improve learning but not give a significant difference in the student's scores.

BIAS

This study was conducted for a specific audience, the students who qualify for the Cal State University of Channel Islands but need help on their Algebra skills. Even though the students were placed in each section randomly, there are several things that could have biased the results. Midterm 1 shows that the sections were statistically similar; there were things that were not taken into consideration that could influence the students' performance. The things that could have affected the scores were:

- Math classes that students took in high school

- Subjects the students enjoyed
- Types of learners
- Student's major
- Last time each student took a math class
- Repeating the algebra class if previously taken at CSUCI
- Experience with art classes
- Students' confidence in graphing
- Students' confidence level in Algebra
- Students' English language skills

Additionally, all sections were taught by the same instructor, hence influence of personality and teaching style cannot be eliminated. Since CSU Channel Islands provides an additional computer lab and tutoring support for algebra students, some influence due to outside of the classroom elements was possible. Also, each section was quite small, hence one-on-one occasional instruction were possible. It is not clear if art activities would work in large sections.

Chapter V

VALIDITY OF RESULTS

This thesis looked at the use of art projects as a special method teaching technique to help students understand Intermediate Algebra. The data shows that the student's performance improved after the use of art projects for all sections that were taught. The results from the special method teaching technique are very encouraging. The results suggest that if the students are allowed to do hands on activities, like art projects in this case, in addition to homework assignments, they will do better in their exams.

There are things that could have influence the validity of the results. Since the author of this thesis was both the instructor in this study and the person who examined the results, the experiment should be repeated with a different instructor, and possibly at a different venue.

The second concern is that the traditional method was always presented before the special method. This could have allowed the students in art activities some additional time to fill in the pieces of information that might have been unclear. Because they were seeing the material twice and in different contexts, positive results of the special method may be due to the extra attention that students received and the fact that art activities in math classes were new to them.

Note that the controlled group was taught during the final semester of this study. Hence there isn't any bias coming from the instructor's experience.

ADDITIONAL INVESTIGATION

This thesis has results that should be looked into further investigated. With the positive results, it would be a good idea to look at ways to further the understanding of students learning through artistic, hands-on projects. The suggestion would be to duplicate the study with a larger population of the students, and if possible, with multiple classes. It might be interesting to see what the results would be if the students were given art projects before they are given traditional homework assignments. Another suggestion would be to try to do projects on possibly a daily basis as part of homework, instead of waiting till the end of each chapter. Other venues should be used as well to eliminate influence of the campus environment.

RECOMMENDATIONS

With the encouraging results that the special methods technique provide, the author believes that using the art projects as supplement to regular teaching techniques will improve test scores. The author believes that this can be used in any math class from a middle school class to a fundamental college math class.

R e f e r e n c e

Austin, J. D. (Fall 1996). Mathematizing String Art. *Mathematics and Computer Education, v30 n3* , 272-77.

Disher, F. (Feb 1995). Activities: Graphing Art. *Mathematics Teacher, v88 n2* , 124-28,134-36.

Entry Level Math (ELM) Exam. (1997). Retrieved November 14, 2010, from CSU Mentor: http://www.csumentor.edu/planning/high_school/elm.asp

McHugh, S. R. (Sep 2007). C[squared] = Creative Coordinates. *Mathematics Teaching in the Middle School, v13 n2* , 82-87 .

Miller, S. (Nov 2001). Understanding Transformations of Periodic Functions through Art. *Mathematics Teacher, v94 n8* , p632-35.

10 Exponents and Radicals

10.1 Radical Expressions and Functions
10.2 Rational Numbers as Exponents
10.3 Multiplying Radical Expressions
10.4 Dividing Radical Expressions
10.5 Expressions Containing Several Radical Terms
10.6 Solving Radical Equations
10.7 The Distance and Midpoint Formulas and Other Applications
10.8 The Complex Numbers

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10.6 Solving Radical Equations

- The Principle of Powers
- Equations with Two Radical Terms

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The Principle of Powers

A **radical equation** is an equation in which the variable appears in a radicand. Examples are

$$\sqrt[4]{5x-1}+4=1 \text{ and } \sqrt{m+2}+\sqrt{m}=9.$$

The Principle of Powers

If $a = b$, then $a^n = b^n$ for any exponent n.

Example Solve: $\sqrt{m} + 3 = 9.$

Solution

$$\sqrt{m} + 3 = 9$$

$$\sqrt{m} = 6 \quad \text{Isolate the radical}$$

$$\left(\sqrt{m}\right)^2 = 6^2 \quad \text{Using the principle of powers}$$

$$m = 36$$

Check:

$\sqrt{m} + 3 = 9$	
$\sqrt{36} + 3$	9
$6 + 3 \stackrel{?}{=} 9$	TRUE

The solution is 36.

Caution!

Raising both sides of an equation to an even power may not produce an equivalent equation. In this case, a check is essential.

To Solve an Equation with a Radical Term

1. Isolate the radical term on one side of the equation.

2. Use the principle of powers and solve the resulting equation.

3. Check any possible solution in the original equation.

Example Solve: $x = \sqrt{x+5} + 1$.

Solution

$$x = \sqrt{x+5} + 1$$
$$x - 1 = \sqrt{x+5}$$
$$(x-1)^2 = \left(\sqrt{x+5}\right)^2$$
$$x^2 - 2x + 1 = x + 5$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$

Solution *continued*

$x-4=0$ or $x+1=0$

$x=4$ or $x=-1$

Check:

$x = \sqrt{x+5}+1$	
4	$\sqrt{4+5}+1$
	$\sqrt{9}+1$
TRUE 4	$3+1$

$x = \sqrt{x+5}+1$	
-1	$\sqrt{-1+5}+1$
	$\sqrt{4}+1$
-1	$2+1$ FALSE

The solution is 4.

Example Solve: $(3x+4)^{1/3}-2=0.$

Solution

$$(3x+4)^{1/3}-2=0$$
$$(3x+4)^{1/3}=2$$
$$[(3x+4)^{1/3}]^3=2^3$$
$$3x+4=8$$
$$3x=4$$
$$x=4/3$$

Because both sides were raised to an *odd* power, it is not *essential* that we check the answer.

Equations with Two Radical Terms

A strategy for solving equations with two or more radical terms is as follows.

To Solve an Equation with Two or More Radical Terms

1. Isolate one of the radical terms.

2. Use the principle of powers.

3. If a radical remains, perform steps (1) and (2) again.

4. Solve the resulting equation.

5. Check the possible solutions in the original equation.

Example Solve: $\sqrt{x+3}-\sqrt{x-2}=1$.

Solution

$$\sqrt{x+3}-\sqrt{x-2}=1$$
$$\sqrt{x+3}=\sqrt{x-2}+1$$
$$\left(\sqrt{x+3}\right)^2=\left(\sqrt{x-2}+1\right)^2$$
$$x+3=(x-2)+2\sqrt{x-2}+1$$
$$4=2\sqrt{x-2}$$
$$2=\sqrt{x-2}$$

Solution *continued*

$$2^2=\left(\sqrt{x-2}\right)^2$$
$$4=x-2$$
$$6=x$$

6 is the solution. The check is left to the student.

10.6 Solving Radical Equations

- The Principle of Powers
- Equations with Two Radical Terms

The Principle of Powers

A ____________________ is an equation in which the variable appears in a radicand. Examples are

$$\sqrt[4]{5x-1}+4=1 \text{ and } \sqrt{m+2}+\sqrt{m}=9.$$

The Principle of Powers

If $a = b$, then $a^n = b^n$ for any exponent n.

Example

Solve: $\sqrt{m} + 3 = 9.$

Caution!

Raising both sides of an equation to an even power may not produce an equivalent equation. In this case, a check is essential.

To Solve an Equation with a Radical Term

1. Isolate the radical term on one side of the equation.

2. Use the principle of powers and solve the resulting equation.

3. Check any possible solution in the original equation.

Example

Solve: $x = \sqrt{x+5} + 1.$

Example

Solve: $(3x+4)^{1/3} - 2 = 0.$

Equations with Two Radical Terms

A strategy for solving equations with two or more radical terms is as follows.

To Solve an Equation with Two or More Radical Terms

1. Isolate one of the radical terms.

2. Use the principle of powers.

3. If a radical remains, perform steps (1) and (2) again.

4. Solve the resulting equation.

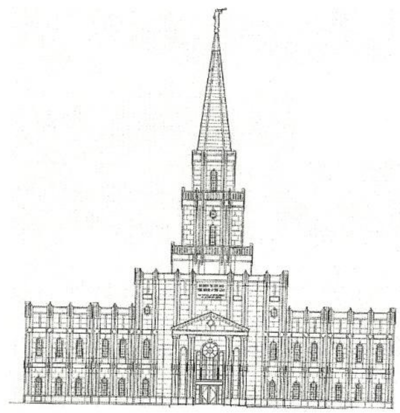
5. Check the possible solutions in the original equation.

Example Solve: $\sqrt{x+3} - \sqrt{x-2} = 1$.

APPENDIX 1C: HOMEWORK ASSIGNMENTS

Homework Assignments	
7.1 Interduction to Functions	10-46 even
7.2 Domain and Range	8-44 even, 54, 56, 58
7.3 Graphs of Function	8-24 even, 42-68 even (you only have to do 7 of these)
7.4 The Algebra of Functions	8 - 30 even, 42 - 58 even, Try 60 - 66 even for extra credit
7.5 Formulas, Applications, and Variation	14-32 even, 44-54 even, 70-76 even
8.1 Systems of Equations in Two Variables	10 - 16 even, 18 - 36 even, 38, 40, 42, 46, 50, 54
8.2 Solving by Substitution or Elimination	8 - 42 even
8.3 Solving Applications: Systems of Two Equations	16, 18, 29, 30, 38, 40
8.4 Systems of Equations in Three Variables	8-36 even
8.5 Solving Applications: Systems of Three Equations	2, 4, 6, 12
9.1 Inequalities and Domain	12-24 even, 30-36 even , 42-52 even
9.2 Intersections, Unions, and Compound Inequalities	12 - 72 every other even (eoe)
9.3 Absolute-Value Equations and Inequalities	16 - 88 eoe
9.4 Inequalities in Two Variables	12- 48 even
10.1 Radical Expressions and Functions	10-16 even, 18-26 even, 42-66 eoe
10.2 Rational Numbers as Exponents	10-30 eoe, 32-48 eoe, 50-62 eoe, 66-78 eoe, 80-96 eoe
10.3 Multiplying Radical Expressions	8 - 40 eoe, 48 - 76 eoe
10.4 Dividing Radical Expressions	10 - 38 even, 42 - 58 even, Extra Credit: 40, 60 - 72 even
10.5 Expressions Containing Several Radical Terms	8 - 68 eoe, 80 - 96 eoe
10.6 Solving Radical Equations	8 - 48 eoe
10.7 The Distance and Midpoint Formulas and Other App.	8 - 18 even, 20, 30 - 44 even, 52 - 76 eoe
10.8 The Complex Numbers	10 - 94 every other even
11.1 Quadratic Equations	40-58 even, 66-70 even, Extra Credit: 8-36 eoe
11.2 The Quadratic Formula	8-36 even
11.3 Studying Solutions of Quadratic Equations	8-56 eoe
11.5 Equations Reducible to Quadratic	1-8 all, 16-52 eoe
11.6 Quadratic Functions and Their Graphs	10 - 58 eoe
11.7 More about Graphing Quadratic Functions	10 - 32 even, 44 - 54 even
12.1 Composite and Inverse Functions	10 - 26 eoe, 32, 34, 36 - 56 eoe
12.2 Exponential and Logarithmic Functions	8 - 38 even
12.3 Logarithmic Functions	10 - 62 eoe, 64, 66, 72, 82, 86, 90
12.4 Properties of Logarithmic Functions	8 - 68 eoe
13.1 Conic Section: Parabolas and Circles	1-8, 14, 20, 78, 80
13.2 Conic Section: Ellipses	22, 24, 26, 28, 30, 32(use hint from 31)

Chapter 7 Art Project
25 points

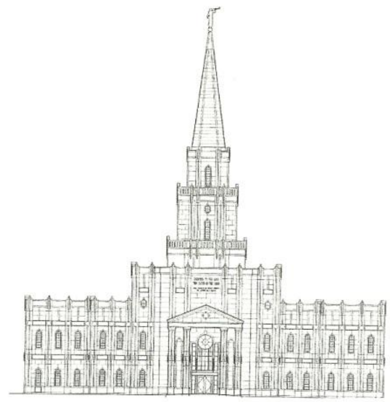


Create your own drawing using the skills you have learned about linear functions. You have to include at a minimum of 6 lines in your picture. Of those 6 lines, there needs to be at a minimum of 2 restrictions. Again these are all minimum values that you have to include, which means that depending on how much above that you do, I would be willing to give you more then just the 20 points. If you do include say 25 lines in your picture, but you only give me the equations for 6, the most points you can get is 20.

So have fun with this, be creative. Just remember that I need 1 paper with the drawing and the lines numbered and a separate paper with the equations to those lines in the numbered order.

Chapter 8 Art Project

25 points

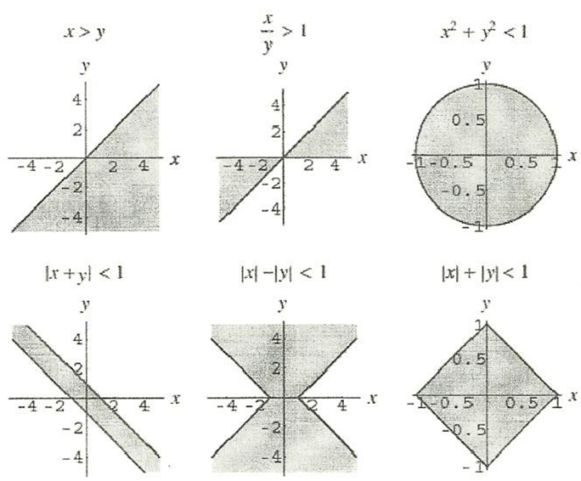


Create your own drawing using the skills you have learned about linear functions and systems of equations. I will give you 6 points at random. You will then have to take those points and create at a minimum 6 lines. Of those 6 lines, there needs to be at a minimum of 4 restrictions. Once you have those lines, you will draw a minimum of 3 more lines in order to create a picture out of all of your lines. Again these are all minimum values that you have to include, which means that depending on how much above that you do, I would be willing to give you more then just the 25 points.

So have fun with this, be creative. Just remember that I need 1 paper with the drawing and the lines numbered and a separate paper with the equations to those lines in the numbered order.

Chapter 9 Art Project

25 points



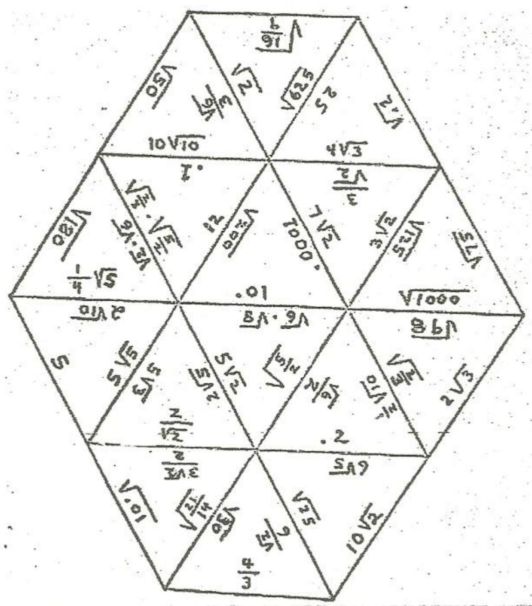
Create your own drawing using the skills you have learned about linear functions, systems of equations and inequalities. I will give you the equation for either a circle or an ellipse. You will then create a minimum of 8 inequalities that can either pass through the circle/ellipse or just touch the circle/ellipse. Of those 8 lines, you will have to have a minimum of 6 restrictions. Again these are all minimum values that you have to include, which means that depending on how much above that you do, I would be willing to give you more then just the 25 points.

So have fun with this, be creative. Just remember that I need 1 paper with the drawing and the lines numbered and a separate paper with the equations to those lines in the numbered order.

Square Root Review

Ch 10

1. Cut out each of the triangles below.
2. Then reassemble them to form one large equilateral traingle in which each pair of adjoining sides is equivalent.

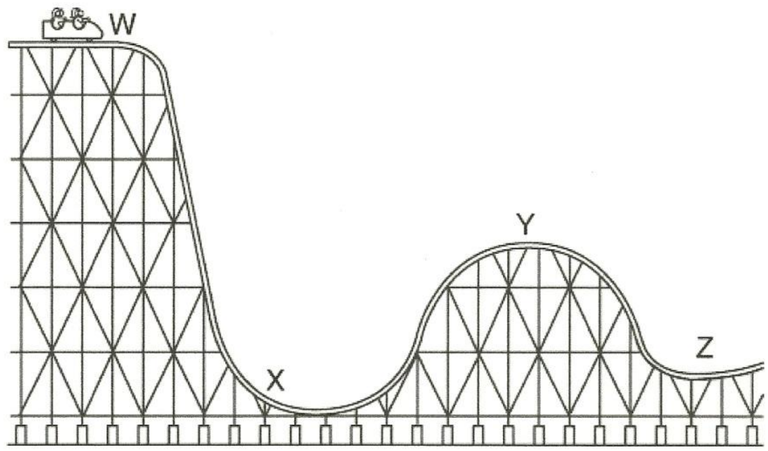


3. Once you assemble the large equilateral triangle glue it onto a piece of paper. (or tape it)

4. Write a paragraph about what you learned from doing this extra credit project.

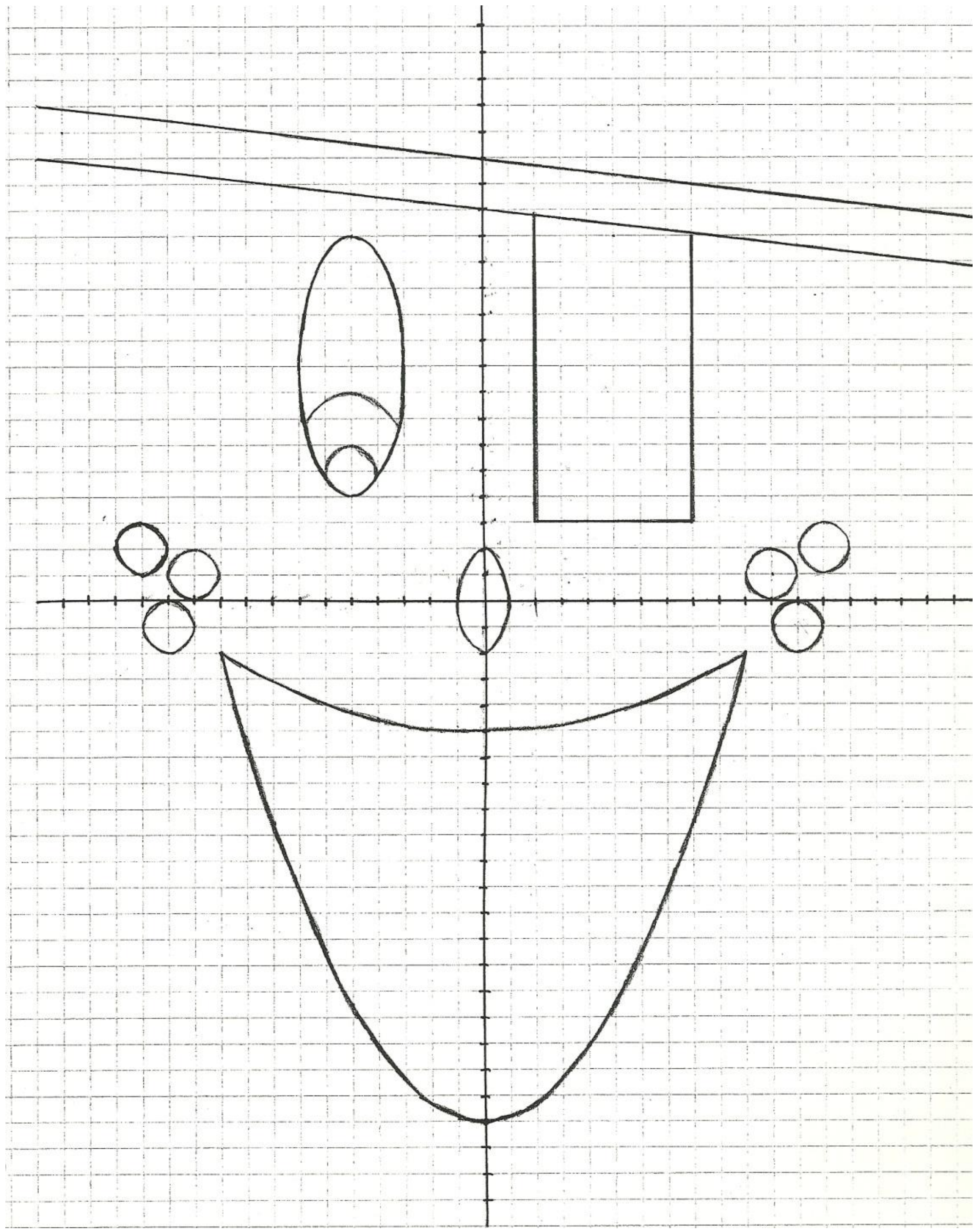
Chapter 11 Art Project

25 points

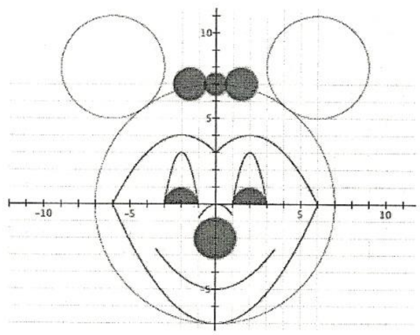


Create your own drawing using the skills you have learned about linear functions, systems of equations and parabolas. You have to include at a minimum of 15 equations for your picture. These are only minimum values that you have to include, which means that depending on how much above that you do, I would be willing to give you more than just the 25 points. If you do include say 25 equations for your picture, but you only give me the equations for 15, the most points you can get is 25.

So have fun with this, be creative. Just remember that I need 1 paper with the drawing and the lines numbered and a separate paper with the equations to those lines in the numbered order.



Chapter 13 Project
25 points



Create your own drawing using the skills you have learned from the Conic section. You have to include at a minimum 3 circles, 2 ellipses, 1 parabola facing up or down, 1 parabola facing left or right, and 3 lines. That is a total of 10 functions at a minimum that you have to have on your picture. Of those 10 functions though, there needs to be at a minimum of 4 restrictions. Again these are all minimum values that you have to include, which means that depending on how much above that you do, I would be willing to give you more then just the 25 points. If you do include say 25 functions in your picture, but you only give me the equations for 10, the most points you can get is 25.

So have fun with this, be creative. Just remember that I need 1 paper with the drawing with the functions numbered and another paper with the equations to those functions in the numbered order.

Mid-term #1

Math 95 Fall 2009
Chapter 7 - 8

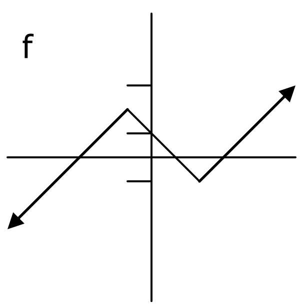
Determine whether the relation is a function. (Write either 'yes, it is a function' or 'no, it is not a function)

	Domain	Correspondence	Range
1.	The players on a team	The number of each player	A set of numbers
2.	The members of a band	An instrument the person can play	A set of instruments
3.	A set of triangles	The area of each triangle	A set of numbers

Find the values of the function $f(x) = 5x^2 + 4x$.

4. $f(-1)$ **5.** $f(2a)$

6. Determine the domain and the range for f.



7. Find the domain of f.

$$f = \frac{2x - 7}{x^2 + 8x + 7}.$$

Find the indicated function values for the function.

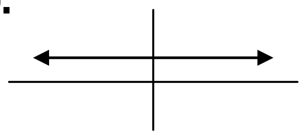
$$f(x) = \begin{cases} 2x^2 - 3, if\ x < 2 \\ x^2, if\ 2 \le x \le 4 \\ 5x - 7, if\ x > 4 \end{cases}$$

8. f(3)

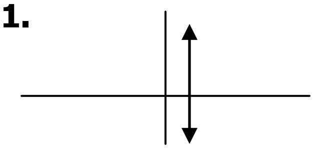
9. f(6)

Determine whether the graph is a function. (Write either 'yes, it is a function' or 'no, it is not a function)

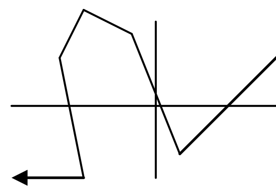
10.



11.

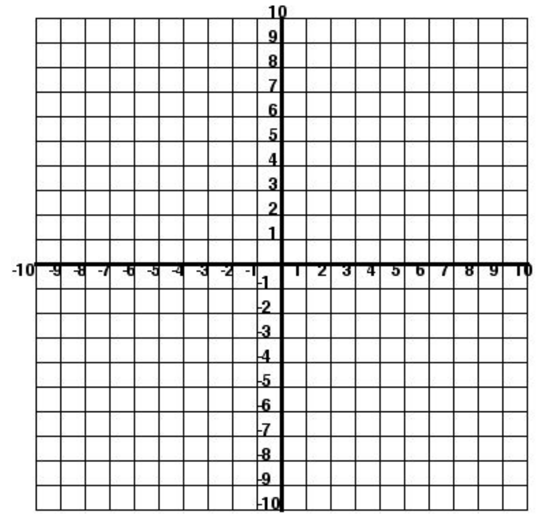


12.



13. Graph the function.

f(x) = -4x – 2



Solve each formula for the specified variable.

14. $= Ld$; d

15. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$; p

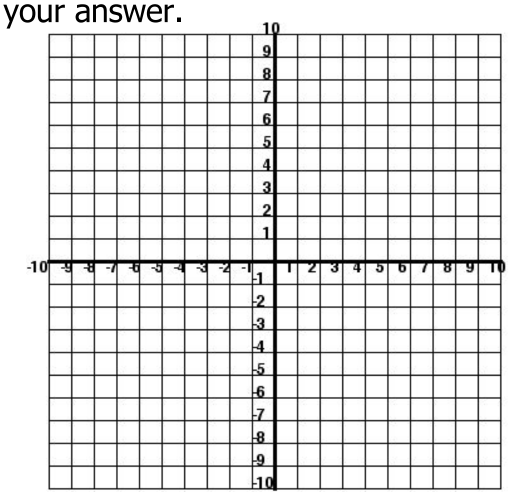
16. Find an equation of variation in which y varies jointly as w and the square of x and inversely as z, and y = 49 when w = 3, x = 7, and z = 12.

Let f(x) = -3x + 1 and g(x) = x^2 + 2. Find the following.

17.(f + g)(-1) **18.**(f • g)(0)

19. Solve the system *graphically*. Be sure to check your answer.

$$x + y = 4$$
$$x - y = 2$$



Solve using the method of your choice

20.
$$x - 2y = 6$$
$$-x + 3y = -4$$

21.
$$3x + 2y = 3$$
$$9x - 8y = -2$$

22. $2a + 2b = 2$

$3a - b = 7$

23. Two angles are complementary. The sum of the measures of the first angle and one half the second angle is 60. Find the measures of the angles.

24. Solve the system.

$$3p + 2r = 11$$
$$q - 7r = 4$$
$$p - 6q = 1$$

Extra Credit:

The sum of three numbers is eighty three. The second number is three more than twice the first. The third number is forty more than the first. Find the three numbers.

An electrician, a carpenter, and a plumber are hired to work on a house. The electrician earns $21 per hour, the carpenter $19.50 per hour and the plumber $24 per hour. The first day on the job, they worked a total of 21.5 hours and earned a total of $469.50. If the plumber worked 2 more hours than the carpenter did, how many hours did each work?

Solve the system of equations.

$$-w + 2x - 3y + z = -8$$
$$-w + x + y - z = -4$$
$$w + x + y + z = 22$$
$$-w + x - y - z = -14$$

Mid-term #2
Math 95 Fall 2010
Chapter 9 - 10

Graph each inequality and write the solution set using both set-builder and interval notation.

1. $-\frac{1}{2}x - \frac{1}{4} > \frac{1}{2} - \frac{1}{8}x$
2. $-2(x - 5) \geq 6(x + 7) - 12$
3. $x - 1 \leq 2x + 3$
4. Let $f(x) = 3x + 2$ and $g(x) = 10 - x$. Find all values of x for which $f(x) \leq g(x)$.

Solve and graph each solution set. Write each solution in either set-builder or interval notation.

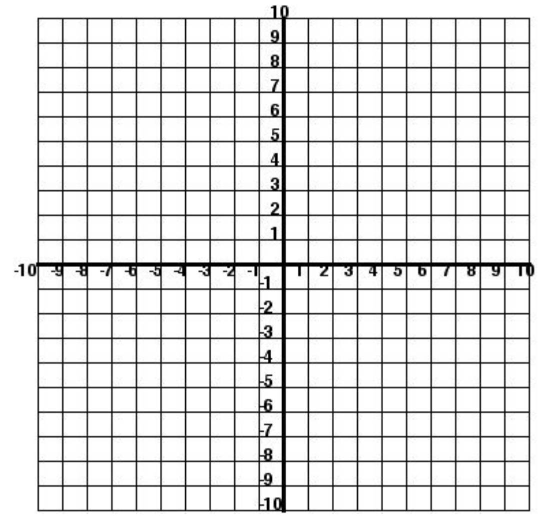
5. $-3 < x + 5 \leq 5$
6. $|a - 8| > 3$
7. $|5n + 6| < -11$
8. $2x + 5 > 6 \; or \; x - 3 \leq 9$

9. You are given the set $s = \{d, i, s, n, e, y\}$ and $r = \{l, a, n, d\}$.
   a. Find the $\cup$ of s and r.

b. Find the ∩ of s and r.

10. Graph the system of inequalities.

$$2y - x \geq -7$$
$$2y + 3x < 15$$
$$y \leq 0$$
$$x \geq 0$$



Simplify . Assume that variables can represent any real number.

11. $\sqrt[4]{3x^5b}\sqrt[3]{9xb^2}$

12. $\left(\sqrt{3} - 3\sqrt{8}\right)\left(\sqrt{5} + 2\sqrt{8}\right)$

13. $\sqrt[4]{16x^{20}y^8}$

14. $\sqrt[4]{\frac{48a^{11}}{c^8}}$

15. $\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$

16. Rationalize the denominator : $\frac{4\sqrt{5}}{\sqrt{2}+\sqrt{3}}$.

17. Solve: $1 + \sqrt{x} = \sqrt{3x - 3}$

18. Simplify: i^{45}

19. Divide and simplify to the form $a + bi$: $\frac{7-2i}{3+4i}$.

Solve. Give exact answers and approximations to three decimal places.

20. One leg of a 30-60-90 right triangle is 12 cm long. Find the posible lengths of the other leg.

Extra Credit

21. Solve: $\sqrt{11x + \sqrt{6+x}} = 6$.

22. Simplify: $\frac{2}{1-3i} - \frac{3}{4+2i}$

APPENDIX 2C: FINAL

Math 95 Final

Name:

This is another opportunity for you to demonstrate your knowledge and familiarity with the material presented in chapter 7 through chapter 12. This material should be familiar since you should have reviewed past quizzes and assignments, in addition to the review day this past week. Take your time and make sure you answer the questions fully and clearly. Circle all the page numbers for extra credit. You must show all necessary steps in order to receive credit. Simplify but do not approximate any of the solutions unless asked to do so. ***If in doubt, write it out!*** Look over the exam and answer the questions that you find easier first. Please do not spend too much time on any one question.
For word problems, make sure you clearly define whatever unknown you identify (*e.g. "Let x represent…."*) and write a clear answer in the form of a sentence. Put an x through all of the even page numbers for more extra credit. Also remember to show that you checked your work so that the answer makes sense to you and convinces me.

- You may only ask questions from the instructor.
- You may only use the paper the instructor gives you, if you need more room, you may use the back of the pages but please number the continuation.
- Put everything into its simplest form.

1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

1. What are the domain and/or range for each function?

a) $f(x) = \dfrac{x+4}{7-x}$

Domain: ______________________________

b) f = {(0, 3), (2, 2), (4, 0)}

Domain:____________________ Range: _________________

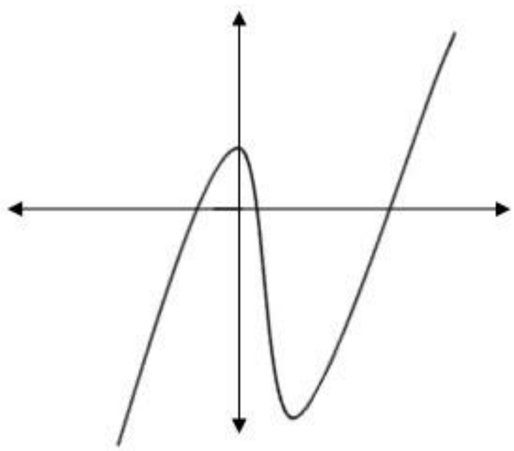
2. Find the indicated function values for the given function.

$$f(x) = \begin{cases} 3x, & \text{if } x < -3 \\ 1 + x, & \text{if } -3 \le x < 7 \\ x^2, & \text{if } x \ge 7 \end{cases}$$

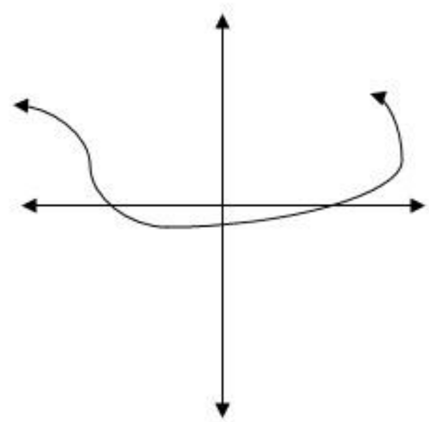
a) f(−3) = **b)** f(7) =

3. Are these functions? Circle your answers.

a) Yes or No



b)Yes or No



c)Yes or No

$f(x) = \left|2x^2 + x - 7\right|$

4. Find the following values given functions f and g.

$$f(x) = 2x \ \ +1$$

$$g(x) = x^2 + 5$$

a) $g(2a)$ "g of 2a" =

b) $(g + f)(-1)$, "g plus f of 1" =

c) $(f \circ g)(3)$, "f of g of 3" =

5. Solve using the Substitution Method or the Elimination Method.

$$9x - 2y = 3$$

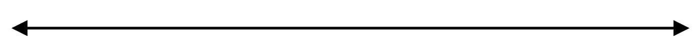
$$3x - 6 = y$$

6. Two cars leave the same point in Bakersfield (honestly, if you were in Bakersfield, wouldn't you be leaving too?), traveling in opposite directions. One car travels at a constant speed of 81 miles per hour and the other at 93 miles per hour. In how many hours will they be 435 miles apart?

7. The sum of three numbers is -5. The first number minus the second number plus five times the third number is -9. The third number plus two times the first number plus the second number is -6. Find the numbers.

8. Solve then graph the following function. Write the solution set using both interval and set-builder notations.

$$\frac{2t - 9}{-3} \geq 7$$



Interval Notation:__________________

Set-Builder Notation:________________________

9. Find the intersection and union.

{w,e,s,t} $\cap$ {c,o,a,s,t}= _______________________________________

{w,e,s,t} $\cup$ {c,o,a,s,t}=____

10. Solve for x.

$$|3x+5|=|x-6|$$

11. Solve for t.

$$3+\sqrt{5-x}=x$$

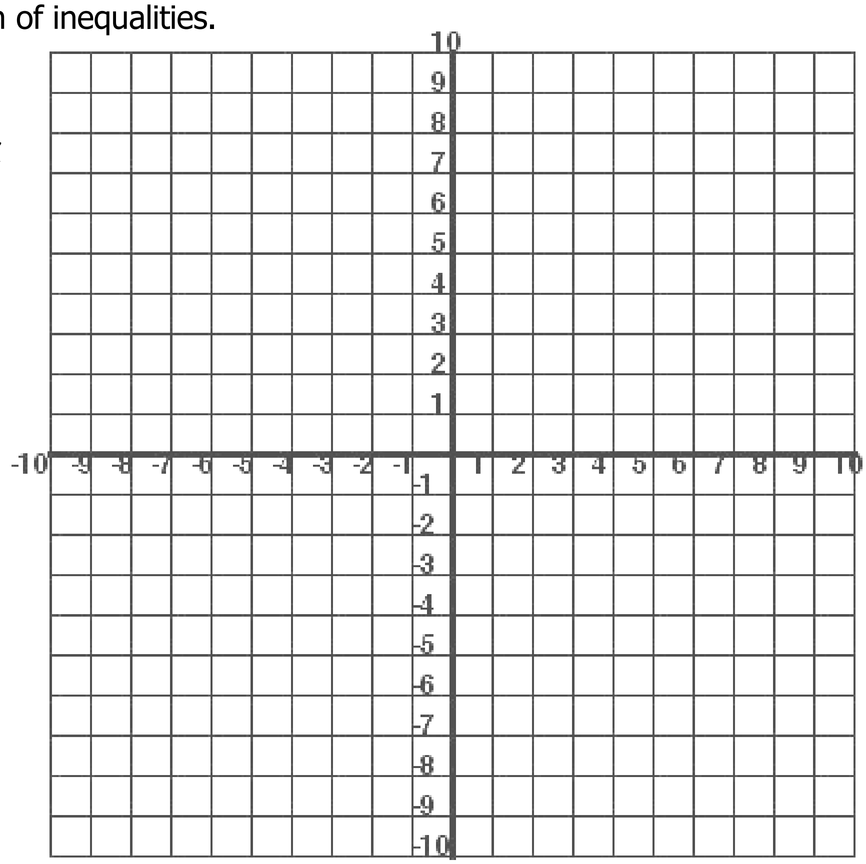
12. Graph the system of inequalities.

$$y \geq 2x+1$$

$$y-1 < -\frac{1}{2}x$$

$$x > -4$$

$$y \geq 0$$



13. Write an equivalent expression using exponential notation (no radicals) with positive exponents for each of the following. Simplify if possible.

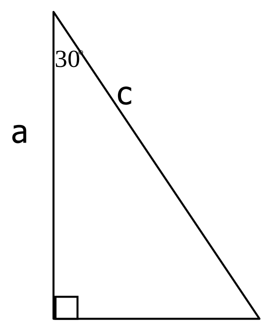
$$\left(\frac{1}{16}\right)^{-\frac{3}{4}}$$

14. Simplify.

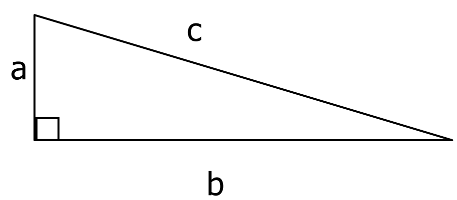
a) $i^6 + i^0 =$

b) $i^{19} =$

15. In each right triangle, find the length of the side not given.



a) a = 5, b = ____________, c = ______________



b) a = 5, b = _____________, c = 13

16. Solve using the quadratic formula. Show your work.

$$25x = 3x^2 + 28$$

17. Simplify.

a) $\log_5 125 =$

Rewrite as a single logarithm.

b) $\log_a(2x+1) + \log_a(x) - \log_a(7z)$

18. Find the following for the quadratic function.

$$f(x) = -2x^2 + 6x - 3$$

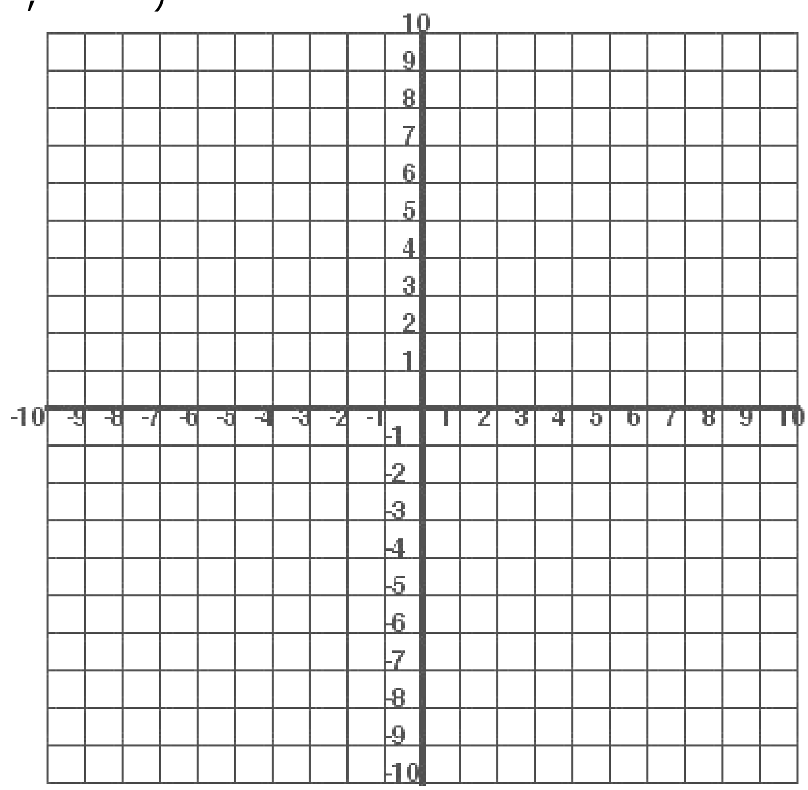
a) Put $f(x)$ into the form $a(x-h)^2 + k$.

b) The vertex: (,)

c) The axis of symmetry: x=__________

d) The y-intercept: (,)

e) Graph the function:



19. Use this function to answer the questions.

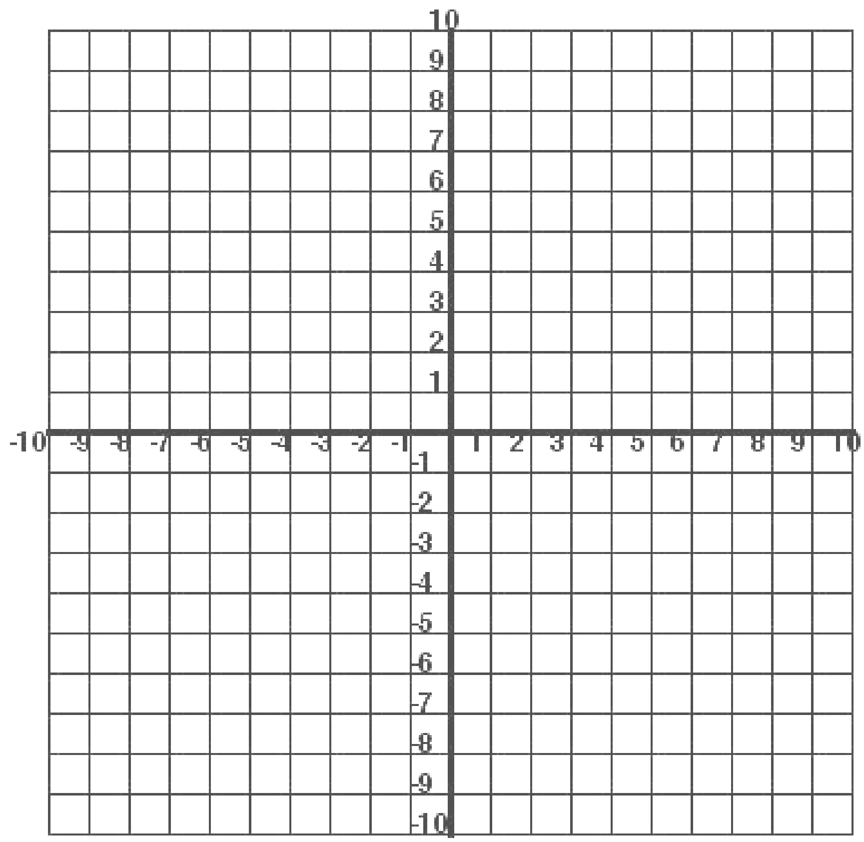
$$f(x) = -\frac{1}{6}x + 1$$

a) Is this function one-to-one?

b) If yes, find a formula for its inverse. If no, why not?

20. Graph $y = 3^{(x-1)}$

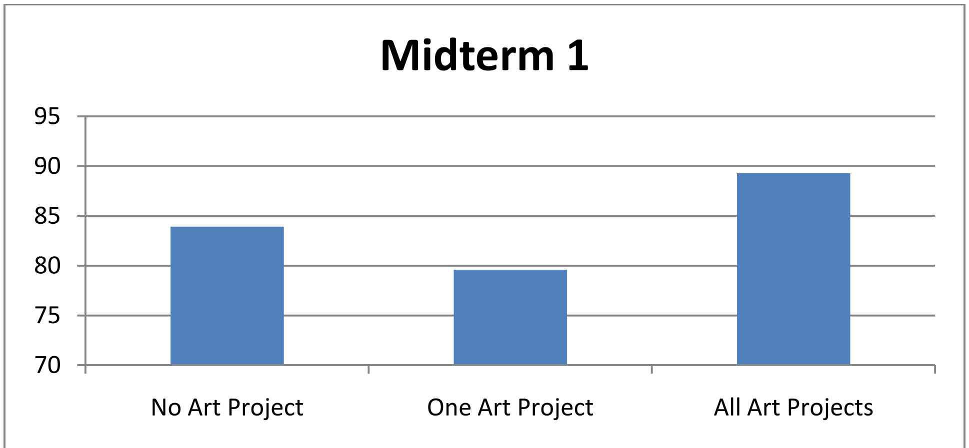
x	y

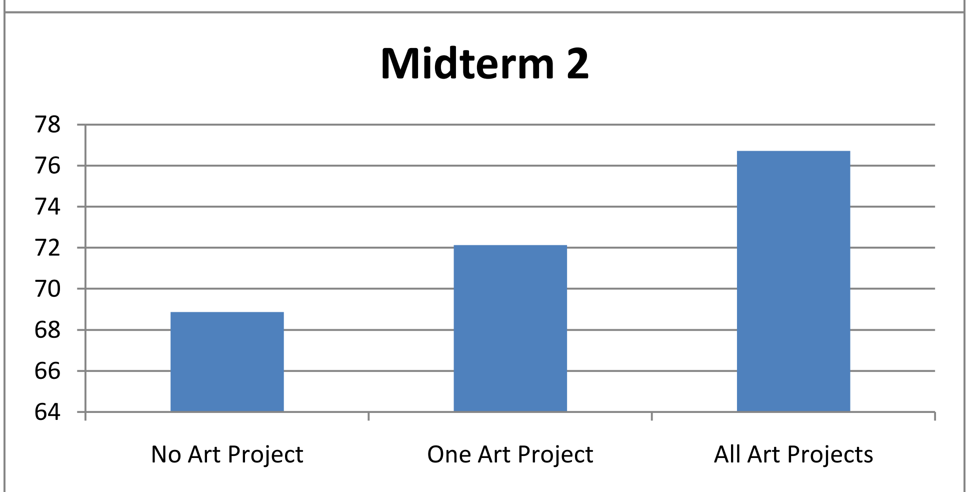


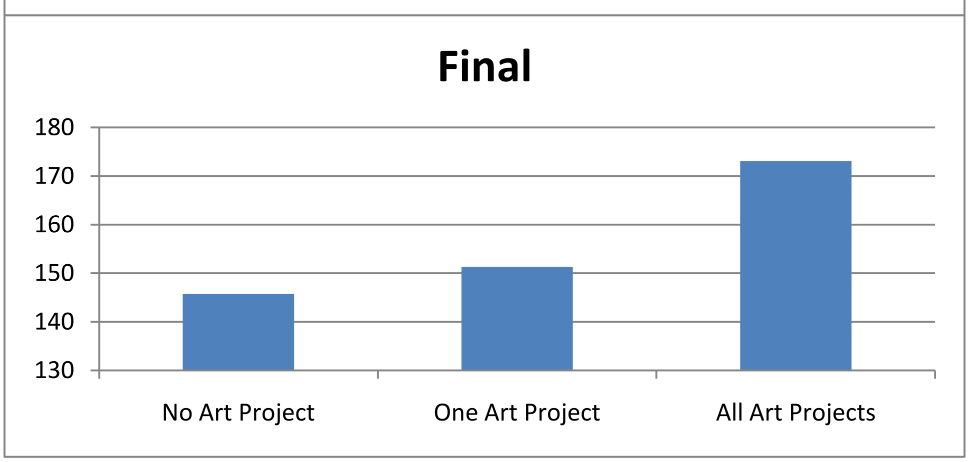
APPENDIX 3A: TEST SCORES

No Art Project				One Art Project				All Art Projects			
Name	**Midterm 1**	**Midterm 2**	**Final**	**Name**	**Midterm 1**	**Midterm 2**	**Final**	**Name**	**Midterm 1**	**Midterm 2**	**Final**
Hip-Hop	96.5	84.5	154	B-Ball	90	71	160	McQueen	86.5	73	160
Yoga	97	68.5	140	H2O	88	60	142	Volley	107.5	88	175
Jam	90	93	140	Bow	83.5	80	158	Sunshine	96	93	208
Fashonista	96	70	149	Tink	91.5	78	144	PomPom	46.5	86	198
Adventure	80.5	62.5	151	Skater	84	58	130	Five	80.5	52	130
Futbul	85.5	68.5	148	Art	79	89	164	Motocross	83	49	202
Boots	31.5	36	76	FIFA	86.5	65	150	Waffles	84.5	76	170
Green Mint	98	71	183	Sports	49.5	56	104	Whatevs	93.5	61	158
Sleepy	96.5	65	141	Cleans	101	83	206	Passport	106	91	202
Gamer	58.5	70	166	Talks	95.5	92	186	Cancun	95	84	168
Perky	81.5	66.5	104	Volley	99.5	80	168	FroYo	103.5	93	144
Crayola	95	86.5	179	Snow	81.5	58	124	Sweets	79	78	170
Crafty	84	67.5	154	Speedy	74.5	58	134	Starburst	100	82	144
Cheeser	100	71	150	Sing	84.5	88	172	BFF	88.5	68	194
Bookworm	67	22.5	102	Camper	98	82	180				
Woodsy	89.5	79.5	148	Shred	88	62	162				
Platypus	99.5	80.5	166	Knitts	98	83	200				
Bubbles	96.5	77	172	Sleepy	70.5	38	86				
Eats	85.5	68.5	151	Sandy	42.5	69	94				
Volley	92.5	64	155	Boyfriend	68.5	72	138				
Thrifty	71.5	71	145	Photo	17	75	158				
Padres	97	59	149	Stretch	79.5	90	168				
Beatles	87	60	142								
Salior Moon	80	78.5	169								
Newbie	78.5	78.5	119								
Mouse	47	71	135								

APPENDIX 3B: MEAN SCORES







APPENDIX 3C: T-TEST SCORES

Midterm		
t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 2*
Mean	83.9231	79.5682
Variance	288.0538	415.1499
Observations	26	22
Hypothesized Mean Difference	0	
df	41	
t Stat	0.7958	
P(T<=t) one-tail	0.2154	
t Critical one-tail	1.6829	

Midterm 1		
t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 3*
Mean	83.92308	89.28571
Variance	288.0538	239.0659
Observations	26	14
Hypothesized Mean Difference	0	
df	29	
t Stat	-1.010647	
P(T<=t) one-tail	0.160272	
t Critical one-tail	1.699127	

Midterm 1		
t-Test: Two-Sample Assuming Unequal Variances		
	Section 2	*Section 3*
Mean	79.56818	89.28571
Variance	415.1499	239.0659
Observations	22	14
Hypothesized Mean Difference	0	
df	33	
t Stat	-1.620792	
P(T<=t) one-tail	0.05729	
t Critical one-tail	1.69236	

Midterm 2 t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 2*
Mean	68.8654	72.1364
Variance	206.0712	194.5996
Observations	26	22
Hypothesized Mean Difference	0	
df	45	
t Stat	-0.7987	
P(T<=t) one-tail	0.2143	
t Critical one-tail	1.6794	

Midterm 2 t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 3*
Mean	68.865385	76.71429
Variance	206.07115	211.2967
Observations	26	14
Hypothesized Mean Difference	0	
df	26	
t Stat	-1.635954	
P(T<=t) one-tail	0.0569496	
t Critical one-tail	1.7056179	

Midterm 2 t-Test: Two-Sample Assuming Unequal Variances		
	Section 2	*Section 3*
Mean	72.136364	76.71429
Variance	194.59957	211.2967
Observations	22	14
Hypothesized Mean Difference	0	
df	27	
t Stat	-0.935673	
P(T<=t) one-tail	0.1788678	
t Critical one-tail	1.7032884	

Final t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 2*
Mean	145.6923	151.2727
Variance	573.0215	972.3983
Observations	26	22
Hypothesized Mean Difference	0	
df	39	
t Stat	-0.6857	
P(T<=t) one-tail	0.2485	
t Critical one-tail	1.6849	

Final t-Test: Two-Sample Assuming Unequal Variances		
	Section 1	*Section 3*
Mean	145.6923	173.0714
Variance	573.0215	611.1484
Observations	26	14
Hypothesized Mean Difference	0	
df	26	
t Stat	-3.378009	
P(T<=t) one-tail	0.001155	
t Critical one-tail	1.705618	

Final t-Test: Two-Sample Assuming Unequal Variances		
	Section 2	*Section 3*
Mean	151.2727	173.0714
Variance	972.3983	611.1484
Observations	22	14
Hypothesized Mean Difference	0	
df	32	
t Stat	-2.325688	
P(T<=t) one-tail	0.013266	
t Critical one-tail	1.693889	