Building Foundations for Pre-Service Teachers in Mathematics

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Masters of Science

by

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Building Foundations for Pre-Service Teachers in Mathematics

Title of Item

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3 to 5 keywords or phrases to describe the item

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Abstract

Recent studies suggest that many pre-service and in-service elementary school teachers are not equipped with the adequate content knowledge required to teach elementary mathematics. This research study compared 3 sections of Mathematics for Elementary School Teachers courses and investigated student understandings of fraction multiplication and division. One of the sections was instructed using a traditional method of teaching, while the other two sections used concrete hands-on models of real-life applications. The analysis shows that the concrete models of real life applications improved pre-service teachers' conceptual understanding of operations of fractions and improved their ability to create fraction story problems.

Introduction

Since the 1960's, the relationship between teacher characteristics, teacher conduct, and student achievement has been investigated in both qualitative and quantitative studies. In 1986, Shulman introduced the idea of pedagogical content knowledge and described the required knowledge for teaching as the intersection of content specific knowledge and pedagogy (Shulman, 1986). Ma identified the deep understandings required by teachers using the phrase, "profound understandings of fundamental mathematics," (Ma, 1999) and Hill, Ball, and Shilling defined a framework describing the required teaching knowledge, distinguishing between subject matter knowledge and pedagogical content knowledge (2008). As explained by Ball and Hill, in order to effectively teach mathematics a competent teacher needs to know the correct knowledge of concepts and procedures, an understanding of the underlying principles and meanings, and an appreciation and understanding of the connection among mathematical ideas; this is known as the knowledge of mathematics for teaching, (Ball, 1990; (Hill & Schilling & Ball, 2004).

It is essential that teachers have the required knowledge of teaching mathematics, as it has been demonstrated that there is a positive correlation between a teacher's mathematical knowledge and student achievement, (Hill & Rowan & Ball, 2005). However, various studies show that many pre-service and in-service elementary school teachers do not meet this standard when teaching fraction operations. At Tel-Aviv University, Tirosh found in her study of enhancing prospective teaches' knowledge that most of them knew how to divide fractions but could not explain the procedure, (Tirosh, 2000). She found that they had not developed a conceptual understanding of fraction division. In research done at the University of Wollongong, Forrester and Chinnappan found that pre-service teachers' understanding of fraction division is predominately procedural, (Chinnappan & Forrester, 2010). They did not demonstrate a deep understanding of fraction concepts and can therefore not pass conceptual knowledge onto their students. These studies are examples that demonstrate that overall, pre-service teachers have technical competence in fraction division and multiplication but are unable to conceptually explain the procedures, and therefore cannot teach these concepts.

Recent mathematics education literature identifies word-problem writing as an avenue for assessing students' conceptual understanding of various mathematical concepts, (Barlow & Cates, 2007; Barlow & Drake, 2008). As proposed by Luo, word-problem writing can also serve as a tool for evaluating the mathematical content knowledge of pre-service teachers, (2009). Various studies have demonstrated that pre-service teachers have significant difficulty in writing word problems. At Washington University, Azim found that only 28% of pre-service teachers were able to describe a situation modeled by multiplication with fractions, (1995). As Ball investigated the mathematical understandings of prospective teachers, she found that very few secondary candidates and no elementary candidates were able to generate a mathematically appropriate representation of fraction division, (Ball, 1990). Also, Simon found that 70% of pre-service teachers were unable to symbolically represent fraction division problem, (1993).

A pre-service teacher's ability to write word problems reflects their mathematics literacy and content knowledge, (Luo, 2009). According to Kaiser and Willander, mathematics literacy can range from illiteracy, "the inability to cope with information regarded as culturally relevant," to multidimensional literacy, "Science literacy extends beyond vocabulary, conceptual schemes, and procedural methods to include other understandings about science," (2005). A pre-service teacher's mathematics content knowledge can be evaluated in terms of their narrative and paradigmatic knowledge, (Bruner, 1985; Chapman, 2006). In relation to word problems, narrative knowing requires a focus on the social context of the problem and paradigmatic knowledge requires focus on mathematical models or mathematical structures that are universal and context-free, (Chapman, 2006).

In order to improve instruction of mathematics, researchers have investigated how to improve pre-service teachers' conceptual understanding. Schram, Wilcox, Lanier, and Lappan found that pre-service teachers developed a conceptual understanding of many facts and formulas they had previously only memorized as a result of taking a class that emphasized problem solving, reasoning, discourse, group work, and the use of multiple representations, (1988).

To ensure elementary students have the mathematical knowledge for conceptually understanding fraction multiplication and division, it is essential that pre-service teachers are equipped with a well-developed understanding of the knowledge of mathematics for teaching including the ability to create fraction multiplication and division word problems, (Luo, 2009).

Motivation

This research was motivated by the acute need to improve instruction in secondary school mathematics courses and the need for students to develop deep understandings of basic concepts. Overall, we hope to develop methods to improve the conceptual understanding of future teachers and encourage their appreciation of mathematics connecting it to real world situations. As students continue on in their education and careers their conceptual understanding of and their disposition toward mathematical concepts will greatly influence their ability to teach mathematics. Since elementary students can only learn as much as their teachers know, pre-service teachers should have the best opportunities to build their conceptual understanding in preparation for transferring their knowledge to their pupils. One of the most common misconceptions concerning mathematics is its abstract nature and lack of applicability toward real world situations, so this research study examined pre-service teachers' understanding of how mathematics concepts are utilized in context. We believe that concrete models of real life applications will provide a deeper conceptual understanding of concepts and allow teachers to create engaging story problems in their classrooms.

Hypothesis

In this paper we study the following hypothesis: If pre-service teachers learn fraction division and multiplication using concrete models and visual representations of real life applications, then they will develop a deep conceptual understanding of the concept and be able to create fraction story problems to use with their students. The statistical hypothesis of this thesis is that students' ability to create story problems will not change during the course of this experiment, and the alternative hypothesis is that the students' ability will significantly improve.

During this research, various additional statistical hypotheses were tested at various stages. Paired t-tests were done with a 95% confidence level between the mean scores of the pre-tests and post-tests of each individual class section in accordance with the following hypotheses:

Control Group:	Study Group1:	Study Group2:
$H_0: \mu_{pre-testCG} = \mu_{post-testCG}$	$H_0: \mu_{pre-testSG1} = \mu_{post-testSG1}$	$H_0: \mu_{pre-testSG2} = \mu_{post-testSG2}$
$H_a: \mu_{pre-testCG} < \mu_{post-testCG}$	$H_a: \mu_{pre-testSG1} < \mu_{post-testSG1}$	$H_a: \mu_{pre-testSG2} < \mu_{post-testSG2}$

Where H_0 represents the statistical hypothesis, H_a represents the alternative hypothesis, and $\mu_{pre-test}$ and $\mu_{post-test}$ represent the mean pretest scores post-test scores, for each section, respectively.

Also, unpaired t-tests were done with a 99% confidence level between the post-tests and pre-tests results of each study group with the control group with the following hypotheses:

Control Group and Study Group 1:

$$H_0: \mu_{pre-testCG} = \mu_{pre-testSG1}$$
$$H_a: \mu_{pre-testCG} < \mu_{pre-testSG1}$$

Control Group and Study Group 2:

$$H_0: \mu_{pre-testCG} = \mu_{pre-testSG2}$$
$$H_a: \mu_{pre-testCG} < \mu_{pre-testSG2}$$

Control Group and Study Group 1:

$$H_0: \mu_{post-testCG} = \mu_{post-testSG1}$$
$$H_a: \mu_{post-testCG} < \mu_{post-testSG1}$$

Control Group and Study Group 2:

$$H_0: \mu_{post-testCG} = \mu_{post-testSG2}$$
$$H_a: \mu_{post-testCG} < \mu_{post-testSG2}$$

Pearson correlations were done between survey questions and post-test results with values based on the scale depicted in Figure 1.

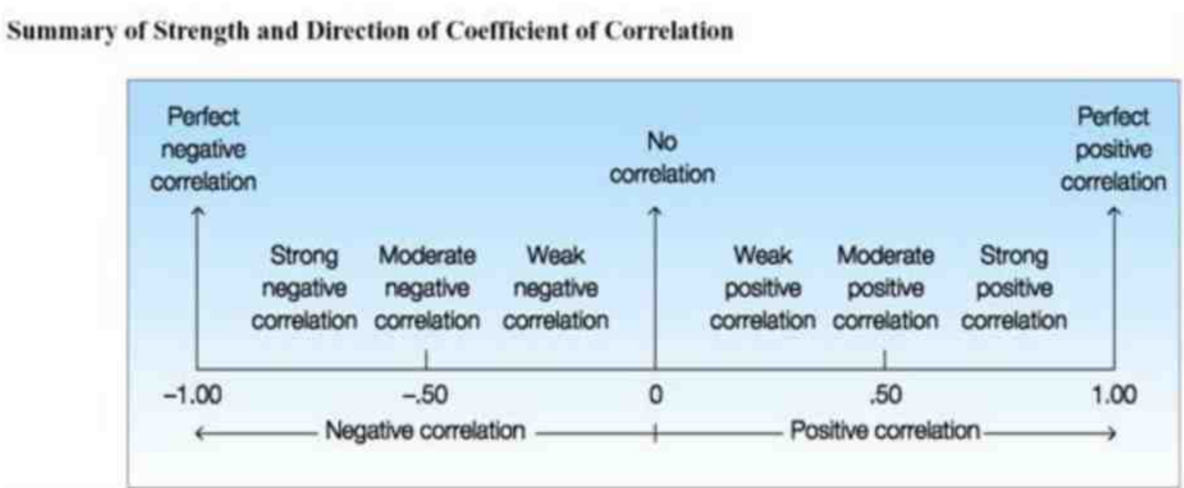


Figure 1

Methodology and Tools

Participants:

Students at CSU Channel Islands who are working toward obtaining their Bachelor of Arts in Liberal Studies: Teaching and Learning Option, and students working toward obtaining a minor in Foundational Mathematics, take the course Mathematics for Elementary School Teachers, as a pre-requisite for Mathematics for Secondary School Teachers. The course is designed to discuss K-8 mathematics curriculum, including abstract thinking and problem solving approaches to teaching.

The participants of this study consist of students enrolled in three different sections of Math for Elementary School Teachers, at CSUCI in Fall 2011 and one section of Mathematics for Elementary School Teachers. The students enrolled range from sophomores to seniors and have all completed or satisfied the requirements for college algebra.

The control group, later denoted CG, consists of 20 students, 18 females and 2 males; study group 1, ST1, consists of 20 students, 19 girls and 1 boy, enrolled in Math for Elementary School Teachers; study group 2, ST2, consists of 18 students. Each section met twice a week for 75 minutes and was in the middle of their mathematics education course when participating in this research and had received instruction in whole number operations and fraction concepts.

Variables:

We considered four main variables for the design of this study. As this area of research relates to specific teaching strategies, we tried to assess the different learning styles of the participants by administering a learning styles survey. Since we are also interested in students' general opinions and feelings toward mathematics and fractions, we administered a Revised Mathematics Anxiety Rating Scales, RMARS, Survey. Participants completed a demographic survey so we could analyze any relationships between a student's background and previous education with their pre- and post-test performance. Finally, to evaluate participants' prior knowledge of fraction division and multiplication, (including their conceptual and procedural understanding of the concepts), students completed a pre-test.

Overview of study design:

Each groups' participation consists of four consecutive sessions.

CG- Control Group: During the first day of the study, CG took the fraction pre-test. During the second session, CG received traditional lecture-based instruction on fraction multiplication and during third session they received traditional lecture-based instruction on fraction division. During the final session, CG completed the fraction post-test.

SG1/ SG2- Study Group1 and Study Group 2: During the first day of the study, SG1 and SG2 took the fraction pre-test and filled out the Learning Styles Survey, Demographic Survey, and RMARS Survey. During the second session, SG1 and SG2 participated in the designed fraction multiplication activities. During the third session, SG1 and SG2 participated in the designed fraction division activities. During the final session, SG1 and SG2 took the fraction post-test.

Description of Surveys:

Groups SG1 and SG2 completed three surveys on the first day of their participation, the Learning Styles Survey (Appendix- A1), the Demographic Survey (Appendix- A2), and the RMARS Survey (Appendix- A3) .The Learning Styles Survey consisted of questions used to identify students as an auditory language, auditory numerical, kinesthetic, visual language, visual numerical, individual, group, expressiveness oral, and/or expressiveness written learner. The results of this survey are then totaled and compared on a scale measuring learning styles as major, minor, or not applicable for each participant. The Demographic Survey consisted of 10 questions identifying each participants' background including, but not limited to, ethnicity, age, math courses completed, work schedule, and future goals. The participants also completed the RMARS Survey where they rated how anxious different mathematical situations made them feel on a scale from 1 to 5, 1 meaning "not at all anxious" and 5 meaning "very anxious." These surveys are used to determine if there is a significant correlation between participants' learning style, demographic information, and level of math anxiety and the post-test results.

Description of Tests:

Groups CG, SG1, and SG2 all completed the fraction pre-test (Appendix- B1) on the first day of their participation and a fraction post-test (Appendix- B2) on the last day of their participation. The questions on these tests were designed to determine the progress students have made during their participation in this study. The pre-test and post-test differed in the number sentences with which the pre-service teachers worked, but were of equal difficulty so improvement could be measured.

The word problems on the pre- and post-test were scored using a rubric (Appendix-B3) developed by Luo, based on Kaiser and Willander's discussion of mathematical literacy and Bruner's description of pragmatic and narrative knowledge, (Luo, 2009). Each word problem was assigned one of five performance levels ranging from 0, a failing level, to a perfect score of 4, where each level described the demonstrated conceptual understanding of the participant. In addition, the computational problems on the pre- and post-test were scored using a rubric (Appendix-B4) developed to measure the procedural competency of the participants. Each computation problem was also assigned one of five performance levels ranging from 0, a failing level, to a perfect score of 4.

Description of Traditional Method Instruction of CG- Control Group

During session 1, CG participants completed the fraction multiplication and division pre-test.

During session 2, participants were taught multiplication of fractions as described in the textbook *Math for Elementary School Teachers,* (Musser & Burger & Peterson, 2006). Participants were presented with three methods/ models of fraction multiplication: the repeated addition approach, the number line approach, and the area model. Participants were provided with the following definition of fraction multiplication:

$$Let\,\frac{a}{b}\,and\,\frac{c}{d}\;be\;any\;fractions. Then\;\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}.$$

Participants were then presented with various examples:

$$Compute\;the\;following\;products\;\frac{2}{3}\times\frac{5}{13},\frac{3}{4}\times\frac{28}{15},2\frac{1}{3}\times 7\frac{2}{5}.$$

Finally, participants were presented with the properties of fraction multiplication: closure, associativity, distributivity, commutativity, identity, and the existence of the multiplicative inverse.

During session 3, participants were taught the three different types of fraction division problems as presented in the textbook *Math for Elementary School Teachers,* (Musser & Burger & Peterson, 2006). Participants were taught division of fractions with common denominators:

$$Let\;\frac{a}{b}\,and\;\frac{c}{b}\,be\;fractions\;with\;c\neq 0, then\;\frac{a}{b}\div\frac{c}{b}=\frac{a}{c}.$$

Participants were also taught the missing factor approach for dividing fractions:

$$Let\ \frac{a}{b} and\ \frac{c}{d} be\ fractions\ with\ c \neq 0.\ If\ \frac{a}{b} \div \frac{c}{d} = \frac{e}{f}, then\ \frac{c}{d} \times \frac{e}{f} = \frac{a}{b}.$$

Students were then taught the method of dividing fractions with uncommon denominators as follows:

$$Let\ \frac{a}{b} and\ \frac{c}{d} be\ fractions\ with\ c \neq 0, then\ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Finally, participants did various examples using each definition and method of fraction division.

During the fourth session, participants took the fraction multiplication and division post-test.

Description of Experimental Method Activities for SG1 and SG2

During the first session, groups SG1 and SG2 participants took the fraction multiplication and division pre-test, and filled out the Demographic, Learning Styles, and RMARS Survey.

During the second session, groups SG1 and SG2 participated in the designed fraction multiplication activities. When students arrived to class there were six stations set up around the classroom, with the following materials:

- Pumpkins and gourds- pumpkins and gourds labeled with associated weights, plastic bags, bowl, scissors, glue, questions sheet, and manipulative cut-outs

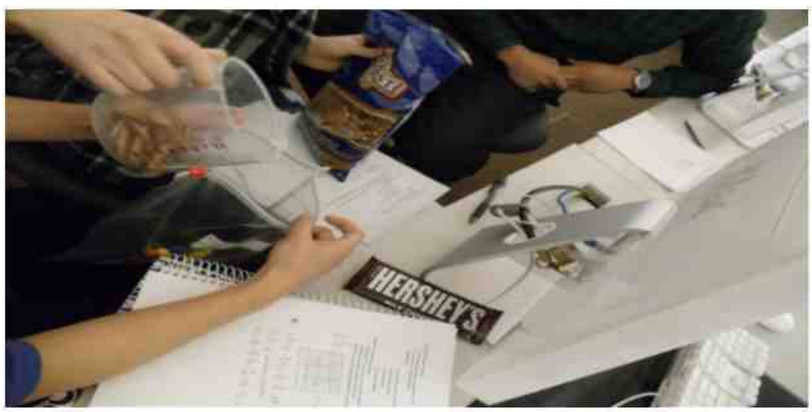
- Coffee Grounds- 8oz of sand (coffee), measuring scoops $\frac{1}{4}, \ldots, 1\ cup$, zip lock bags, scissors, glue, questions sheet, and manipulative cut-outs

- Ribbons and Tiaras- Tiara, ribbon, measuring tape, ruler, scissors, glue, questions sheet, and manipulative cut-outs

- Hershey's Chocolate- Hershey's Chocolate bars, dominos, glue, scissors, questions sheet, and manipulative cut-outs

- Police Academy- policemen, scissors, glue, questions sheet, and manipulative cut-outs

- Chex Party Brain Mix- Box of Chex mix cereal, M&Ms, pretzels, animal crackers, measuring scoops $\frac{1}{4}, \ldots, 1$ cup, zip lock bags, scissors, glue, questions sheet, and manipulative cut-outs



The following consisted of the problems related to each activity (Appendix-C1):

Pumpkins and Gourds Activity

- You need $\frac{9}{4}$ pounds of pumpkins and gourds to make a delicious pumpkin (and gourds) pie. If the pumpkins and gourds cost $\$2$ per pound; how much money will you spend buying the pumpkins and gourds?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the pumpkin and gourds and dollar bill manipulatives. Demonstrate your process using the pumpkins and dollar bill paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the repeated addition approach.

Coffee Grounds Activity

- You purchase at the store a 6 cup bag of coffee. You want to portion out $\frac{1}{3}$ of the coffee you bought into individual servings. How much coffee will you be portioning out?

Answer the following questions:
1. What number sentence can be used to solve this problem?
2. Solve the problem using the "coffee grounds" and measuring cup manipulatives. Demonstrate your process using the zip lock bag cut out, glue, and scissors.
3. Solve the number sentence in part 1 using the repeated addition approach.

Ribbons and Tiaras Activity

- You are creating princess tiaras for a princess themed birthday party and each tiara requires 10 feet of ribbon. This ribbon is then cut into strips and tied onto the tiara. If each strip is $\frac{1}{5}$ of the total required ribbon length, how long is each strip?

Answer the following questions:
1. What number sentence can be used to solve this problem?
2. Solve the problem using the ribbon and measuring tape manipulatives. Demonstrate your process using ribbon paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the measurement approach.

Hershey Bars Activity

- You have $\frac{3}{4}$ of a candy bar left over from last night's trip to the movies. If you want to eat $\frac{1}{3}$ of your leftovers, how much of the candy bar will you eat?

Answer the following questions:
1. What number sentence can be used to solve this problem?
2. Solve the problem using the Hershey Bar (or dominos) manipulatives. Demonstrate your process using the Hershey Bar paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using an area model.

Police Academy Activity

- $\frac{1}{3}$ of a class of police academy students will be out of class and participating in physical fitness course. If there are 30 students in the class, how many students are participating in the course?

Answer the following questions:
1. What number sentence can be used to solve this problem?
2. Solve the problem using the police officer figure manipulatives. Demonstrate your process using the police officer figure paper cut-outs, glue, and scissors.

3. Solve the number sentence in part 1 using the measurement approach.

Chex Brain Power Party Mix Activity

- The following consists of ingredients for Chex Brain Party Mix.
  Serving Size: 9 Hungry Students
  Ingredients:
    - $4\frac{1}{2}$ cups Chex cereal (any variety)
    - 1 cup Animal Crackers
    - $2\frac{1}{4}$ cups pretzels
    - $\frac{3}{4}$ cup peanut M&M's

Answer the following questions:

1. Create your Chex party mix for 3 students instead of 9. Solve the M&M portioning fraction multiplication problem using an area model.
2. Develop your own fraction multiplication story problem. What number sentence is used to solve it?

At each station there were various cut-out manipulatives for participants to use as they completed activities including: pumpkins, dollar bills, zip lock bags, ribbons, chocolate bar, number lines, and police man number lines. (For activity manipulative cut-outs see Appendix-C2)

Participants worked in small teams of 3 or 4 for 15 minutes at each station on the above fraction multiplication real world activities.

During session three, SG1 and SG2 participated in the designed fraction division activities. When participants arrived to class, there were five stations set up around the classroom with the following materials:

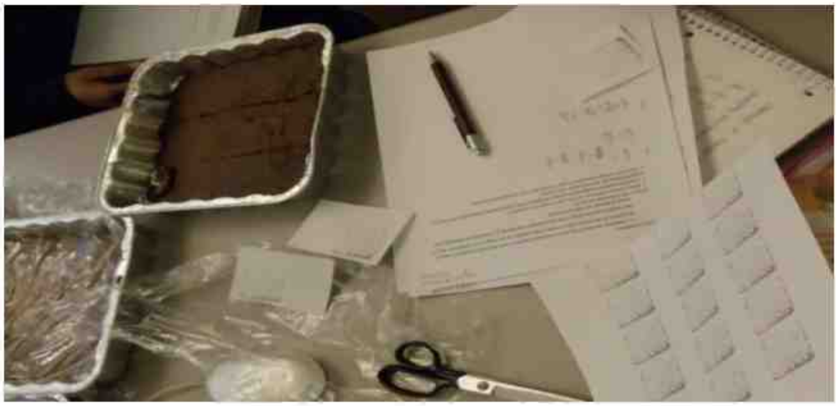
- Pumpkins and gourds- pumpkins and gourds labeled with associated weights, plastic bags, bowl, scissors, glue, questions sheet, and manipulative cut-outs

- Coffee Grounds- 8oz of sand (coffee), measuring scoops $\frac{1}{4}, \ldots, 1\ cup$, zip lock bags, scissors, glue, questions sheet, and manipulative cut-outs

- Ribbons and Tiaras- Tiara, ribbon, measuring tape, ruler, scissors, glue, questions sheet, and manipulative cut-outs

- Hershey's Chocolate- Hershey's Chocolate bars, dominos, glue, scissors, questions sheet, and manipulative cut-outs

- Police Academy- brownie pan, plastic knife, scissors, glue, questions sheet, and manipulative cut-outs



- Chex Party Brain Mix- Box of Chex mix cereal, M&Ms, pretzels, animal crackers, measuring scoops $\frac{1}{4}, \ldots, 1\ cup$, zip lock bags, scissors, glue, questions sheet, and manipulative cut-outs

The following consisted of the problems related to each activity (Appendix-C1):

Pumpkins and gourds

- You are decorating your living room table for Thanksgiving dinner and you go to the store to buy some pumpkins and gourds. You have 9 guests coming for dinner and you want a $\frac{1}{4}$ pound pumpkin or gourd for each guest, so you need to purchase $\frac{9}{4}$ pounds of pumpkins and gourds. The grocery bags used to purchase the pumpkins and gourds can each hold $\frac{3}{4}$ of a pound. How many bags will you need to purchase your nine pumpkins and gourds?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the pumpkin and gourds and pumpkin grocery bag manipulatives. Demonstrate your process using the pumpkins and bag paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the common denominator approach.

Coffee Grounds

- From the last class, we have $\frac{6}{3}$ cups of coffee to portion out. If we want to make daily servings in zip lock bags of $\frac{2}{3}$ cup portions, how many bags will we fill?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the coffee grounds and measuring scoop manipulatives. Demonstrate your process using the zip lock bag cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the common denominator approach.

Ribbons and Tiaras

- Suppose you have $2\frac{4}{12}$ feet of ribbon, and you want to cut it into strips $\frac{4}{3}$ feet long. How many strips will you have?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the ribbon and measuring tape manipulatives. Demonstrate your process using the ribbon cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the numerator/denominator multiples approach.

Hershey's Chocolate

- If you have $1\frac{1}{2}$ Hershey chocolate bars, and you want to give s $\frac{1}{4}$ of a chocolate bar to each of your friends, how many friends can you give chocolate to?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the Hershey bar (or dominos) manipulatives. Demonstrate your process using the Hershey Bar cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the invert and multiply approach.

Police Academy

- To celebrate the end of the police academy program, one of the students brings in a pan of brownies. As the future officer drove to the academy he got hungry at a red light and ate $\frac{2}{8}$ of the brownie pan he made. If he wants to give each person $\frac{3}{16}$ of the left over brownies, how many students can have a brownie?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the brownies and plastic knife manipulatives. Demonstrate your process using the brownie cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the invert and multiple approach.

Chex Party Brain Power Mix

- Measure out $\frac{5}{3}$ cups of the Chex Party Brain Mix prepared the previous day. Suppose you want to make $\frac{1}{4}$ cup portions of your party mix in your small zip lock bags. How many portions can you make?

Answer the following questions:

1. What number sentence can be used to solve this problem? Solve this number sentence in using the invert and multiple approach.

3. Develop your own fraction division story problem. What number sentence is used to solve it?

At each station there were various cut-out manipulatives for participants to use as they completed each activity including pumpkins, plastic bags, zip lock bags, ribbons, chocolate bars, and brownies. (For activity manipulative cut-outs see Appendix- C2)

Participants worked in small teams of 3 or 4 for 15 minutes at each station on the above fraction division real world activities.

During session 4, participants took the fraction multiplication and division post-test.

Data and Analysis

We start with descriptive statistical analysis of data collected from the pre- tests of groups CG, SG1, and SG2. Recall that the pre-test consisted of two word problems and two computational problems worth four points each. The following graphs show that all students scored low, typically 1 or 0 out of 4, on each question of the pre-tests, demonstrating that they needed additional instruction to master fractions. The following Figures 2.1-2.12 shows the individual scores on each question.

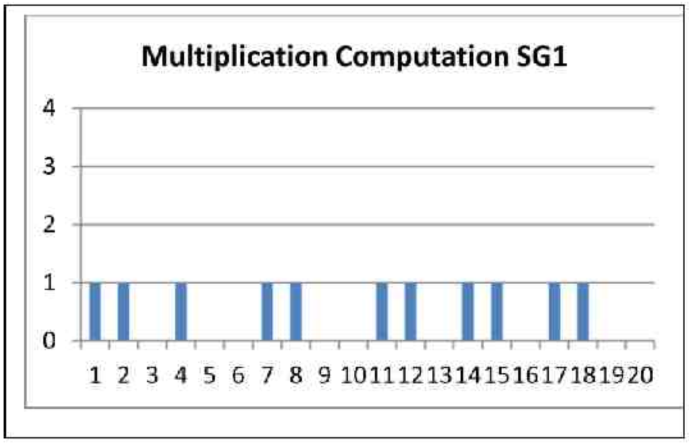


Figure 2.1

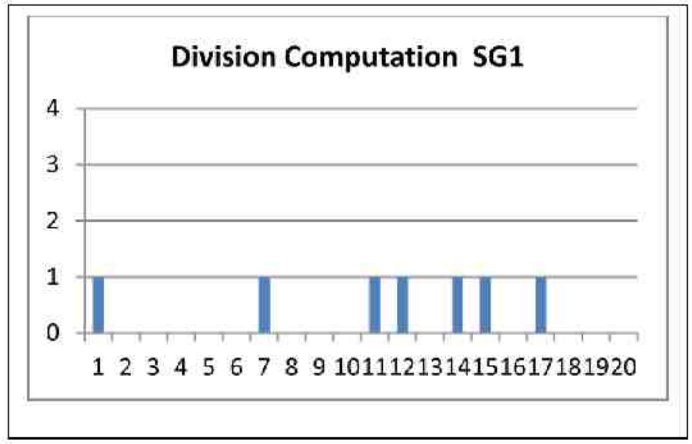
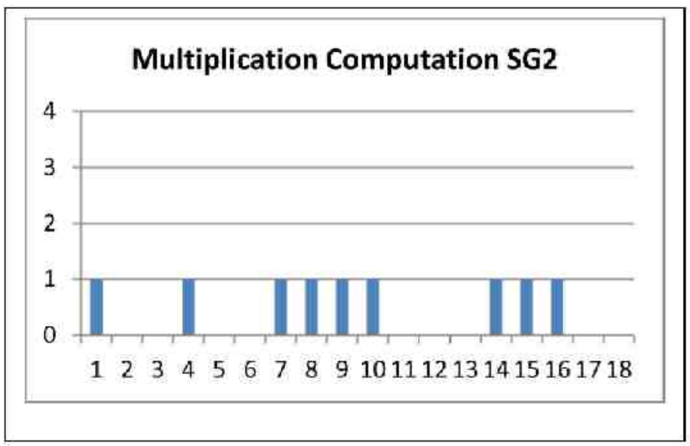


Figure 2.2



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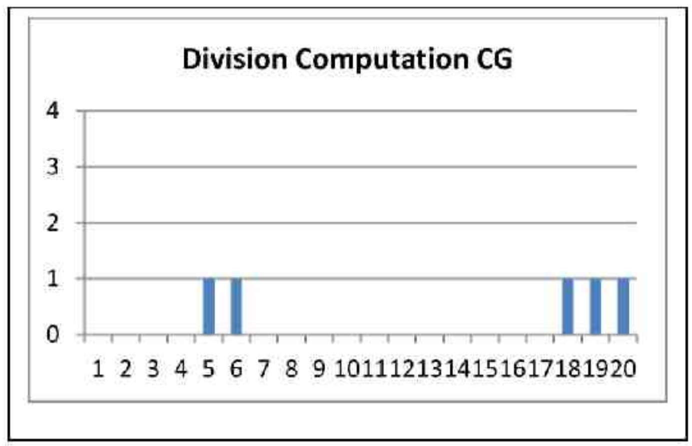
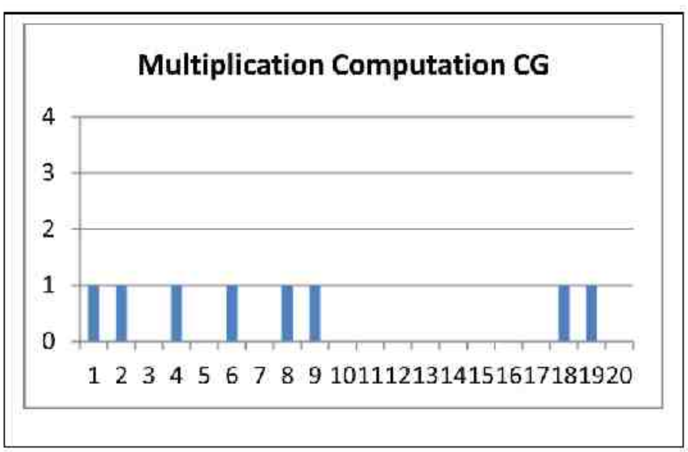


Figure 2.4



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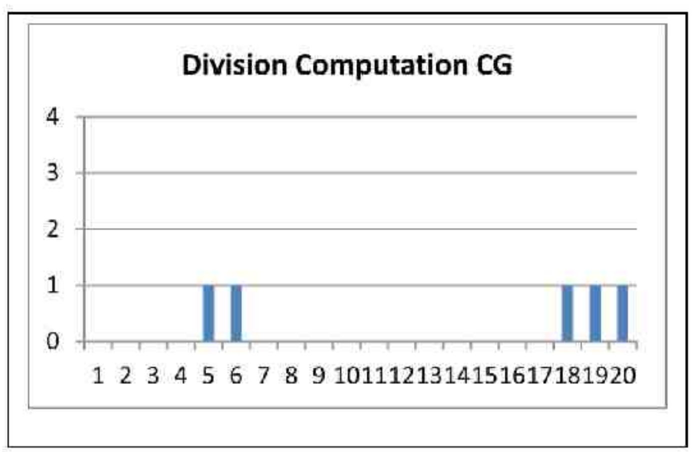
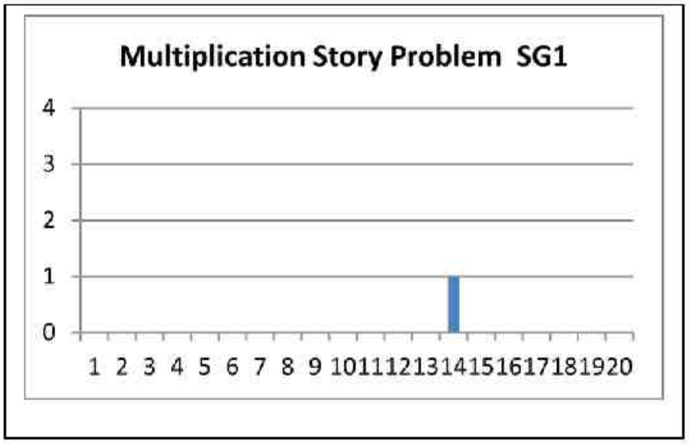


Figure 2.6



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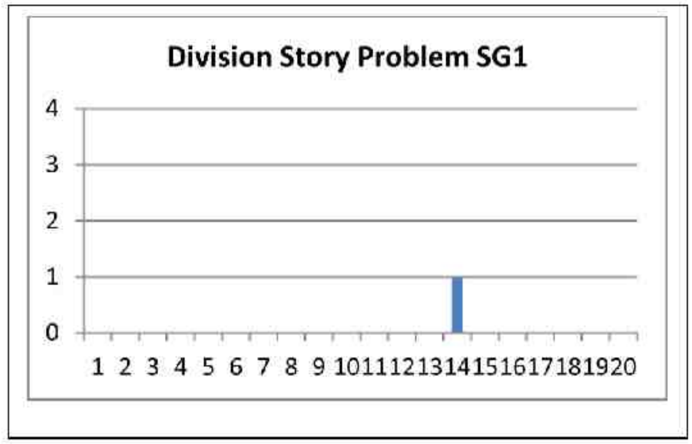

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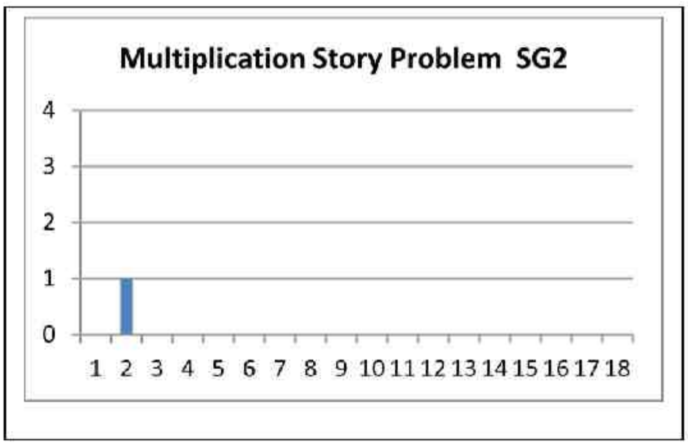

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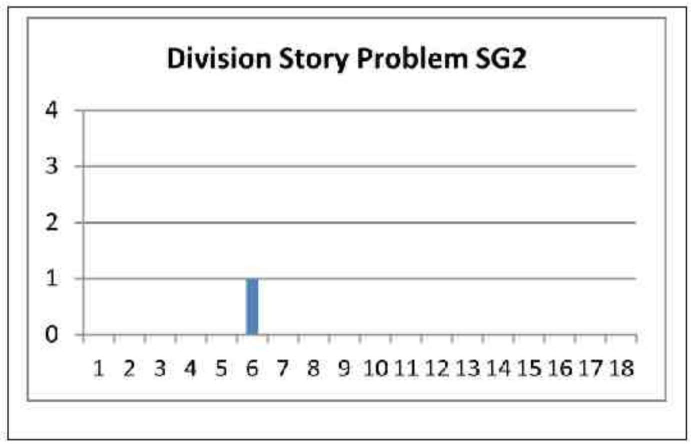

Figure 2.10

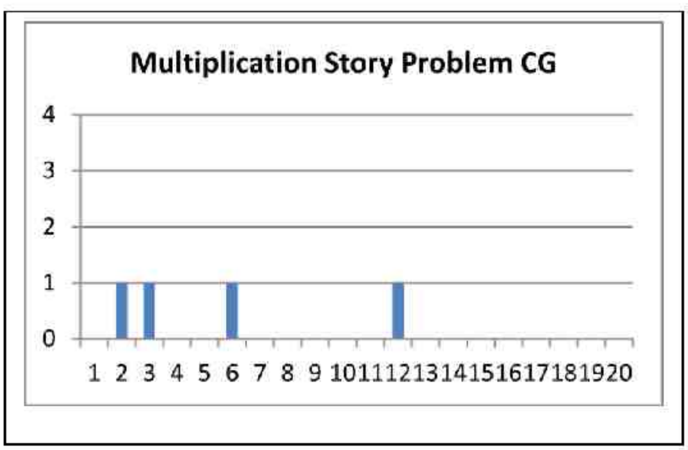

Figure 2.11

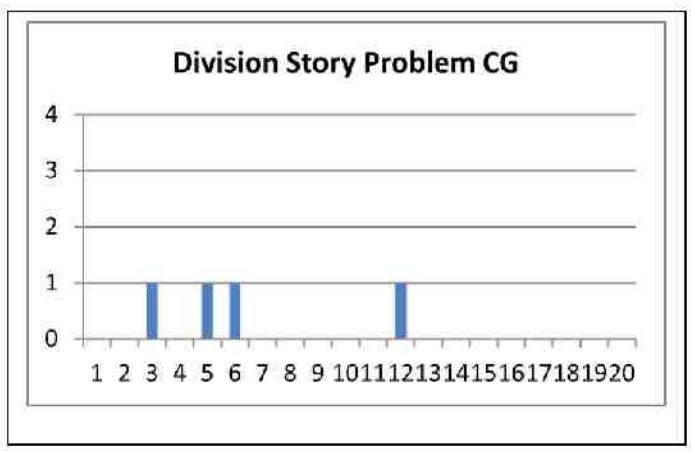

Figure 2.12

The following Figures 2.13, 2.14, and 2.15 show total scores for each student on the pre-test. (Score out of 16 points)

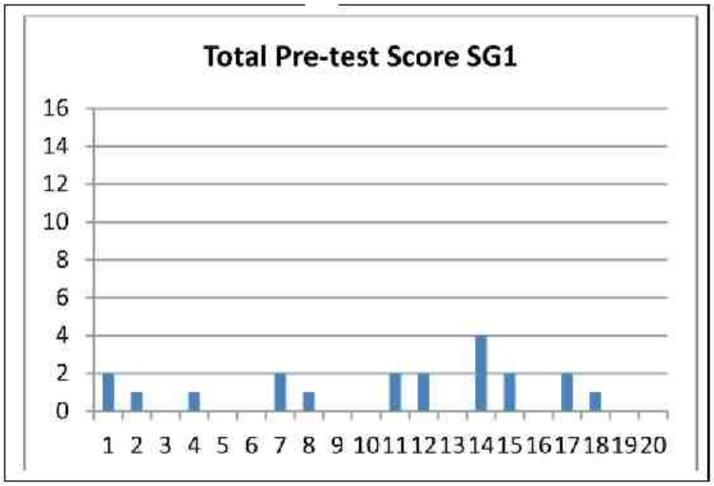

Figure 2.13

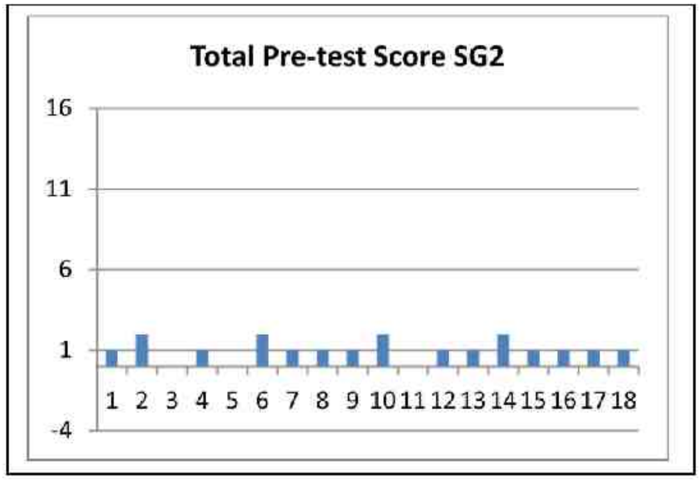

Figure 2.14

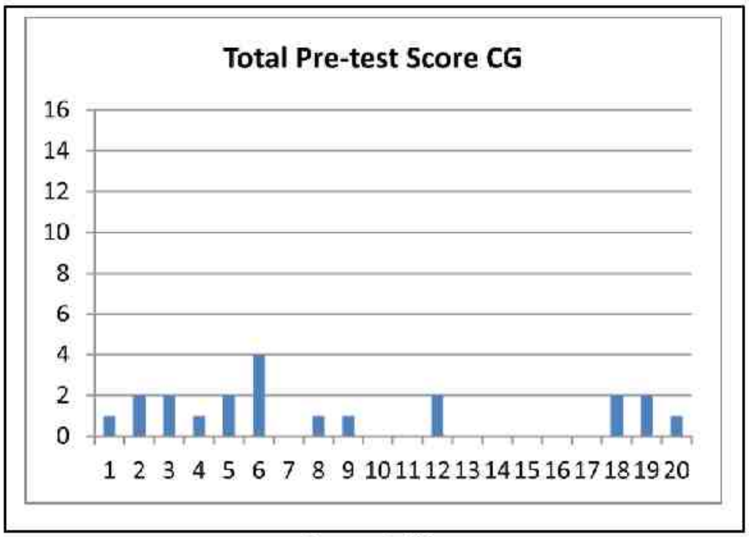

Figure 2.15

The statistics below confirms that all three sections were initially very similar. Figures 3.1- 3.3 show descriptive parameters.

SG1	
Mean	1
Median	1
Mode	0
Standard Dev.	1.12
Variance	1.26
Range	4
Minimum	0
Maximum	4
Sum	20
Count	20

Figure 3.1

SG2	
Mean	1.06
Median	1
Mode	1
Standard Dev.	.64
Variance	.41
Range	2
Minimum	0
Maximum	2
Sum	19
Count	18

Figure 3.2

CG	
Mean	1.05
Median	1
Mode	0
Standard Dev.	1.09
Variance	1.21
Range	4
Minimum	0
Maximum	4
Sum	21
Count	20

Figure 3.3

We performed un-paired t-tests to compare the three sections. The results displayed in Figure 3.4 show no significant difference between the three groups

Un-Paired T-test Results (1-tail)	
SG1, SG2	p=.425
SG1, CG	p= .444
SG2, CG	p= .492

Figure 3.4

Since p-values are close to .5, T-test initial comparisons, with significance level of 100% , show that there were no initial differences between the three sections and they were therefore comparable.

The following consists of descriptive statistical analysis of data collected from the post- tests of CG, SG1, and SG2. Recall that the post-test also consisted of two word problems and two computation problems worth four points each. From the graphs below we can see that the study groups SG1 and SG2 performed much better than the control group. Figures 4.1-4.12 show scores on individual questions (out of 4).

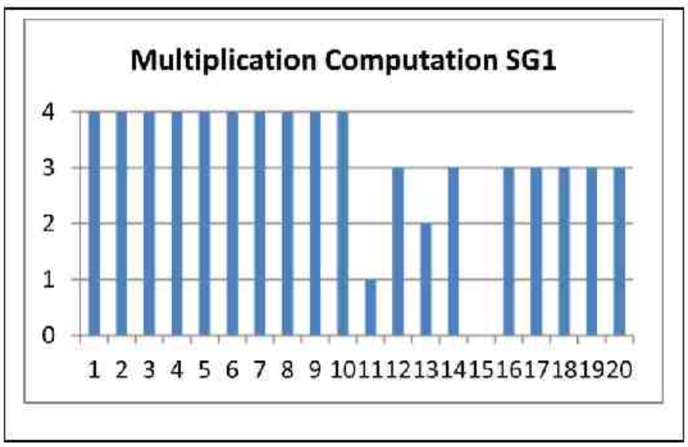

Figure 4.1

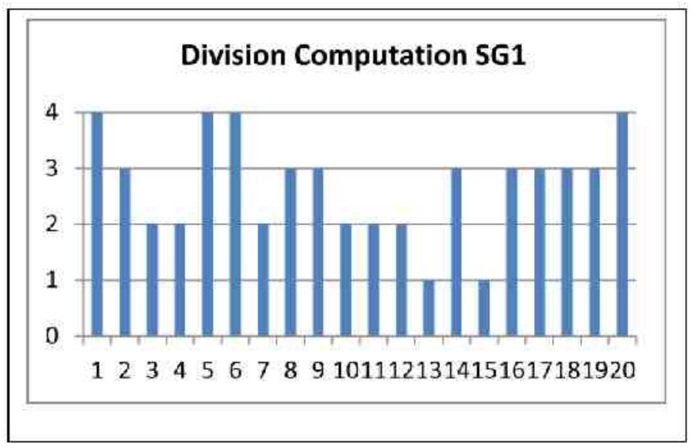

Figure 4.2

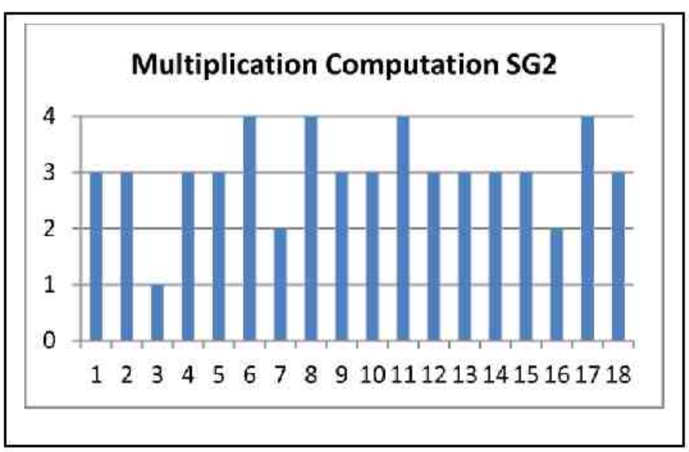

Figure 4.3

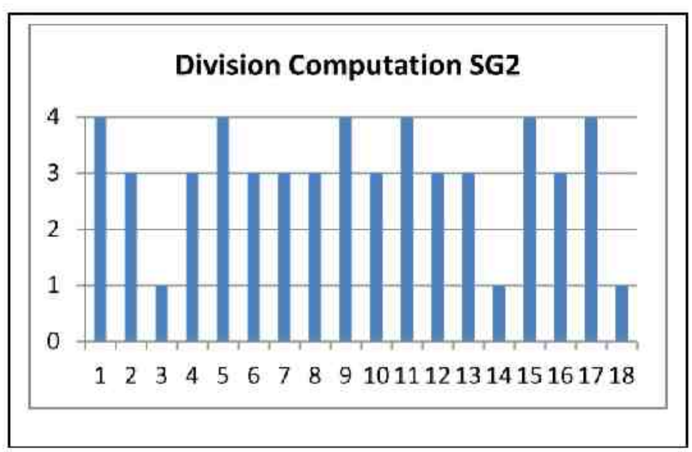

Figure 4.4

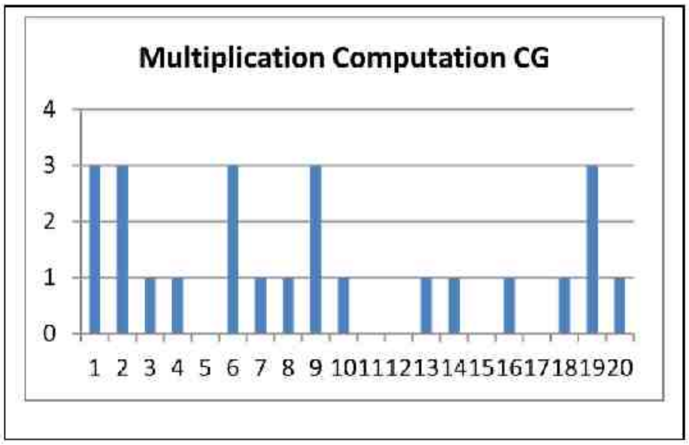

Figure 4.5

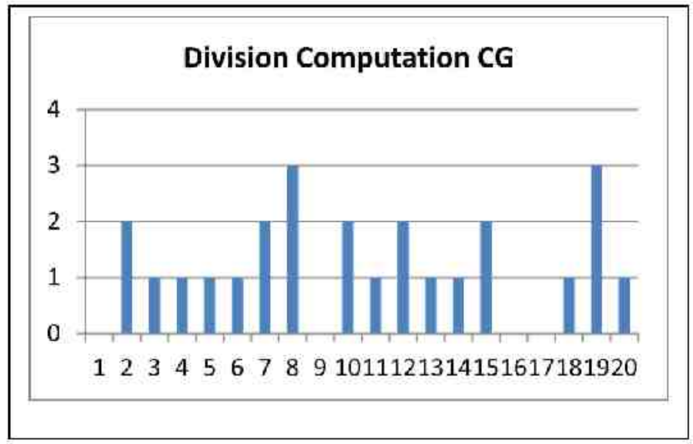

Figure 4.6

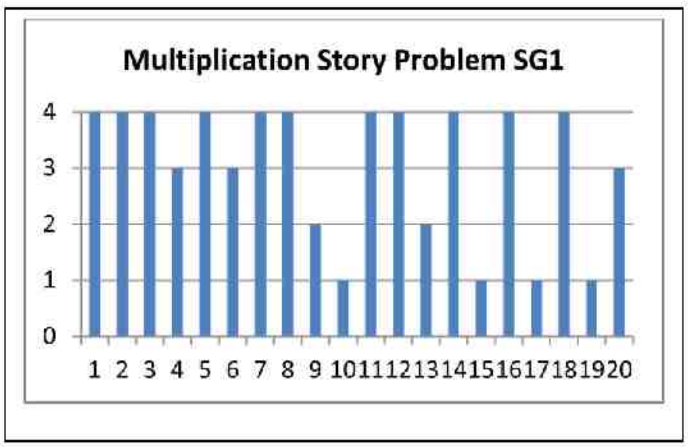

Figure 4.7

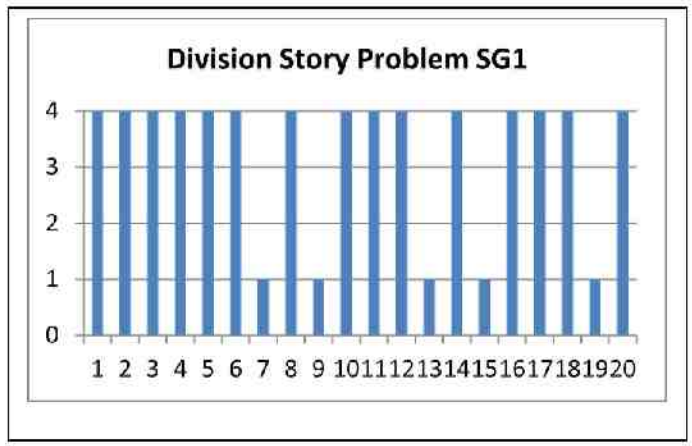

Figure 4.8

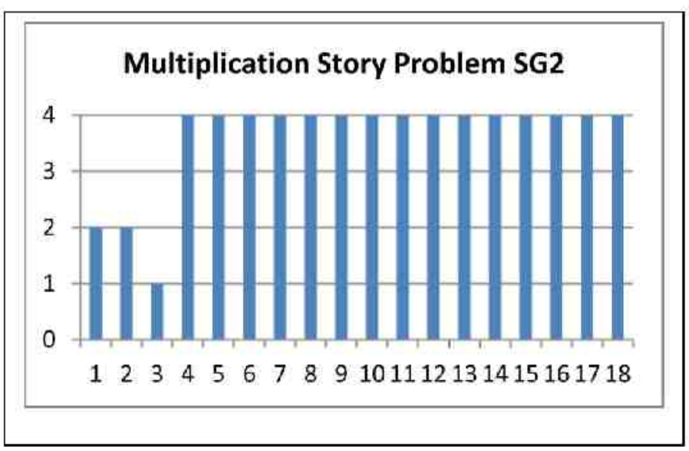

Figure 4.9

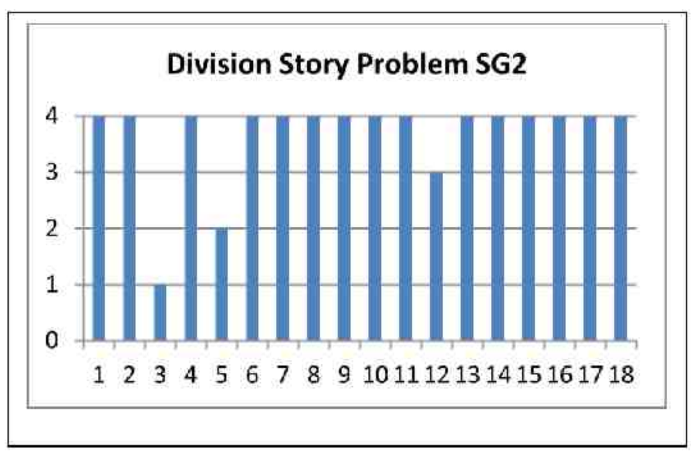

Figure 4.10

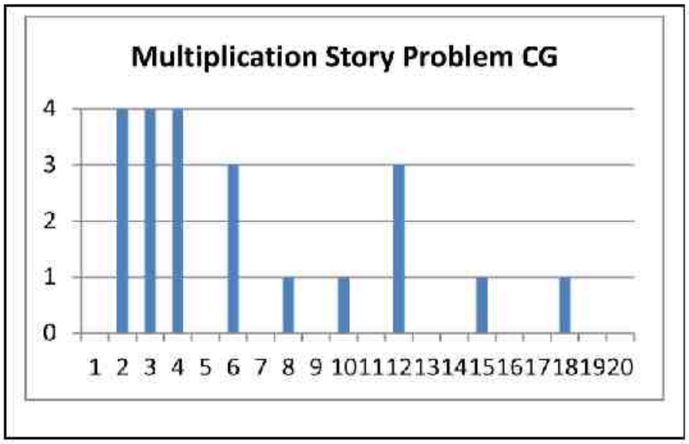

Figure 4.11

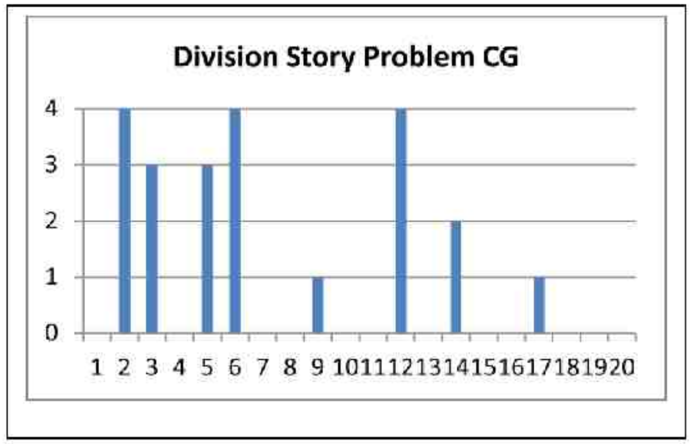

Figure 4.12

Figures 4.13, 4.14, and 4.15 show the post-test results for each section, out of 16.

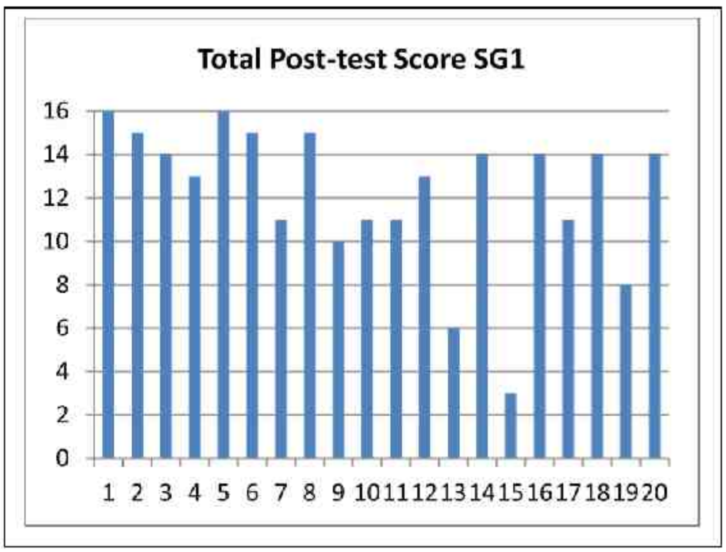

Figure 4.13

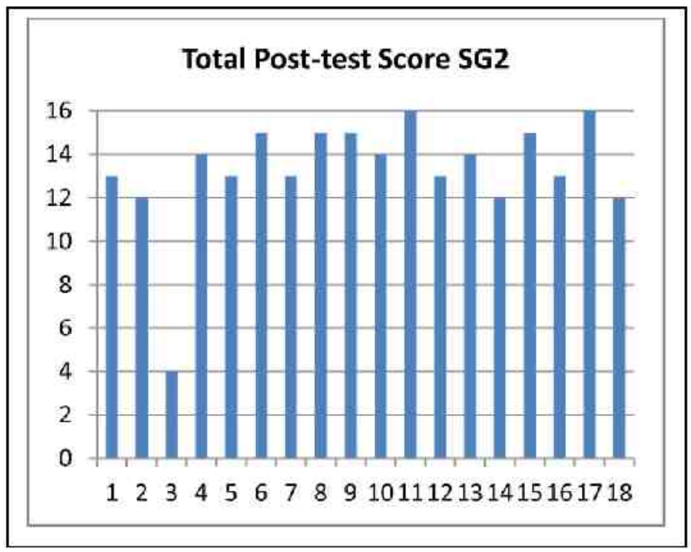

Figure 4.14

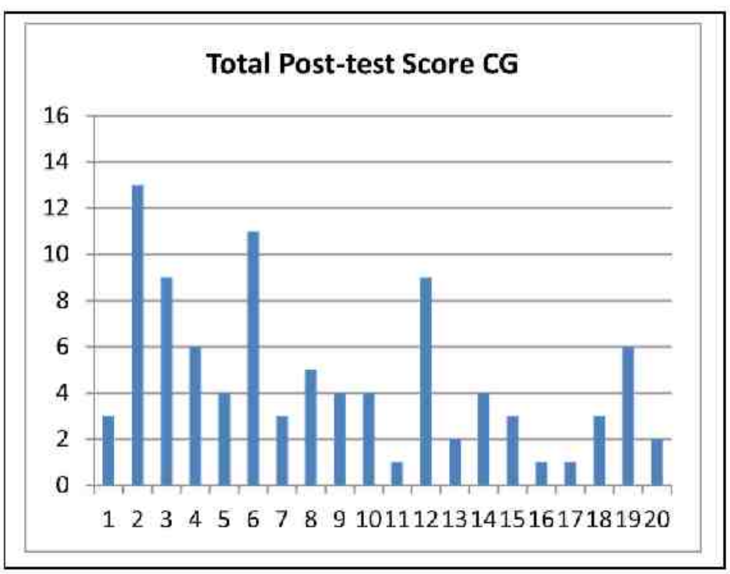

Figure 4.15

The following descriptive statistics shows that group CG's post-test scores were much lower than groups SG1 and SG2, shown in Figures 5.1-5.3.

SG1	
Mean	12.2
Median	13.5
Mode	14
Standard Dev.	3.41
Variance	11.6
Range	13
Minimum	3
Maximum	16
Sum	244
Count	20

Figure 5.1

SG2	
Mean	13.3
Median	13.5
Mode	13
Standard Dev.	2.7
Variance	7.03
Range	12
Minimum	4
Maximum	16
Sum	239
Count	18

Figure 5.2

CG	
Mean	4.7
Median	4
Mode	3
Standard Dev.	34
Variance	11.5
Range	12
Minimum	1
Maximum	13
Sum	94
Count	20

Figure 5.3

We performed paired t-tests to evaluate the improvement for each section from pre-test to post-test. The following chart, Figure 5.4, displays the results:

Paired t-test Results	
SG1	1.06052E-11
SG2	1.37597E-13
CG	3.38205E-06

Figure 5.4

Note that p-values are lower than .01, hence our statements are valid at 99% confidence level. Recall the following hypotheses:

Study Group1:
$H_0: \mu_{pre-testSG1} = \mu_{post-testSG1}$
$H_a: \mu_{pre-testSG1} < \mu_{post-testSG1}$

Study Group2:
$H_0: \mu_{pre-testSG2} = \mu_{post-testSG2}$
$H_a: \mu_{pre-testSG2} < \mu_{post-testSG2}$

Control Group:
$H_0: \mu_{pre-testCG} = \mu_{post-testCG}$
$H_a: \mu_{pre-testCG} < \mu_{post-testCG}$

For group SG1, with a $p - value = 1.06052E - 11 < .01$, at the 99% confidence level we reject the null hypothesis (Ho) and we accept the alternative hypothesis (Ha), and the mean scores for the post-test is greater than the mean scores for the pre-test. Therefore, participants' scores in this section improved significantly through the designed activity.

For group SG2, with a $p - value = 1.37597\text{E} - 13 < .01$, at the 99% confidence level we reject the null hypothesis (Ho) and we accept the alternative hypothesis (Ha), and the mean scores for the post-test is greater than the mean scores for the pre-test. Therefore, participants' scores in this section improved significantly through the designed activity.

For group CG, with a $p - value = 3.38205\text{E} - 06 < .01$, at the 99% confidence level we reject the null hypothesis (Ho) and we accept the alternative hypothesis (Ha), and the mean scores for the post-test is greater than the mean scores for the pre-test. Therefore, participants' scores in this section improved significantly through the traditional method.

In addition, unpaired t-tests were performed to evaluate the level of improvement of participants' post-test scores compared with each section. The following chart, Figure 5.5, displays the results:

Unpaired t-test Results	
SG1 and CG	1.33067E-08
SG2 and CG	1.40724E-10

Figure 5.5

Recall our hypotheses:

Control Group and Study Group 1:

$H_0: \mu_{post-testCG} = \mu_{post-testSG1}$

$H_a: \mu_{post-testCG} < \mu_{post-testSG1}$

Control Group and Study Group 2:

$H_0: \mu_{post-testCG} = \mu_{post-testSG2}$

$H_a: \mu_{post-testCG} < \mu_{post-testSG2}$

For SG1 and CG, with a $p - value = 1.33067\text{E} - 08 < .01$, at the 99% confidence level we reject the null hypothesis (Ho) and we accept the alternative hypothesis (Ha). Therefore, SG1's post-test scores were significantly greater than CG's post-test scores.

For SG2 and CG, with a $p - value = 1.40724\text{E} - 10 < .01$, at the 99% confidence level we reject the null hypothesis (Ho) and we accept the alternative hypothesis (Ha). Therefore, SG2's post-test scores were significantly greater than CG's post-test scores.

Now we will study the influences of qualitative variables collected on the Demographic, RMARS, and Learning Styles Survey and the post-test results. The following chart, Figure 6, consists of the results from comparing the strength and direction of a linear relationship between the random variables using a Pearson correlation. The assigned significance is based on Figure 1:

Correlations Between Survey Questions and Post Test Results		
Survey Question	Correlation Value	Correlation Significance
Learning Styles: Visual	.2329	Weak Positive Correlation
Learning Styles: Auditory	-.1798	Weak Negative Correlation
Learning Styles: Kinesthetic	.5509	Moderate Positive Correlation
RMARS	.3352	Weak Positive Correlation
Demographic: Gender	-.1105	Weak Negative Correlation
Demographic: Age	.09941	Weak Positive Correlation
Demographic: Ethnicity	.00030	Weak Positive Correlation
Demographic: Employment	-.1796	Weak Negative Correlation
Demographic: Semesters on campus	.1127	Weak Positive Correlation
Demographic: Level of education	-.2132	Weak Negative Correlation
Demographic: # of units	.0089	Weak Positive Correlation
Demographic: # of math classes	.1224	Weak Positive Correlation
Demographic: First education class	-.1647	Weak Negative Correlation
Demographic: Goal	.0436	Weak Positive Correlation

Figure 6

Therefore there is a moderate positive correlation between students who are identified as Kinesthetic Learners and post-test results. Since other correlations are weak, we conclude that other variables were irrelevant in this study.

Results

In summary, pre-test results demonstrate that students in all sections were comparable in terms of their ability to create fraction multiplication and division story problems and performing fraction computations. Post-test results demonstrate that students in all sections improved significantly following the instruction they received on fraction multiplication and division. However, further statistical analysis demonstrates that pre-service teachers that learn fraction division and multiplication using concrete models of real life applications improved significantly more than students taught with a traditional method. Hence, we claim that there is sufficient evidence that the designed activities produce better results than the traditional method.

Note that this included a specific group of individuals, students working toward becoming elementary school teachers. Although participants were placed in sections randomly, there were various factors that could have created bias in the results. The following factors that may have influenced the validity of our results include:

- Previous math courses taken, students major
- Students confidence level in math
- Students English Language skills

- Physical state on testing days
- Previous experience with fraction division and multiplication

In addition, the study groups were taught by a different instructor then the control group so teaching style and personality might have influenced the results. There is a free tutoring center on campus at CSU Channel Islands, and some students might have used private tutors; hence there is also the possibility that the results were affected by this and other outside resources. Finally, each section studies was small; so it is unclear whether real world application stations would work in large sections.

Conclusions

We studied specific hands-on real world applications as a special teaching technique to evaluate their influence on pre-service teachers' ability to create fraction division and multiplication story problems. The data shows that pre-service teachers' performance improved significantly after participating in the designed activities. This research highlights the need for implementing word problem writing into the pre-service teacher curriculum, as well as demonstrates the need to improve pre-service teachers' conceptual understanding of fraction multiplication and division. The results from this study suggest that if pre-service teachers have the opportunity to explore math concepts using hands-on explorations their ability to create story problems will improve.

Since there are various factors that may have influenced our results, this experiment should be repeated under different circumstances with different instructors and possibly a different venue.

Future Plans

There are various possible extensions of this study. Other concepts can be taught using hands-on real world activities and pre-service teachers' ability to create story problems of these concepts can be measured. It would be also interesting to conduct a study where students' residual learning of creating fraction story problems is measured at the end of the semester as opposed to directly after instruction and activities. It would be valuable to investigate whether students participating in a class designed around word problem writing would increase their conceptual understanding of concepts including fraction operations. It is also worth knowing whether an emphasis on the transition between whole-number and non-whole number operations would improve pre-service teachers' ability to create word problems. Finally, a study should be conducted with a larger group of students to evaluate the effectiveness of this method in bigger classrooms.

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Learning Styles Survey				
Question	**Most Like Me**			**Least Like Me**
1. Making things for my studies helps me to remember what I have learned.	4	3	2	1
2. I can write about most of the things I know better than I can tell about them.	4	3	2	1
3. When I really want to understand what I have read, I read it softly to myself.	4	3	2	1
4. I get more done when I work alone.	4	3	2	1
5. I remember what I have read better than what I have heard.	4	3	2	1
6. When I answer questions, I can say the answer better than I can write it.	4	3	2	1
7. When I do math problems in my head, I say the numbers to myself.	4	3	2	1
8. I enjoy joining in on class discussions.	4	3	2	1
9. I understand a math problem that is written down better than one that I hear.	4	3	2	1
10. I do better when I can write the answer instead of having to say it.	4	3	2	1
11. I understand spoken directions better than written ones.	4	3	2	1
12. I like to work by myself.	4	3	2	1
13. I would rather read a story than listen to it read.	4	3	2	1
14. I would rather show and explain how something works than write about how it works.	4	3	2	1
15. If someone tells me three numbers to add, I can usually get the right answer without writing them down.	4	3	2	1
16. I prefer to work with a group when there is work to be done.	4	3	2	1
17. A graph or chart of numbers is easier for me to understand than hearing the numbers said.	4	3	2	1
18. Writing a spelling word several times helps me to remember it better.	4	3	2	1
19. I learn better if someone reads a book to me than if I read it silently to myself.	4	3	2	1
20. I learn best when I study alone.	4	3	2	1
21. When I have a choice between reading and listening, I usually read.	4	3	2	1
22. I would rather tell a story than write it.	4	3	2	1
23. Saying the multiplication tables over and over helps me remember them better than writing them over and over.	4	3	2	1
24. I do my best work in a group.	4	3	2	1
25. I understand a math problem that is written down better than one I hear.	4	3	2	1
26. In a group project, I would rather make a chart or poster than gather the information to put on it.	4	3	2	1
27. Written assignments are easy for me to follow.	4	3	2	1
28. I remember more of what I learn if I learn it alone.	4	3	2	1
29. I do well in classes where most of the information has to be read.	4	3	2	1
30. I would enjoy giving an oral report to the class.	4	3	2	1
31. I learn math better from spoken explanations than written ones.	4	3	2	1
32. If I have to decide something, I ask other people for their opinions.	4	3	2	1
33. Written math problems are easier for me to do than oral ones.	4	3	2	1
34. I like to make things with my hands.	4	3	2	1
35. I don't mind doing written assignments.	4	3	2	1
36. I remember things I hear better than things I read.	4	3	2	1
37. I learn better by reading than by listening.	4	3	2	1
38. It is easy for me to tell about the things I know.	4	3	2	1
39. I make it easier when I say the numbers of a problem to myself as I work it out.	4	3	2	1
40. If I understand a problem, I like to help someone else understand it , too.	4	3	2	1
41. Seeing a number makes more sense to me than hearing a number.	4	3	2	1
42. I understand what I have learned better when I am involved in making something for the subject.	4	3	2	1
43. The things I write on paper sound better than when I say them.	4	3	2	1
44. I find it easier to remember what I have heard than what I have read.	4	3	2	1
45. It is fun to learn with classmates, but it is hard to study with them.	4	3	2	1

Appendix A1

Learning Styles Scoring Rubric

Visual Language 5 -- ____ 13 -- ____ 21 -- ____ 29 -- ____ 37 -- ____ Total ____ x 2 =___(Score)	**Individual Learner** 4 -- ____ 12 -- ____ 20 -- ____ 28 -- ____ 45 -- ____ Total ____ x 2 =___(Score)	**Auditory Numerical** 7 -- ____ 15 -- ____ 23 -- ____ 31 -- ____ 39 -- ____ Total ____ x 2 =___(Score)
Visual Numerical 9 -- ____ 17 -- ____ 25 -- ____ 33 -- ____ 41 -- ____ Total ____ x 2 =___(Score)	**Group Learner** 8 -- ____ 16 -- ____ 24 -- ____ 32 -- ____ 40 -- ____ Total ____ x 2 =___(Score)	**Kinesthetic -Tactile** 1 -- ____ 18 -- ____ 26 -- ____ 34 -- ____ 42 -- ____ Total ____ x 2 + ___(Score)
Auditory Language 3 -- ____ 11 -- ____ 19 -- ____ 36 -- ____ 44 -- ____ Total ____ x 2 =___(Score)	**Expressiveness - Oral** 6 -- ____ 14 -- ____ 22 -- ____ 30 -- ____ 38 -- ____ Total ____ x 2 =___(Score)	**Expressiveness - Written** 2 -- ____ 10 -- ____ 27 -- ____ 35 -- ____ 43 -- ____ Total ____ x 2 =___(Score)
	Score: 33-40 = Major Learning Style 20-32 = Minor Learning Style 5 -20 = Learning Style N/A	

Appendix A2

Demographic Survey

1. What is your gender?

Male Female

2. What is your age?

18 19 20 21-24 25-29 30-39 40-49 50 or above

3. What is your ethnic origin?

Native American (including Alaskan Native)

Asian (including Oriental, Pacific Islander and Filipino)

African American

Hispanic

White

Other Race

4. What is your employment status?

Not employed Employed part-time Employed full-time

Self-employed More than one job (how many) _____

5. Is this your first semester enrolled at CSUCI? Yes No

6. What is the highest level of education that you have completed?

Some high school High school graduate Some college

Associate's Degree Bachelor's Degree

7. How many units are you taking (including this course) this semester?

1-3 4-6 7-9 10-12 13 or more

8. How many math courses are you taking this semester?

1 2 3 or more

9. Is this your first math education course?

Yes No

10. What is your goal in attending college?

Seeking AA Degree Working Toward Bachelor's Degree Personal Growth

Multiple Subject Teaching Credential Single Subject Teaching Credential

Appendix A3

RMARS Survey

Rate how anxious the following situations make you feel on a scale from 1 to 5, where 1 represents "not at all" anxious, and 5 represents "very much" anxious

Question	Rating of 1-5
1. Studying for a math test .	
2. Taking the mathematics section of college entrance exam	
3. Taking an exam (quiz) in a math course	
4. Taking an exam (final) in a math course	
5. Picking up math textbook to working on a homework assignment	
6. Being given homework assignments of many difficult problems that are due the next class meeting	
7. Thinking about an upcoming math 1 week before	
8. Thinking about an upcoming math test 1 day before	
9. Thinking about an upcoming math test 1 hour before	
10. Realizing you have to take a certain number of math classes to fulfill requirements in your major	
11. Picking up math textbook to begin a difficult reading assignment	
12. Receiving your final math grade in the mail	
13. Opening a math or stat book and seeing a page full of problems	
14. Getting ready to study for a math test	
15. Being give a "pop" quiz in a math class	
16. Reading a cash register receipt after your purchase	
17. Being given a set of numerical problems involving addition to solve on paper	
18. Being given a set of subtraction problems to solve	
19. Being given a set of multiplication problems to solve	
20. Being given a set of division problems to solve	
21. Buying a math textbook	
22. Watching a teacher work on an algebraic equation on the blackboard	
23. Signing up for a math course	
24. Listening to another student explain a math formula	
25. Walking into a math course	
Total________	

Appendix A3

RMARS Scoring Rubric

Range of Total Score	Level of math Anxiety
25-45	Not anxious
45-65	Slightly Anxious
65-85	Moderately Anxious
85-105	Anxious
105-125	Very Anxious

Appendix B1

Pre-Test

1. **Demonstrate these division problems a, b, and c using three different methods of fraction division.**

   a. $\frac{12}{13} \div \frac{6}{13} =$

   b. $\frac{21}{40} \div \frac{7}{8} =$

   c. $\frac{6}{15} \div \frac{4}{7} =$

   d. **Explain why and how the invert and multiply method works.**

____

____

____

____

2. **Demonstrate these multiplication problems a,b, and c using three different models/ methods of fraction multiplication. Identify the method/ model you chose.**

   a. $3 \times \frac{1}{4} =$

   b. $\frac{1}{2} \times 6 =$

   c. $\frac{1}{3} \times \frac{5}{7} =$

   d. **Use pattern blocks to show how to this multiplication problem can be computed:** $\frac{1}{3} \times 2\frac{1}{2}$

3. **Imagine that you are teaching multiplication with fractions. Create a story problem that can be solved with the number sentence** $1\frac{3}{4} \times \frac{2}{3} =$

4. **Imagine that you are teaching division with fractions. Create a story problem that can be solved with the number sentence** $1\frac{1}{3} \div \frac{1}{4} =$

Appendix B2

Post Test

1. **Demonstrate these division problems a, b, and c using three different methods of fraction division.**

    a. $\frac{15}{17} \div \frac{5}{17} =$

    b. $\frac{27}{45} \div \frac{9}{5} =$

    c. $\frac{7}{13} \div \frac{3}{8} =$

    a. **Explain why and how the invert and multiply method works.**

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2. **Demonstrate these multiplication problems a,b, and c using three different models/ methods of fraction multiplication. Identify the method/ model you chose.**

    d. $4 \times \frac{1}{2} =$

    e. $\frac{1}{3} \times 6 =$

    f. $\frac{1}{4} \times \frac{2}{5} =$

    a. **Use pattern blocks to show how to this multiplication problem can be computed:** $\frac{2}{3} \times 2\frac{1}{3}$

3. **Imagine that you are teaching multiplication with fractions. Create a story problem that can be solved with the number sentence** $1\frac{1}{4} \times \frac{1}{2} =$

4. **Imagine that you are teaching division with fractions. Create a story problem that can be solved with the number sentence** $2\frac{1}{2} \div \frac{3}{4} =$

Appendix B3

	Multiplication Word Problem	Division Word Problem
Failing (Score=0) *Sample :* $1\frac{2}{3} \times 4 =?$ Catherine $1\frac{2}{3}$ oranges. She wants to share it among her friends. How much oranges does each friend get?	(a)An individual does not write a word problem; or (b)An individual ignores basic mathematical concepts and methods; his or her word problem is not solvable by using the given multiplication sentence.	(a)An individual does not write a word problem; or (b)An individual ignores basic mathematical concepts and methods; his or her word problem is not solvable by using the given division sentence.
Poor (Score=1) *Sample :* $\frac{1}{2} \times \frac{1}{3} =?$ Joe made brownies and split the pan in $\frac{1}{2}$ for his sister. She ate $\frac{1}{3}$ of the $\frac{1}{2}$ of her part. If you took the total pan, how much did she eat?	(a)An individual demonstrates a basic understanding of mathematical concepts and methods, but does not demonstrate the ability to unify and relate central mathematical concepts and methods. (b)The word problem is solvable by using a multiplication sentence, but does not match with the given numerical value.	(a)An individual demonstrates a basic understanding of mathematical concepts and methods, but does not demonstrate the ability to unify and relate central mathematical concepts and methods. (b)The word problem is solvable by using a division sentence, but does not match with the given numerical value.
Weak (Score=2) *Sample :* $1\frac{2}{3} \times 4 =?$ Four children have each got $1\frac{2}{3}$ of a sheet cake. How many cakes do they have all together?	(a)An individual demonstrates an ability to unify and relate central mathematical concepts and methods, but does not demonstrate a contextual understanding and application of mathematical concepts and methods. (b)The word problem matches the given multiplication sentence, but includes a logical error or misleading context.	(a)An individual demonstrates an ability to unify and relate central mathematical concepts and methods, but does not demonstrate a contextual understanding and application of mathematical concepts and methods. (b)The word problem matches the given division sentence, but includes a logical error or misleading context.
Fair (Score=3) *Sample :* $\frac{1}{2} \times \frac{1}{3} =?$ Mom made a cookie. Sally frosted $\frac{1}{3}$ of the cookie. On the frosted part, she put sprinkles on $\frac{1}{2}$. How much has sprinkles?	(a)An individual demonstrates a contextual understanding and application of mathematical concepts and methods, but does not demonstrate it in a clear or coherent manner sufficiently. (b)The word problem is logically and contextually correct, but does not show sufficient clarity or coherence.	(a)An individual demonstrates a contextual understanding and application of mathematical concepts and methods, but does not demonstrate it in a clear or coherent manner sufficiently. (b)The word problem is logically and contextually correct, but does not show sufficient clarity or coherence.
Good (Score=1.00) *Sample :* $1\frac{2}{3} \times 4 =?$ Mary had $1\frac{2}{3}$ pieces of pie. Joe had 4 times as many pieces. How many pieces of pie does Joe have?	(a)An individual demonstrates a contextual understanding and application of mathematical concepts and methods in a clear and coherent manner. (b)The word problem is logically and contextually correct, and clearly and coherently described.	(a)An individual demonstrates a contextual understanding and application of mathematical concepts and methods in a clear and coherent manner. (b)The word problem is logically and contextually correct, and clearly and coherently described.

Appendix B3

Points	Multiplication Computation	Division Computation
0	Student is unable to correctly compute fraction multiplication computation and cannot demonstrates the multiplication problem using patter blocks	Student is unable to compute fraction division computation or explain why the invert and multiply method works
1	Student correctly computes fraction multiplication computations using 1 approach or correctly demonstrates the multiplication problem using patter blocks.	Student correctly computes fraction division computations using one approach or explains why the invert and multiply method works
2	Student correctly computes fraction multiplication computations using 2 approaches or computes using 1 model correctly and student correctly demonstrates the multiplication problem using patter blocks.	Student correctly computes fraction division computations using two different approaches or 1 approach and explains why the invert and multiply method works
3	Student correctly computes fraction multiplication computations using the number line approach, measurement approach, and the area model or computes 2 models and student correctly demonstrates the multiplication problem using patter blocks.	Student correctly computes fraction division computations using the common denominator approach, multiplies approach, and invert and multiply approach or computes 2 approaches and explains why the invert and multiply method works
4	Student correctly computes fraction multiplication computations using the number line approach, measurement approach, and the area model and student correctly demonstrates the multiplication problem using patter blocks.	Student correctly computes fraction division computations using the common denominator approach, multiplies approach, and invert and multiply approach and student correctly explains why the invert and multiply method works.

Appendix C1

Pumpkins and gourds

- You need $\frac{9}{4}$ pounds of pumpkins and gourds to make a delicious pumpkin (and gourds) pie. If the pumpkins and gourds cost $2 per pound. How much money will you spend buying the pumpkins and gourds?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the pumpkin and gourds and dollar bill manipulatives. Demonstrate your process using the pumpkins and dollar bill paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the repeated addition approach.

Appendix C1

Coffee Grounds

- You purchase at the store a 6 cup bag of coffee. You want to portion out $\frac{1}{3}$ of the coffee you bought into individual servings. How much coffee will you be portioning out?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the "coffee grounds" and measuring cup manipulatives. Demonstrate your process using the zip lock bag cut out, glue, and scissors.
3. Solve the number sentence in part 1 using the repeated addition approach.

Appendix C1

Ribbons and Tiaras

- You are creating princess tiaras for a princess themed birthday party and each tiara requires 10 feet of ribbon. This ribbon is then cut into strips and tied onto the tiara. If each strip is $\frac{1}{5}$ of the total required ribbon length, how long is each strip?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the ribbon and measuring tape manipulatives. Demonstrate your process using ribbon paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the measurement approach.

Appendix C1

Hershey Bars

- You have $\frac{3}{4}$ of a candy bar left over from last night's trip to the movies. If you want to eat $\frac{1}{3}$ of your leftovers, how much of the candy bar will you eat?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the Hershey Bar (or dominos) manipulatives. Demonstrate your process using the Hershey Bar paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using an area model.

Appendix C1

Police Academy

- $\frac{1}{3}$ of a class of police academy students will be out of class and participating in physical fitness course. If there are 30 students in the class, how many students are participating in the course?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the police officer figure manipulatives. Demonstrate your process using the police officer figure paper cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the measurement approach.

Appendix C1

Chex Brain Power Party Mix

- The following consists of ingredients for Chex Brain Party Mix.

  Serving Size: 9 Hungry Students

  Ingredients:

    - $4\frac{1}{2}$cups Chex cereal (any variety)
    - 1 cup Animal Crackers
    - $2\frac{1}{4}$ cups pretzels
    - $\frac{3}{4}$ cup peanut M&M's

Answer the following questions:

3. Create your Chex party mix for 3 students instead of 9. Solve the M&M portioning fraction multiplication problem using an area model.
4. Develop your own fraction multiplication story problem. What number sentence is used to solve it?

Appendix C1

Pumpkins and gourds

- You are decorating your living room table for Thanksgiving dinner and you go to the store to buy some pumpkins and gourds. You have 9 guests coming for dinner and you want a $\frac{1}{4}$ pound pumpkin or gourd for each guest, so you need to purchase $\frac{9}{4}$ pounds of pumpkins and gourds. The grocery bags used to purchase the pumpkins and gourds can each hold $\frac{3}{4}$ of a pound. How many bags will you need to purchase your nine pumpkins and gourds?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the pumpkin and gourds and pumpkin grocery bag manipulatives.

Demonstrate your process using the pumpkins and bag paper cut-outs, glue, and scissors.

3. Solve the number sentence in part 1 using the common denominator approach.

Appendix C1

Coffee Grounds

- From the last class, we have $\frac{6}{3}$ cups of coffee to portion out. If we want to make daily servings in zip lock bags of $\frac{2}{3}$ cup portions, how many bags will we fill?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the coffee grounds and measuring scoop manipulatives. Demonstrate your process using the zip lock bag cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the common denominator approach.

Appendix C1

Ribbons and Tiaras

- Suppose you have $2\frac{4}{12}$ feet of ribbon, and you want to cut it into strips $\frac{4}{3}$ feet long. How many strips will you have?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the ribbon and measuring tape manipulatives. Demonstrate your process using the ribbon cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the numerator/denominator multiples approach.

Appendix C1

Hershey's Chocolate

- If you have $1\frac{1}{2}$ Hershey chocolate bars, and you want to give $\frac{1}{4}$ of a chocolate bar to each of your friends, how many friends can you give chocolate to?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the Hershey bar (or dominos) manipulatives. Demonstrate your process using the Hershey Bar cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the invert and multiply approach.

Appendix C1

Police Academy

- To celebrate the end of the police academy program, one of the students brings in a pan of brownies. As the future officer drove to the academy he got hungry at a red light and ate $\frac{2}{8}$ of the brownie pan he made. If he wants to give each person $\frac{3}{16}$ of the left over brownies, how many students can have a brownie?

Answer the following questions:

1. What number sentence can be used to solve this problem?
2. Solve the problem using the brownies and plastic knife manipulatives. Demonstrate your process using the brownie cut-outs, glue, and scissors.
3. Solve the number sentence in part 1 using the invert and multiple approach.

Appendix C1

Chex Party Brain Power Mix

- Measure out $\frac{5}{3}$ cups of the Chex Party Brain Mix prepared the previous day. Suppose you want to make $\frac{1}{4}$ cup portions of your party mix in your small zip lock bags. How many portions can you make?

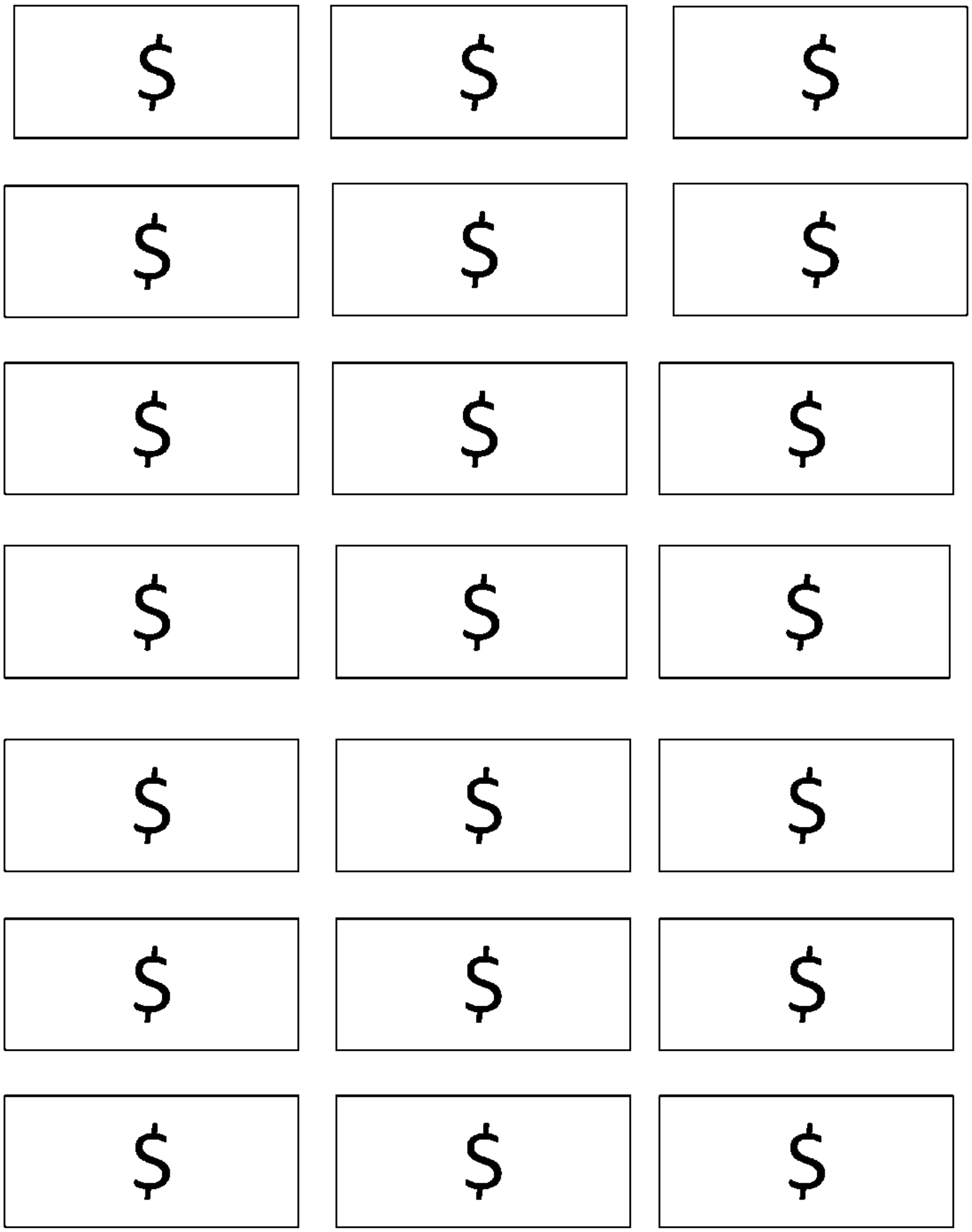
Answer the following questions:

1. What number sentence can be used to solve this problem? Solve this number sentence in using the invert and multiple approach.

3. Develop your own fraction division story problem. What number sentence is used to solve it?

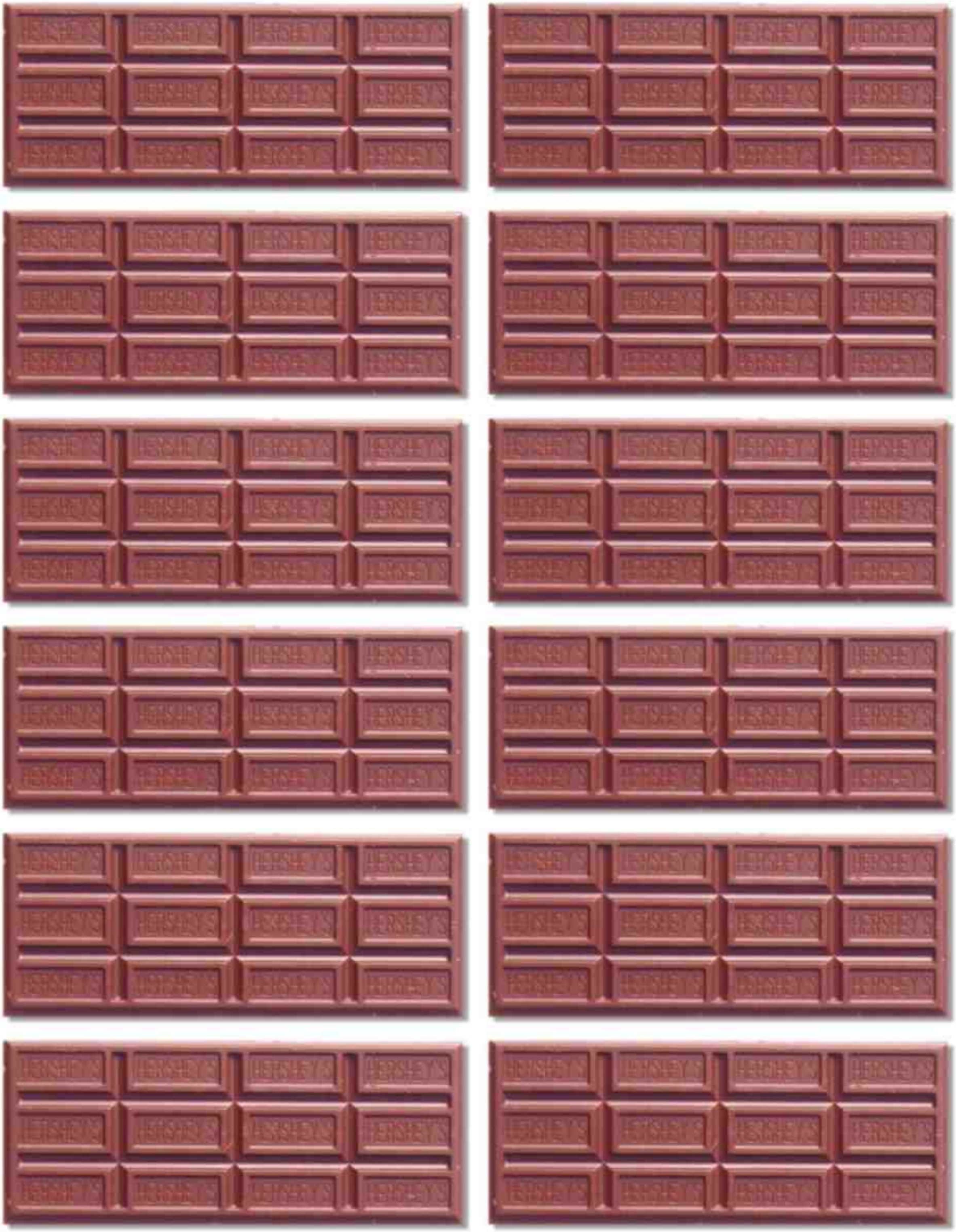
Appendix C2

Appendix C1



Appendix C2

Appendix C2



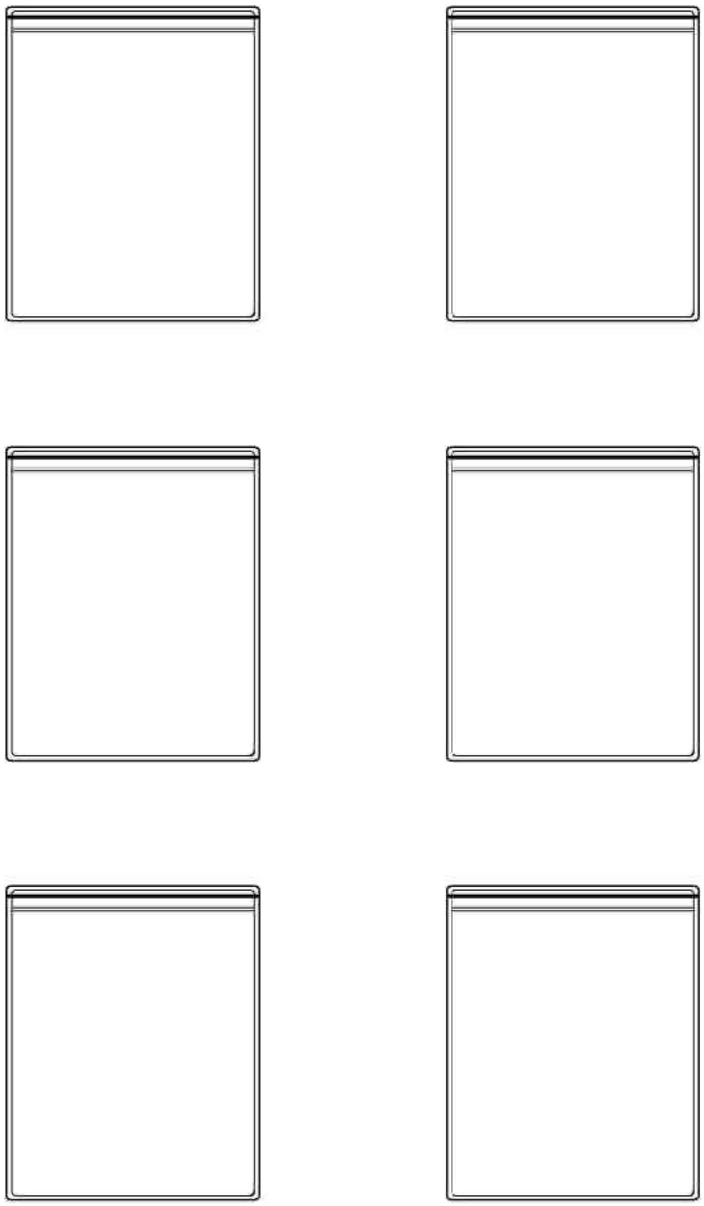
Appendix C2








Appendix C2



Appendix C2


4 CUPS
32oz
1 Quart
3 CUPS
28oz
24oz
3 CUPS
20oz
2 CUPS
16oz
2 CUPS
12oz
1 CUP
8oz
1 CUP
4oz


4 CUPS
32oz
1 Quart
3 CUPS
28oz
24oz
3 CUPS
20oz
2 CUPS
16oz
2 CUPS
12oz
1 CUP
8oz
1 CUP
4oz


4 CUPS
32oz
1 Quart
3 CUPS
28oz
24oz
3 CUPS
20oz
2 CUPS
16oz
2 CUPS
12oz
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8oz
1 CUP
4oz


4 CUPS
32oz
1 Quart
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28oz
24oz
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8oz
1 CUP
4oz


4 CUPS
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4 CUPS
32oz
1 Quart
3 CUPS
28oz
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3 CUPS
20oz
2 CUPS
16oz
2 CUPS
12oz
1 CUP
8oz
1 CUP
4oz

Appendix C2