Effects of Incorporation

Mayan and Modular Mathematics

into Algebra Curriculum

By

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A Thesis Presented to The Faculty of the Mathematics Program
California State University Channel Islands
2012

In Partial Fulfillment
Of the Requirements of the Degree
Master of Science

Signature page for the Masters in Mathematics Thesis of Cindy Enrique

MS THESIS BY CINDY ENRIQUE
APPROVED FOR THE MATHEMATICS PROGRAM

12/21/12

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12-21-12

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Effects of incorporating Mayan and modular mathematics into algebra Curriculum

Title of Item

Maya Modular Mathematics

3 to 5 keywords or phrases to describe the item

Cindy Enrique

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Cindy Enrique 1-20-13

Author(s) Signature Date

ACKNOWLEDGEMENTS

I would like to express my gratitude to Professor Ivona Grzegorczyk, who directed my research, for her encouragements and support. Her thoughtfulness, understanding, and insight were invaluable for me during this research. She has been a great mentor, motivator, and a great source of ideas throughout my graduate studies.

I would like to thank Dr. Brian Rasnow for meticulously reading my thesis and many valuable comments.

Finally, I would like to express my gratitude to my family. Without their support and love I could not have completed my Master Degree in Mathematics.

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Abstract

In this research we explored how to enrich current California curriculum in algebra. There are four components of this study. The first part focused on understanding Mayan mathematics in modern context of moduli arithmetic. The second included design and implementation of Mayan based mathematical activities into algebra classroom. The third part concentrated on collection and analyzing of data on students' learning through the activities and the changes in their attitudes. Lastly, we analyzed examples of mathematical artifacts we have collected in Guatemala.

1 Introduction

Improvements in curriculum and innovative teaching methods are necessary when addressing the needs of an increasingly growing population. According to the U.S. Census Bureau [28], people reporting Hispanic/Latino origin accounted for 36.6% of California's total population. In this study we explored how to incorporate a Latino cultural framework into algebra courses that prepare students for a college level curriculum. Mayan mathematics served as a vehicle to provide a cultural context for students that may improve the mathematical development and educational pursuits of various social groups. The population of interest were college freshman enrolled in California institutes of higher education. Using a scientific approach this research focuses on analysis of the effects of teaching Mayan mathematical concepts in introductory courses. We expected that lessons on modular arithmetic within a classic Maya theoretical framework, would improve students' performance and attitudes towards college mathematics.

One of the many goals of this study aims to better understand how to increase the number of students in STEM degrees in California. Currently, "[...] only 10% of Latinos in California older than the age of 25 have a bachelors degree compared to 47% and 38% for Asian/Pacific Islander and white Californians, respectively. [Furthermore,] only 15% of bachelors degrees awarded to Latinos in 2007 were in STEM fields, compared to 22% of degrees awarded to whites and 34% of degrees awarded to Asians/Pacific Islanders[26]." Mathematical understanding within the Latino/a community relies on educational instructors to communicate the importance of mathematics in a societal context, as there is no traditional course that incorporates their heritage into the classroom.

It is necessary for educators to understand the society of the students they are teaching in order to optimize student potential. Furthermore, it is important to distinguish that a common label such as "Hispanic" does not capture the true meaning of the cultural base of the population. "Hispanic" is a "convenient term to apply to all Spanish-speaking people in the United States, especially in the context of health, education, and welfare programs" [7]. For the purposes of the study we define "Latino" as a term " to refer all peoples of Latin American descent in the US, including Mexican Americans" [7]. Examining mathematics within the rich Mayan culture of their ancestors can possibly create a special interest within the population as they are often struggling with the politics of identity and the worth of scientific pursuits.

Mayan mathematics was analyzed and translated into an algebra lesson plan. The Mayans utilized a base twenty counting system that counted days and calculated Maya calendar dates. Modern modular arithmetic can be used to provide a deeper understanding of such Maya calculations. A lesson plan concentrating in modular arithmetic and ethnomathematics was created and implemented into college level algebra courses. The research was intended as an opportunity to improve the quality of instruction in California's higher education. We interpreted the results and conclusions found in the last section of the paper. In this study, the test subjects were not only limited to algebra students. As a graduate student at CSUCI, I had the opportunity to further research Maya mathematics in Guatemala with the Mathematical Association of America in 2011.

The research can be compartmentalized into four distinct components as follows:

Understanding Mayan mathematics in modern context of moduli arithmetic.

Design & Implementation of Mayan mathematical activities into algebra classroom.

Collection and analysis of data on student learning Maya activities.

Examples of mathematical artifacts collected in Guatemala.

2 Mayan Mathematics and Moduli Arithmetic

2.1 Background on Mayans

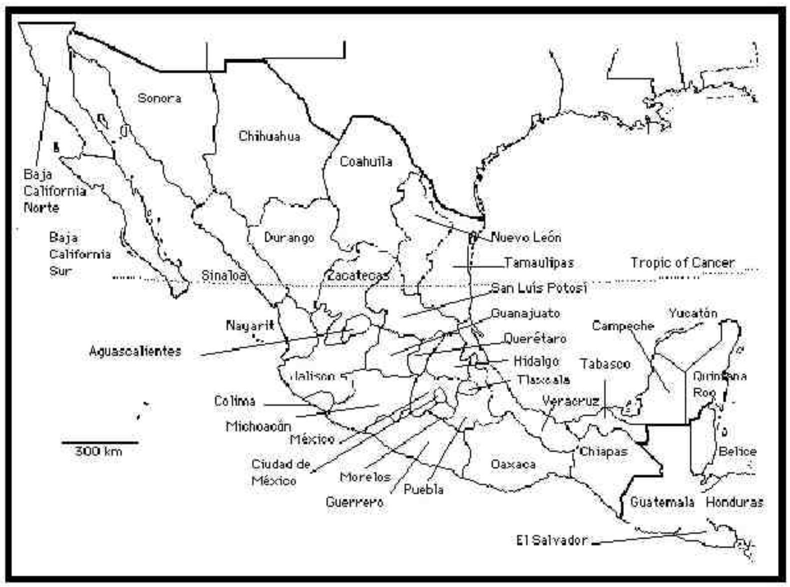


Figure 1: Map of MesoAmerica

The Mayan civilization flourished during the Classic period (250 A.D. - 900 A.D.) in the eastern one third of Mesoamerica, mainly on the Yucatan Peninsula [22]. The Mayans whom have contributed to the development of mathematics inhabited a region currently encompassing Guatemala, Belize, Honduras, El Salvador, Yucatan, Campeche, Quintana, Roo, Tabasco and Chiapas [4]. The map of this region is found in Figure 1.

2.2 Maya Mathematics

The Mayans have had numerous contributions to the scientific world as many of their activities depended on fundamental mathematical understanding. From the few surviving artifacts it is evident that they were masters of modular arithmetic and their calendar, lunar eclipses, history, astronomy findings and much more were recorded in codices. These four remaining codices decipher the scientific understanding of this early society. The Classic Maya had a symbol for zero and developed a number system in base 20, called a vigesimal counting system. Their positional notation was the most modern approach during its time period, and currently adopted decimal system uses the same general idea to display numbers.

Recently decoded texts have proven that the Maya were a sophisticated society in many respects, despite the Conquistadores attempts to destroy their religion and culture in the sixthteen century. The most famous Conquistador in Mexico was Hernando Cortes who arrived in Mexico City in 1519 [4], and who influenced the native colonization process. The Spanish invaders burnt most of the Maya's books, for fear that they were instruments of the devils [11], as the sophisticated literature frightened them. From the recovered texts it is evident that Mayans could represent very large numbers by using only three symbols. Since, our modern decimal system uses ten symbols it is often challenging to decipher the Mayan computations. The following passage by Michael Coe describes the early Maya numerical system. [5], "The Maya usually operated with only three such symbols: the dot for one, the bar for five and a stylized shell for nought. Unlike our system adopted from the Hindus, which is decimal and increasing in value from right to left, the Maya was vigesimal and increased from bottom to top in vertical columns. Thus, the first and lowest place has a value of one; the next above it the value of twenty ; then 400; and so one."

The following examples illustrate the numerical system described by Coe[5].

Example: 1. *Representing the number 28 in the vigesimal counting system.*

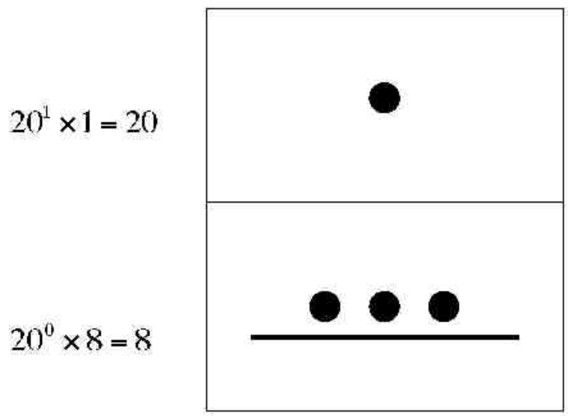


Figure 2: Maya Number 28

From the Figure on the left, we can see the vertical spaces representing the Maya number 28. The vigesimal system has one count of 20 as well as eight counts of 1, hence $20 + 8 \quad 28$. This simple example shows how to convert modern numbers into the vigesimal counting system. Furthermore the visual representation would consists of two vertical spaces as shown in Figure 2.

Example: 2. *Representing the following numbers: 0, 1, 5, 6, 25 411, and 8420, respectively in Figure 3.*

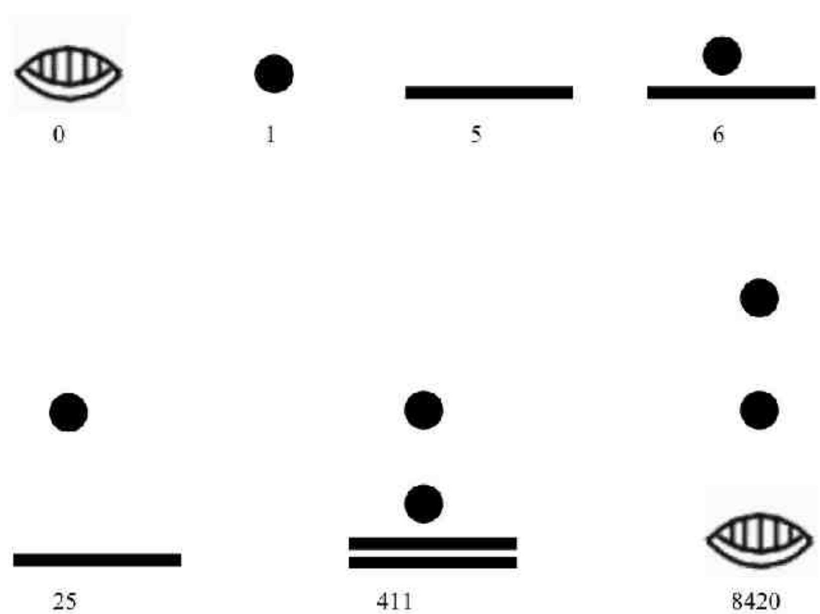


Figure 3: Basic Maya Numbers

Example: 3. *Representing the number* 196, 263 *in the vigesimal counting system.*

$$20^4 \times 1 = 160,000$$

$$20^3 \times 4 = 32,000$$

$$20^2 \times 4 = 4,000$$

$$20^1 \times 13 = 260$$

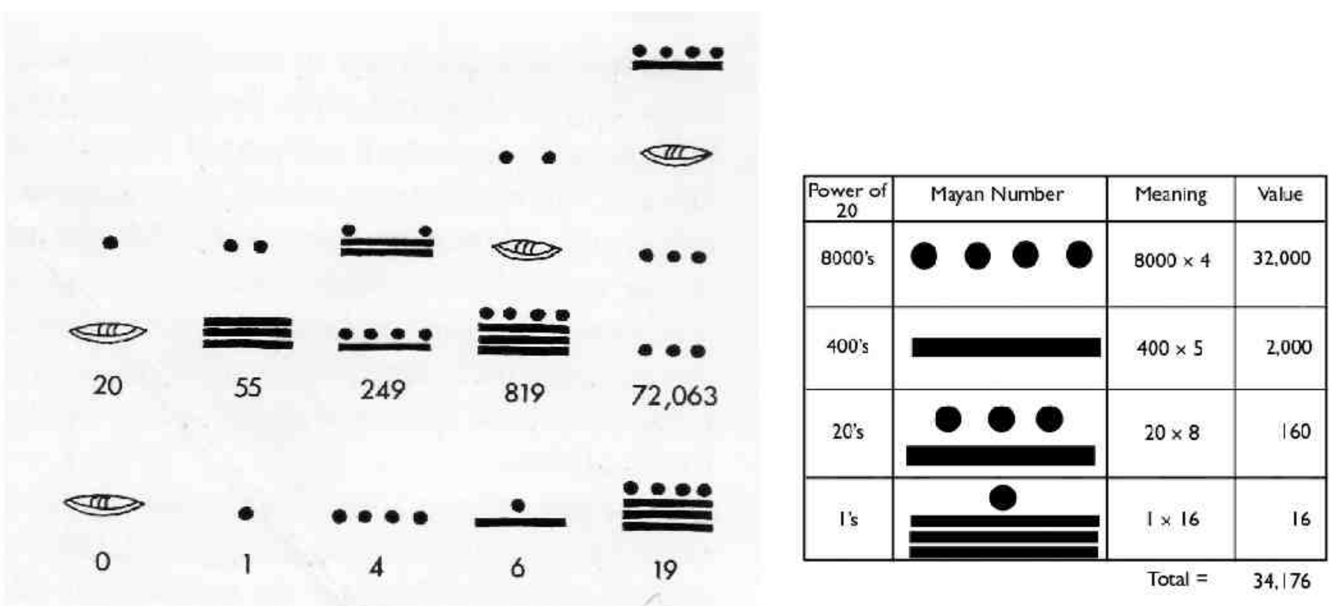
$$20^0 \times 3 = 3$$

We have formulated the following equation :

$$3 + 260 + 4,000 + 32,000 + 160,000 = 196,263.$$

This example illustrates how to write a large number in base 20. Understanding conversions of natural numbers into the vigesimal system is necessary to learn Maya mathematics.

The Figure 4a, helps identify numbers in the Mayan system, whereas, Figure 4(b) captures the calculation converting into the Maya number 34, 176 , see [26].



(a) Vigesimal Numeration (b) Numerical Computation

Figure 4: Maya Numbers

Example: 4. *Representing the number 419 in the Maya number system.*

The Figure to the right deciphers how to convert the notation 419 from base 20 into the Mayan number. The Mayans would represent this number by three distinct vertical spaces stacked one upon the other. The bottom space consisting of 3 bars and four dots, the middle space defined by a shell and the top space with a single dot. Each bar has a value of five and each dot has a value of one. Hence, we have $400 + 0 + 19 = 419$.

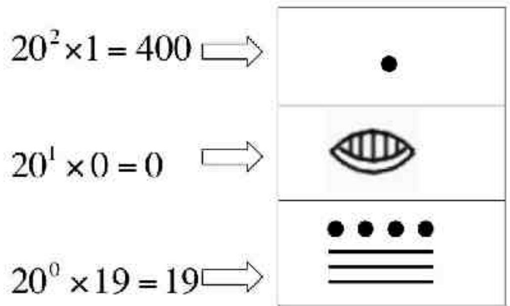


Figure 5: Maya Number 419

Example: 5. *Calculating in Maya arithmetic.*

(1.) $11,131 + 7520 = 18651$

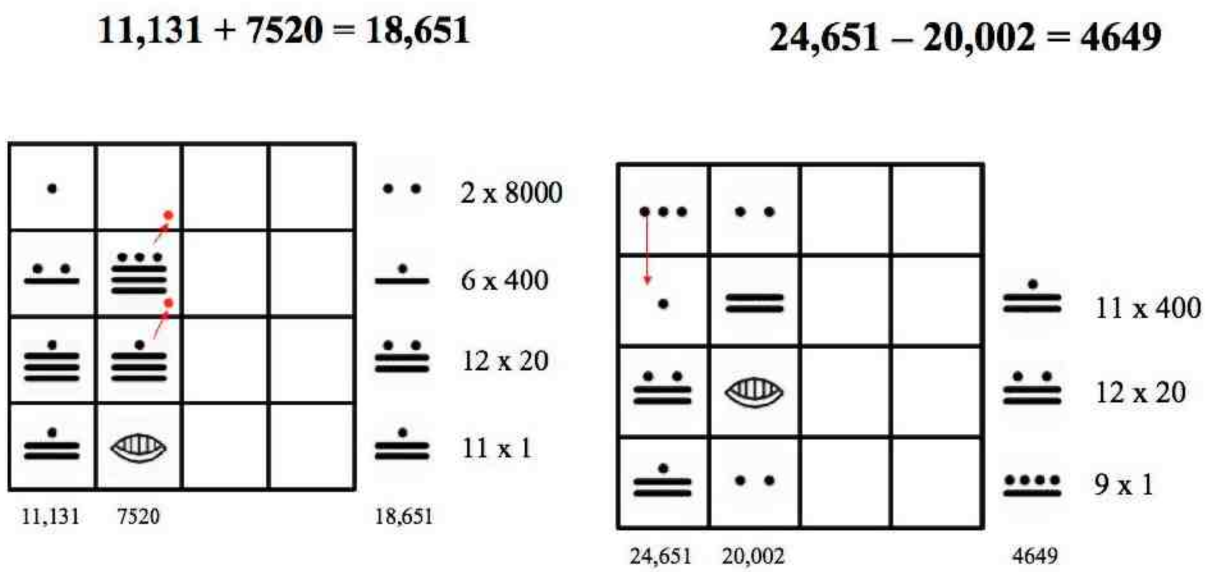
(2.) $24,651 - 20,002 = 4649$

To perform the addition we need to find a way to sum the numbers in the first two columns. The rightmost column in Figure 6a represents the number 11,131 and the column next to that is the number 7520. Thus, the two equations are represented in the following way in Maya notation:

First number: $(20^3 \times 1) + (20^2 \times 7) + (20^1 \times 16) + (20^0 \times 11) = 11,131$

Second number : $(20^2 \times 18) + (20^1 \times 16) + (20^0 \times 0) = 7520$

The vertical spaces for this example represent $20^3, 20^2, 20^1$ & 20^0. The next step is to perform the addition of the columns is to sum corresponding powers of 20. Addition starts with the bottom vertical spaces and continues its algorithm upwards. This algorithm can be represented as follows:



(a) Addition (b) Subtraction

Figure 6: Arithmetic

Powers	20^0	20^1	20^2	20^3
Equation 1	11	16	7	1
Equation 2	0	16	18	0
Total	11	32	25	1
Vigesimal total	11	12	6	2

We note that the vertical space of 20^0 remains as 11 since $11 + 0 = 11$. However, the next vertical space, 20^1, has too many elements as it equals 32. We have $20^1 \times 32 = 640 \implies (20^2 \times 1) + (20^1 \times 12)$. The red dot in this figure represents the shifting of 640 into a value of $20^2 = 400$, and the remainder. In our table we had the vertical space of 20^2 to equal 25, but now we add one from previous calculation. The calculation gives $20^2 \times 26 = 10400 \implies (20^3 \times 1) + (20^2 \times 6)$. Similarly, the second red dot represents the conversion into a third power of 20 and the new total value for the vertical space 20^2 is 6. It follows that the new vertical space of 20^3 is 2. We obtain the following new calculation in base 20: $(20^3 \times 2) + (20^2 \times 6) + (20^1 \times 12) + (20^0 \times 11) = 18,651$.

2.3 Calendar

The calendar is one of the most interesting achievements of the Mayans. It is uncertain if they have developed the calendar with or without the influence of other non-mesoamerican cultures. According to Aldana, the Epi-Olmec (Veracruz region in Mexico) culture had a vigesimal system and calendar that strongly influenced the Mayans [1]. Every calendar represents human approximation of counting days and observations of the stars, sun, moon, equinoxes, and eclipses. According to Harrington[18], "Many ancient cultures, including the Greek, Chinese, Islamic, and Mayan had legends that associated lunar eclipses with plagues, earthquakes, and other disasters." Harrington further describes the ancient Chinese (2800 B.C.) being afraid of solar eclipses because they believed a "ferocious dragon was devouring the Sun!"

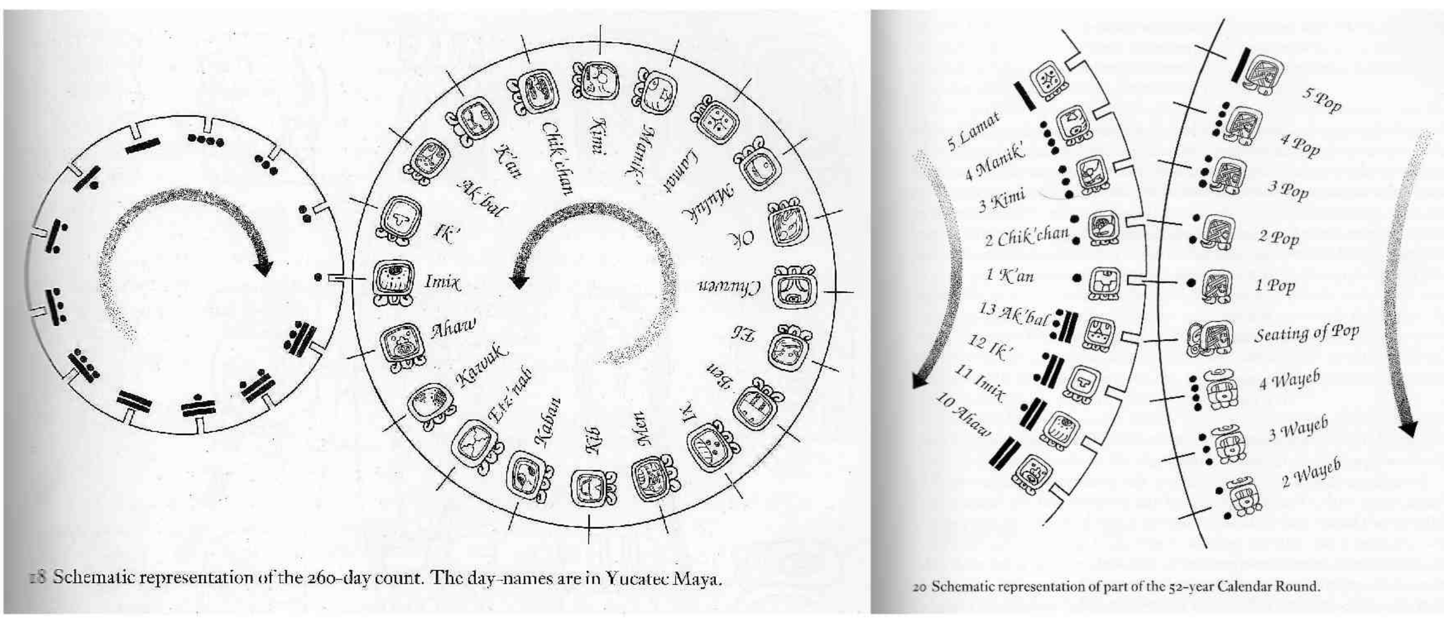
According to Barnhart and Powell, the 16th century chronicles of Yucatan's first Bishop, Fray Diego de Landa, has helped modern scholars recover the most fundamental cycles of the Maya Calendar [26]. These cycles they are described in the following table:

Maya Calendar	Day/Year Count
The Tzolk'in	260 days
The Haab	365 days
The Long Count	5126 years
The Lunar Series	174-180 days
The Lords of the Night	9 days

The Maya similar to other early cultures intertwined science, religion, nature and culture. Their Calendar system provided evidence of combining several beliefs so that the measure of time was accurately recorded. According to Powell [20], the cultivation and harvest of corn heavily influenced the length of the Tzolk'in. The ancient Maya believed that cultivation of corn and the human gestation period both have a 260-day count. This pattern of days between corn cultivation and human gestation period lead them to believe that they were the people of corn. They believed that the spirit of the corn, the "guzano" or worm, is the soul of the people [20]. The horoscope-like significance in human characteristics is found in the following statement, "[...] 260-day Tzolk'in, not the Haab (Solar Year) was believed to define personality traits and destiny[26]. The Tzolk'in has

260-day count, each 20 day names with the associated numbers 1 thru 13 as sequential coefficients date back to 600 B.C.E.[26]. The Tzolk'in can be interpreted in a modern mathematical approach with modular arithmetic and we will discuss it further in details in the Modular Arithmetic section of thesis paper. The calendar year that was based on solar observations was called the Haab, or "vague year," and had 365 days [26], (note that this is a very close approximation to the modern western calendar). The 365-day count included a sequence of 18 months of 20 days and a short month of 5 "unlucky days" at the end [5]. Therefore, The Haab calendar using modern notation had: $18(20) + 5 = 365$ days, 18 months of 20 days with an additional 5 days.

The Calendar Round combined both the Tzolk'in and Habb and created a cycle of 52 years. (Note that the Least Common Multiple (LCM) of the Tzolk'in and Habb is: $LCM = 2^2 \cdot 5 \cdot 13 \cdot 73$, which counts 18,980 days equal to 52 years).



(a) Tzolk'in (b) Tzolk'in & Habb

Figure 7: Calendar Round

Figure 7(a) shows two tangent circles-one with names and another with a sequence of numbers from 1 to 13. Together the tangent circles represent 260 day count. Figure 7(b) shows the Calendar Round as two separate components (circles) that combine to create the cycle of 52 years. Understanding the cycles of the Calendar Round provides evidence that the Mayans had a deep understanding of modular arithmetic.

The Long Count was a cycle of about 5126 years, and was used to record large intervals of time. This cycle is made up of 5 different subcycles named: bak'tun, k'atun, tun, winal, and k'in. The corresponding glyphs are represented below by Figure 8.

145 Glyphs for the cycles of the Long Count. *a*, Introducing Glyph; *b*, bak'tun; *c*, k'atun; *d*, tun; *e*, winal; *f*, k'in.

Figure 8: Long Count

The Long Count was described by Aldana [1], as follows:

"The Long Count may be conceptualized as an odometer of days. Each day adds a unit to the lowest register, and as each register fills up, the count carries forward into the next higher register. In order to facilitate calendric computations, the Long Count did not follow a strictly vigesimal format (i.e. it is not strictly base twenty). Instead, Maya scribes counted days according to the following units:

1 bak'tun = 20 k'atun (= 144,000 days)

1 k'atun = 20 tun (= 7,200 days)

1 tun = 18 winal (= 360 days)

1 winal = 20 k'in (= 20 days)

1 k'in = 1 day "

Modern standard notation for writing a Long Count date is written as follows: *bak'tun. ka'tun. tun. winal. k'in* [1]. It is important to note that 0.0.0.0.0, signified the initial Maya calendar date. Hence, each Long Count date counted how many days past the initial Maya date. Hence, 1 bak'tun was represented as 1.0.0.0.0, which represents 144,000 days past initial date. Amongst scholars there is debate on the actual initial base date but we recognize the archaeologist (Jenkins) [17] in our calendar. The Mayans had calendrical cycles recorded on stone monuments and were deciphered by modern scholars. (The picture below in Figure 9, is a current stelae in Tikal, Guatemala that has the oldest recorded ancient Maya date).

Figure 9: Maya Stone Monument

The Christian calendar had a start date of January 1 AD and the Maya calendar had a start date of August 11, 3114 BC. Theoretically, the Maya calendar has an end date of December 2, 2012 AD.

In order to align Mayan and our current Gregorian calendar dates it is important to analyze the ambiguity of calculating dates. “Due to the Gregorian Calendar reform, the day after 1582 Oct 04 (Julian calendar) is 1582 Oct 15 (Gregorian calendar). Since the Julian calendar does not include the year 0, the year 1 BCE[1] is followed by the year 1 CE. This is awkward for arithmetic calculations. (..). Thus, the astronomical year 0 corresponds to 1 BCE, and year -100 corresponds to 101 BCE, etc.. ”[12]

Moreover, the actual count of a year can vary depending on leap years. “Over the course of a century, the average calendar year consists of 365.25 days (accounting for leap years), while the more accurate tropical year (the average time between successive vernal equinoxes) is roughly 365.2422 days” [24]. Scholars that studied the carved dates in Maya monuments agree that their calendars were very accurate to the true length of a tropical year.

Example: 6. *Calculation of the Maya Long Count date 9.2.3.7.5*

Each notational position found in the Long Count has a unique modulus count of days. The calculation below illustrates how to count the days past the initial Maya base date.

9 bak'tun $\implies 9 \times 144,000 = 1,296,000$ days

2 k'atun $\implies 2 \times 7,200 = 14,400$ days

3 tun $\implies 3 \times 360 = 1,080$ days

7 winal $\implies 7 \times 20 = 140$ days

5 k'in $\implies 5 \times 1 = 5$ days

Hence, 9.2.3.7.5 is $1,296,000 + 14,400 + 1,080 + 140 + 5 = 1,311,620$ days past the initial Maya base date. Using the base year as 3114 BCE and estimating that one year is 365.2422 days we can convert the Long Count date 9.2.3.7.5 into a Gregorian date. Since, 1,311,620 days is approximately 3591.1 years which is approximately the year 476 CE. (We took into account that the Gregorian calendar changed from the Julian Calendar in 1582)

Example: 7. *Representing Gregorian calendar dates into Long Count date [26].*

(a.) The Long Count date for December 20, 2012 AD → 12.19.19.17.19 3 *Kawak* 2 *Kankin*

(b) The Long Count date for December 21, 2012 AD → 13.0.0.0.0 4 *Ahau* 3 *Kankin*

(c) The Long Count date for December 22, 2012 AD → 0.0.0.0.1 5 *Imix* 4 *Kankin*

For examples (a.) - (c.), we assumed that the initial base date of August 11, 3114 BC. The algorithm below counts the number of days past the initial base date.

(a.) December 20, 2012 AD

Assuming that there is 1,871,999 days between August 11, 3114 BC and December 20, 2012 AD, the following algorithm helps compute Long Count date.

$$1,871,999 = 144,000(B) + 7200(K) + 360(T) + 20(W) + 1(k)$$

Where Baktun, Katun, Tun, Winal and Kin are represented as B, K, T, W, k, respectively. Simple arithmetic manipulation produces

$1,871,999 = 144,000(12) + 7200(19) + 360(19) + 20(17) + 1(19)$.

Hence the Maya date that represents the modern western calendar date of December 20, 2012 AD is : 12.19.19.17.19

(b.) December 21, 2012 AD

We know that the Gregorian date December 21, 2012 AD is one day after December 20, 2012 AD , so from part (a) the calculation of days is $1,871,999+1$ from the initial Maya base date. Hence,

$$1,872,000 = 144,000(13) + 7200(0) + 360(0) + 20(0) + 1(0).$$

Therefore the Maya date that represents the modern western calendar date of December 21, 2012 AD is : 13.0.0.0.0

(c.) December 22, 2012 AD

We know that the Gregorian date December 22, 2012 AD is one day after December 21, 2012 AD , so from part (b) the calculation of days is $1,872,00+1$ from the initial Maya base date. Hence, we compute the Long Count date as follows,

$$1,872,001 = 144,000(0) + 7200(0) + 360(0) + 20(0) + 1(1).$$

Therefore Maya date that represents the modern western calendar date of December 21, 2012 AD is : 0.0.0.0.1

According to Powell [26], "There are scholars who believe that the count would not reset to 0.0.0.0.1, but rather continue with 13 as the bak'tuns being 13.0.0.0.1, and then 13.0.0.0.2, and so on up until the 14th bak'tus, being represented as 1.0.0.0.0. Since we have no texts presenting dates within the first bak'tuns range of 400 tuns, this particular questions remains an academic debate." In a sense, December 22, 2012 AD is a new count of 5126 years since the Long Count is considered to have at most 13 baktuns which is about 5125.366127 years (using tropical year of 365.2422). In fact it is this Maya calculation of dates that has triggered some exaggeration about the predicted end of world in 2012. Maya calenderics additionally included cycles referred to as Lunar Series and Lords of the Night. According to Aldana[1], both these cycles were the two most frequent components of the Supplementary Series. One of the surviving pieces of Maya literature, Dresden Codex, has been examined methodically by many scholars who agree that it contains solar and lunar eclipse tables. The Classic Maya had an alternating lunar count of 29 and 30 days [?]. Recall, that the Maya vigesimal system takes into account only natural numbers, thus fractions are not considered.

"A solar eclipse is the result of the Moon coming between the Earth and the Sun, and that a lunar eclipse is the result of the Earth coming between the Sun and the Moon [18]. Moreover, we know how the Moon orbits the Earth every 27.3 days and the Earth orbits the Sun in 365.2 days. By combining these two movements it implies that the Moon takes 29.5 days to complete a set of phases. This is referred to as a synodic month or lunation [18]. Because the Mayans did not use rational numbers, created an alternating sequence of 29 and 30 days for the lunar count which is a good approximation without the use of rational numbers. Perhaps, a reason why the ancient Mayans so carefully observed eclipses was the fact that they looked directly at the vanishing sun during such eclipses, and many went blind. In fact, according to Harrington [18],

" The sun is the only celestial object that can harm someone who looks at it directly. It is dangerous because the Sun radiates visible light and.." its photoshphere alsoemits intense infrared (IR) and ultraviolent (UV) radiation. Just as ultraviolent light radiation casuses sunburn to exposed skin, so too wil it damage your eyes' retinas and at a much faster rate."

The picture in Figure 10 and paragraph below provide Maya representation of the moon.

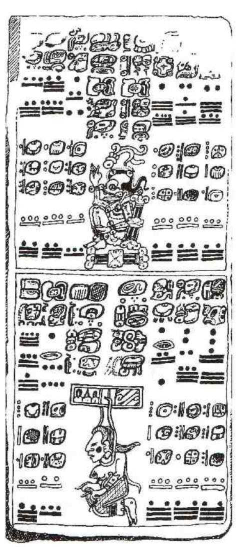


Figure 10: Moon Goddess

" The pictures in the eclipse table include representations of the Sun God, a death god, Venus, and the goddess Ix Tab, who is portrayed hanging from a sky band in the lower section of Figure 10. The goddess, with her eye closed, is marked by black spots on her cheek and nipple, iconography associated with death and eclipses (Closs 1989). In Yucatec Maya, the eclipsed body (sun or moon) is said to have its eye extinguished or blinded (Barrera Vasquez 1980:824), echoing the closed-eyed portraits of the Moon Goddess in the Dresden Codex."

According to Barnhart and Powell [26], "..connected to the Lunar Series is a 9-day cycle called the Lords of the Night. Little is known about the significance or origin of this 9-day cycle, but it is recognized as the smallest cycle the Maya recorded. [..] In their communities of the Guatemala highlands, it is said that counting nine days forward and nine days backwards from one's tzolk'in birthday gives the identity of their protector spirits."

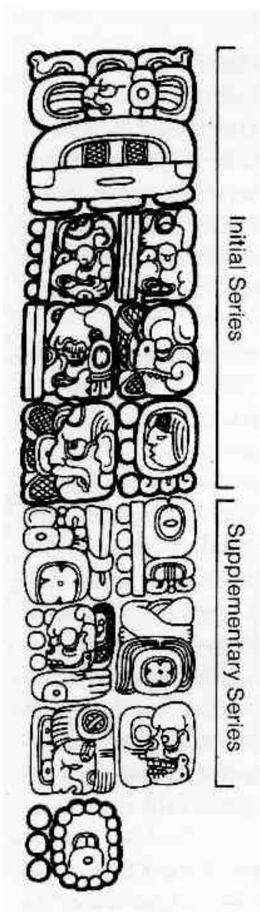


Figure 11: Stela 10

Now we can combine Maya hieroglyphic inscription of the Long Count dates with Calendar rounds and the Supplementary Series. Figure 11 illustrates the methodology of decoding a glyph [6]. The additional Long Count glyphs found in the initial series count a total of 1,407,600 days. Figure 11 identifies the Maya date 9.15.10.0.0. which is located on Stela 10 in Piedras Negras.

2.4 Modular Arithmetic

Even though Mayans had introduced modular contributions to mathematics and sciences, their calculations are often omitted. The reason for the majority of books on the subject of the history of mathematics do not include a section in Mayan Modular arithmetic is because the culture was isolated and records destroyed. Furthermore, little documentation has survived and what is left remains difficult to decode and read even using modern terminology. Here we will show one more important modular example.

We notice that The Calendar Round of 260 days can be thought of as arithmetic in modulus 13. Recall, the Tzolkin, calculates 260 days by combining 20 day names with a sequence of numbers from 1 to 13. For example the sequence of days in the Tzolkin is given below for days 1 thru 20.

1. Imix	8. Lamat	15. Men
2. Ik	9. Muluc	16. Cib
3. Akbal	10. Oc	17. Caban
4. Kan	11. Chuen	18. Etz'nab
5. Chicchan	12. Eb	19. Cauac
6. Cimi	13. Ben	20. Ahau
7. Manik	14. Ix	

For example, here is a short sequence of days combined with selected months and days.

12 *Eb*, 13 *Ben*, 1 *Ix*, 2 *Men*, 3 *Cib*

In this counting system each day name will eventually have all the distinct numbers from 1 to 13. Hence, there will eventually have 1 Imix, 2 Imix, 3 Imix,..., 13 Imix within the 260-day count. As an example, the table below expresses the first 39 days of the Maya Calendar, Tzolk'in. Note: the left-hand side as the day count with corresponding modulus, whereas, the right-hand side of the table is the corresponding Tzolk'in date.

$1 \pmod{13} \to 1$ Imix	$14 \equiv 1 \pmod{13} \to 1$ Ix	$27 \equiv 1 \pmod{13} \to 1$ Manik
$2 \pmod{13} \to 2$ Ik	$15 \equiv 2 \pmod{13} \to 2$ Men	$28 \equiv 2 \pmod{13} \to 2$ Lamat
$3 \pmod{13} \to 3$ Akbal	$16 \equiv 3 \pmod{13} \to 3$ Cib	$29 \equiv 3 \pmod{13} \to 3$ Muluc
$4 \pmod{13} \to 4$ Kan	$17 \equiv 4 \pmod{13} \to 4$ Caban	$30 \equiv 4 \pmod{13} \to 4$ Oc
$5 \pmod{13} \to 5$ Chicchan	$18 \equiv 5 \pmod{13} \to 5$ Etz'nab	$31 \equiv 5 \pmod{13} \to 5$ Chuen
$6 \pmod{13} \to 6$ Cimi	$19 \equiv 6 \pmod{13} \to 6$ Cauac	$32 \equiv 6 \pmod{13} \to 6$ Eb
$7 \pmod{13} \to 7$ Manik	$20 \equiv 7 \pmod{13} \to 7$ Ahau	$33 \equiv 7 \pmod{13} \to 7$ Ben
$8 \pmod{13} \to 8$ Lamat	$21 \equiv 8 \pmod{13} \to 8$ Imix	$34 \equiv 8 \pmod{13} \to 8$ Ix
$9 \pmod{13} \to 9$ Muluc	$22 \equiv 9 \pmod{13} \to 9$ Ik	$35 \equiv 9 \pmod{13} \to 9$ Men
$10 \pmod{13} \to 10$ Oc	$23 \equiv 10 \pmod{13} \to 10$ Akbal	$36 \equiv 10 \pmod{13} \to 10$ Cib
$11 \pmod{13} \to 11$ Chuen	$24 \equiv 11 \pmod{13} \to 11$ Kan	$37 \equiv 11 \pmod{13} \to 11$ Caban
$12 \pmod{13} \to 12$ Eb	$25 \equiv 12 \pmod{13} \to 12$ Chicchan	$38 \equiv 12 \pmod{13} \to 12$Etz'nab
$13 \pmod{13} \to 13$ Ben	$26 \equiv 13 \pmod{13} \to 13$ Cimi	$39 \equiv 13 \pmod{13} \to 13$ Cauac

This continues until 260 days have elapsed, one complete cycle of the Tzolkin. The Maya were able to calculate calendar dates by performing operations in modulo 13.

2.5 Modern Moduli Arithmetic

Although, the Mayas developed sophisticated modular mathematics their culture and achievements are rarely discussed in math classrooms in California. According to Michael Closs [4],

"Books on the history of mathematics do not always refer to Mayan developments and whey they do so it is almost in a cursory manner. For example, it may be mentioned that the Maya utilized a system of positional notation which incorporated zero and perhaps a brief description of Maya bar and dot numeration may be attempted. However, many other incredibly attractive Maya mathematical notations are never discussed and contexts and purposes of Maya mathematics are neglected."

However, in many undergraduate mathematics courses students have the opportunity to discover a contributions of various cultures. For example, according to Eves [13], "After the decline of classical Greek mathematics, the mathematics of China became one of the most prosperous in the world." Furthermore, Eves credits the mathematician Sun-tzi for the discovery of the Chinese Remainder Theorem in about A.D. 600. Moreover, many abstract algebra texts do not give any credit for modular arithmetics to Mayans:

"The Chinese Remainder Theorem was known to ancient Chinese astronomers, who used it to date events from observations of various periodic astronomical phenomena. It is used in this computer age as a tool for finding integer solutions to integer equations and for speeding up arithmetic operations in a computer."

Let's analyze the Chinese Remainder Theorem here [15].

Theorem: 1. *Let $m_1, m_2 \ldots m_r \in \mathbb{N}$ where $GCD(m_i, m_j) = 1$ if $i \neq j$.*
Then the system of r simultaneous equations

$$x \equiv a_1 \pmod{m_1}, \qquad x \equiv a_2 \pmod{m_2}, \cdots \qquad x \equiv a_r \pmod{m_r}$$

always has integer solutions. Moreover, if b is one solution, the complete solution is the set of integers satisfying $x \equiv b \pmod{M}$, where $M = m_1 \cdot m_2 \cdots m_r$

Example: 8. *Special Cases:*

We prove the Chinese Remainder Theorem for $r = 2$ and $r = 3$.

Proof 1. *Let* $r = 2$.

Then $m_1, m_2 \in \mathbb{N}$ where $GCD(m_1, m_2) = 1$. Then, we have a system of simultaneous congruences as follows:

(1.) $x \equiv a_1 \pmod{m_1}$

(2.) $x \equiv a_2 \pmod{m_2}$

Let $d_1 = n_1 \pmod{n_2}$. We can find e_1 such that $d_1 \cdot e_1 \equiv \pmod{n_2}$ because n_1 and n_2 are co-prime. Let $b_2 = n_1 \cdot e_1$. Then let $d_2 = n_2 \pmod{n_1}$. We can find e_2 such that $d_1 \cdot e_2 \equiv 1 \pmod{n_2 = 1}$, where we define $b_1 = n_2 \cdot e_1$.

Then,

$b_1 = d_2 \cdot e_2 \equiv 1 \pmod{m_1}$ and $\qquad b_2 = d_1 \cdot e_2 \equiv 1 \pmod{m_2}$

Hence the solution is as follows,

$x \equiv b_1 \cdot a_1 + b_2 \cdot a_2 \pmod{M}$ where $M = m_1 \cdot m_2$

<u>Example</u>

(1) $x \equiv 2 \pmod{3}$

(2) $x \equiv 4 \pmod{5}$

Here $d_1 \equiv 3 \pmod{5} = 3$ and $d_2 \equiv 5 \pmod{3} = 2$.

It follows that $d_1 \cdot e_1 = 3 \cdot 2 \equiv 1 \pmod{5}$. (ie: $e_1 = 2$ and $b_2 = 6$).

Similarly, $d_2 \cdot e_2 = 2 \cdot 2 \equiv 1 \pmod{3}$. (ie: $e_2 = 2$ and $b_1 = 10$).

Hence the solution is as follows,

$x \equiv 2 \cdot 10 + 4 \cdot 6 \equiv 44 \pmod{15} \equiv 14 \pmod{15}$.

Where $14 \equiv 2 \pmod{3}$ and $14 \equiv 4 \pmod{5}$.

Proof 2. *Let* $r = 3$

Then $m_1, m_2, m_3 \in \mathbb{N}$ where $GCD(m_1, m_2, m_3) = 1$

Then, we have a system of simultaneous congruences as follows:

(1.) $x \equiv a_1 \pmod{m_1}$

(2.) $x \equiv a_2 \pmod{m_2}$

(3.) $x \equiv a_3 \pmod{m_3}$

Let $d_1 = n_2 \cdot n_3 \pmod{n_1}$ and let $d_1 \cdot e_1 \equiv 1 \pmod{n_1}$. Then, $b_1 = n_2 \cdot n_3 \cdot e_1$ and follows previous case.

Hence, from modular congruence properties we can find b_1 , b_2, b_3 s.t.

$b_1 \equiv 1 \pmod{m_1}$, $\quad b_2 \equiv 1 \pmod{m_2}$, $\quad b_3 \equiv 1 \pmod{m_3}$

Then the solution is as follows:

$x \equiv b_1 \cdot a_1 + b_2 \cdot a_2 + b_3 \cdot a_3 \pmod{M}$ where $M = m_1 \cdot m_2 \cdot m_3$

Proof 3. *General Case (following Gilbert and Nicholson [15]).*

Proof: Let $M = m_1 \cdot m_2 \cdots m_r$, where $GCD(m_i, m_j) = 1$ if $i \neq j$. Then $\mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \cdots \times \mathbb{Z}_{m_r}$ is a ring isomorphic to $\mathbb{Z}_m$. Consider the following ring isomorphism

$$f : \mathbb{Z}_M \to \mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \cdots \times \mathbb{Z}_{m_r}$$

defined by $f([x]_m) = ([x]_{m_1}, [x]_{m_2}, \cdots, [x]_{m_r},)$. The integer x is a solution of the simultaneous congruences if and only if $f([x]_{m_1}) = ([a_1]_{m_1}, [a_2]_{m_2}, \cdots, [a_r]_{m_r})$. Since there is only a finite number of conditions and $GCD(m_i, m_j) = 1$ if $i \neq j$, there exists a solution, and the solution set consists of exactly one congruence class modulo M.

Example: 9. *Let x be congruent to 1 mod 3, 2 mod 5 and 3 mod 7.*

We have the following congruences:

(1.) $x \equiv 1 \pmod{3}$

(2.) $x \equiv 2 \pmod{5}$

(3.) $x \equiv 3 \pmod{7}$

We know from number theory that if the modulus are pairwise relatively prime then $M_i | b_i$ and $b_i = c_i M_i$. Where M_i corresponds to the following:

$M_1 = 5 \times 7 = 35$

$M_2 = 3 \times 7 = 21$

$M_3 = 3 \times 5 = 15$ Hence we can write $c_i M_i \equiv 1 \pmod{M}_i$

Congruences (1.) - (3.) require the following, respectively

(a) $c_1 35 \equiv 1 \pmod 3$

(b) $c_2 21 \equiv 1 \pmod 5$

(c) $c_3 15 \equiv 1 \pmod 7$

Using the Euclidean Algorithm we have the following computation for congruence for (a), $1 = 35(-1) + 3(12)$. Then $b_1 = (-1)35$, so $b_1 = -35$.

Repeating the process above we find, $b_2 = 21$, $b_3 = 15$.

Hence, the solution to the congruence is as follows:

$x \equiv -35 \cdot 1 + 21 \cdot 2 + 15 \cdot 3 \pmod{105} \rightarrow x \equiv 52 \pmod{105}$

Remark 1. *Note that when the moduli are not pairwise relatively prime there may be no solution and the algorithm does not work in general.*

Example: 10. *Let x be congruent to 1 mod 6 and 2 mod 15.*

We have the following congruences:

(1.) $x \equiv 1 \pmod 6$

(2.) $x \equiv 2 \pmod{15}$

We find that $LCM[6, 15] = 30$

Hence, from the first congruence it follows that:

$x = 1 + 6k$

we substitute this into the second congruence and obtain:

$1 + 6k \equiv 2 \pmod{15} \rightarrow 6k \equiv 1 \pmod{15}$

But $GCD(6, 15) = 3 \neq 1 \implies 3 \nmid 1 \implies$ no solution$\in k \implies$ no solution for x.

These examples show that the essence of modular arithmetic is evident in the Maya calendar cycles. For example, the Tzolk'in was discussed in working modulus 13 in order to create a cycle of 260-days. Furthermore, their vigesimal system can be easily computed using modulus of the powers of 20. In modern times, we use the Chinese Remainder Theorem, to calculate modular arithmetic problems, but these methods were used by Mayans. There is no evidence that Mesoamerican Mayans had any contact with other ancient civilizations of that time period, hence we can assume that they have developed the modular arithmetic independent of other cultures.

3 Student Activities

We incorporated the scientific contributions of the Mayans into algebra problems to provide a mathematical understanding that incorporates history, culture and identity into the classroom. We develop activities based on Mayan mathematics to serve California's diverse population as a tool to teach mathematics in a cultural context. In particular, we introduce students to arithmetic ideas found in the Mesoamerican calendrics and compare them to the achievements based on the Chinese Remainder Theorem. We surveyed students to explore whether mathematical instruction in a cultural Mayan context produces a sense of pride and/or relevance to Latinos and improve attitudes towards higher education in the science, technology, engineering, and mathematics (STEM) disciplines.

CSUCI students are referred to as Class 1 and SBCC students are referred to as Class 2. The student activities introduced modular arithmetic in a theoretical Mayan framework. Both samples of students were given an in-class powerpoint presentation of Mayan arithmetic and Calendars. The duration of the presentation was an estimated 60 minutes per class. Both classes were given seashells, toothpicks and dots to help visualize the Mayans numerical representation. Class 1 was given a separate pre-test and questionnaire. Class 2 was given a combined pre-test and questionnaire and an additional, post-test. The questionnaire allowed students to identify with a culture, whereas, the Maya-lecture integrated the Latino culture into the algebra classroom. The post-test analyzed mathematical understanding of modular arithmetic while capturing information about student attitude and pride in cultural achievements.

4 Data Analysis

The two comparable algebra courses, CSUCI and SBCC, are referred to as Class 1 and Class 2, respectively. All data collected can be found in Appendix. Class 2 has a diverse student population whose age distribution was wider than Class 1. The demographics of the community college represents a mini-population in classrooms. Whereas, the age of freshman students ranged between 18 and 20 whom were enrolled in an introductory algebra course at CSUCI.

The Pre-Test was graded as follows for quantitative questions,
5 pts: Correct answer and correct work to support answer.
4 pts: Correct answer and incorrect work to support answer.
3 pts: Incorrect answer but correct work to support answer.
2 pts: Incorrect answer and incorrect work to support answer attempted.
1 pts: left blank.

For qualitative questions and grading, all students were given qualitative questions and answered them by circling the answer. Most of these questions were given a scale of 1 to 5 , where 5 is the strongest association for the answer. Several questions were evaluated on a scale from 1 to 4.

Description of Pre-Test

Class 2 received a pre-test that combined data found in the questionnaire and pre-test that was given to Class 1. A questionnaire of 18 questions and a pre-test of 4 questions was distributed to intermediate algebra students in Class 1. The pre-test given to Class 2 students had 10 questions. The questionnaire was completed by 26 students in Class 1, whereas 24 students in Class 2 completed the pre-test. Quantitative and qualitative questions can be found in Appendix.

4.1 Descriptive Analysis

This section summarizes the statistical observations from the pre- test for Class 2. The data was calculated with sample size n= 25. The questions in the pretest for Class 2 are labeled Q1-Q10.

Q1: On a scale of 1 to 5, (with 5 being the highest) what is your current interest level in math?

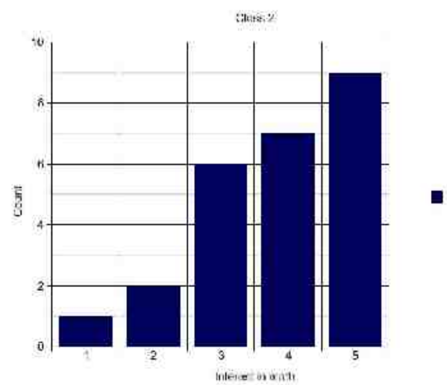


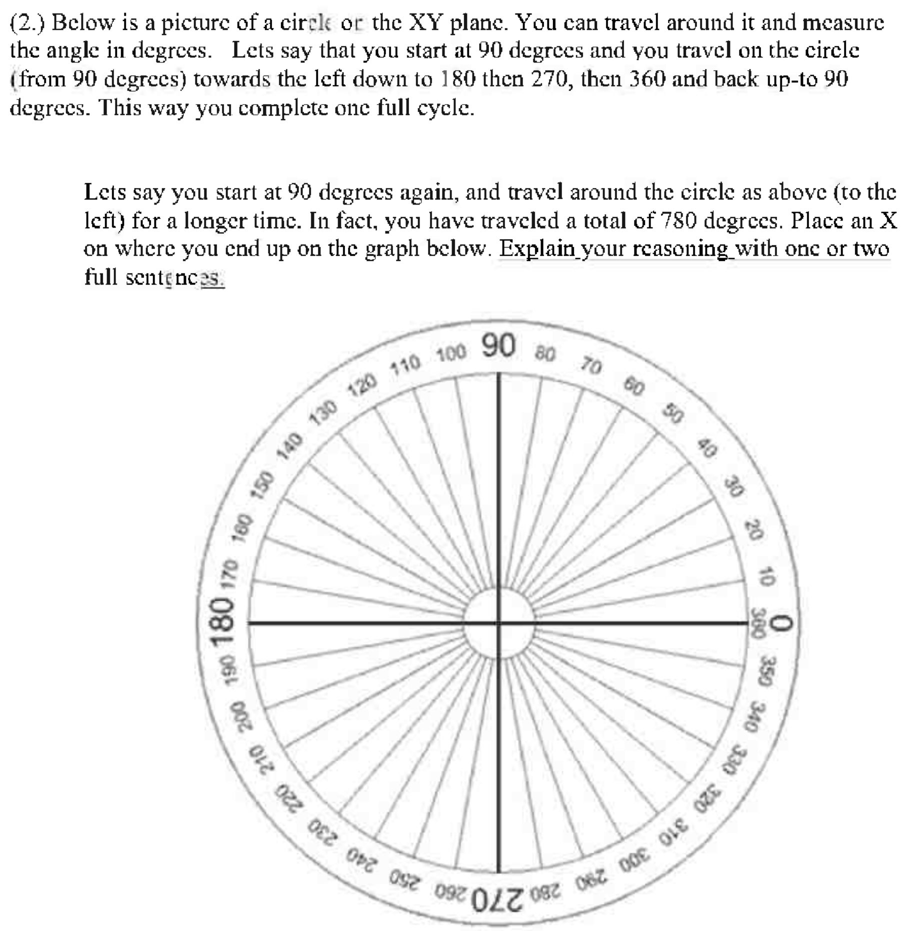
Figure 12: Interest in Math (Class 2)

The statistical results indicate that students in this group were interested in math. Using a scale of $R|1, 5|$, we used Excel to calculate the following, $\mu = 3.8$, $\sigma = 1.1$, $M = 4$ and mode 5. Since the interest was measured as qualitative data, the mode at 5, suggests that the average student is highly interested in math. Interest in mathematics is a component of motivation in the classroom. If a student lacks interest in a subject or topic then as a result, the student may be less likely to be motivated to do well. There was only one student that was not interested in math and two were not sure if they were interested in math. The questionnaire for Class 1, had a similar question that aimed to determine if students were interested in math. Students were asked the following,

On a scale of 1 to 5, (with 5 being the highest), what is your confidence level in the math class?

The results were as follows, $\mu = 3.88$, $\sigma = 0.85$, $M = 4$ and mode $= 4$. Data suggests that students were highly confident in Class 1. The results are interesting because math degrees are not being pursued despite the strong interest and confidence in the subject.

The figure below demonstrates how the second question was presented on the pretest.

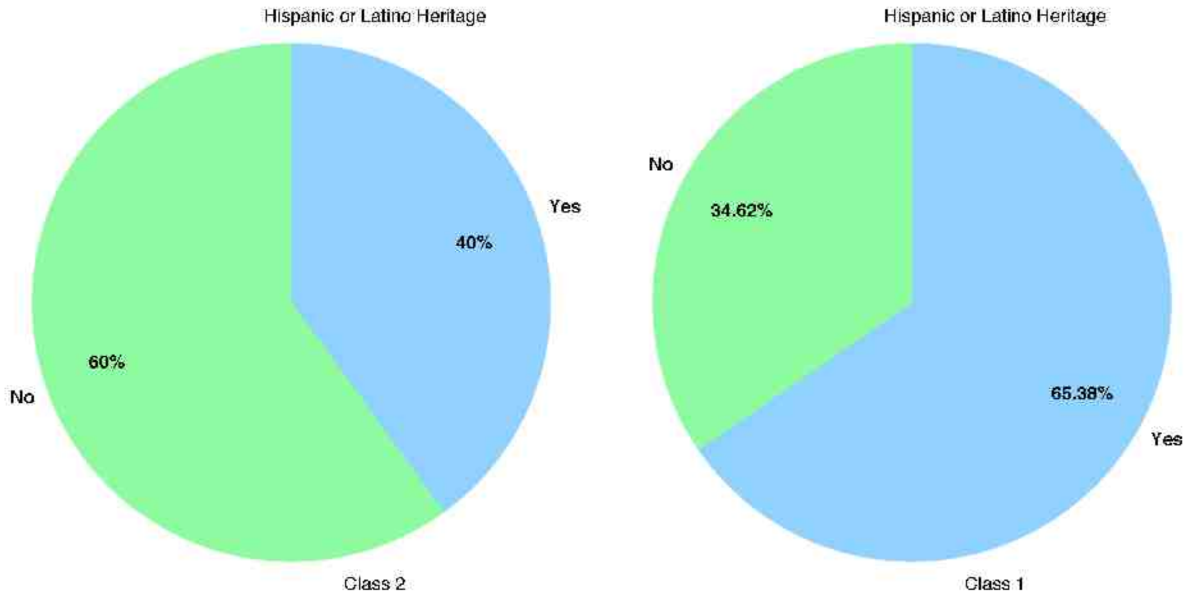

(2.) Below is a picture of a circle on the XY plane. You can travel around it and measure the angle in degrees. Lets say that you start at 90 degrees and you travel on the circle (from 90 degrees) towards the left down to 180 then 270, then 360 and back up-to 90 degrees. This way you complete one full cycle.

Lets say you start at 90 degrees again, and travel around the circle as above (to the left) for a longer time. In fact, you have traveled a total of 780 degrees. Place an X on where you end up on the graph below. Explain your reasoning with one or two full sentences.

Figure 13: Moduli 360 (Class 2)

Using a scale of $R[1,5]$, for quantitative data the results were as follows, $\mu = 3.63$, $\sigma = 0.96$, $M = 4$ and mode $= 4$, therefore the distribution shape is symmetric at 4 with a small standard deviation for Class 2. Here, the majority of students knew how to conceptualize modular numbers when placed into a context of using degrees on a graph. A total of nine students demonstrated they could arrive at the correct answer of 150° using logical reasoning. However, the pretest in Class 1 did not include the following sentence, " explain your reasoning with one or two full sentences." Class 2 had the following results, $\mu = 2.8$, $\sigma = 1.08$ $M = 2$ and mode $= 2$, therefore the distribution shape is symmetric at 2. Data suggests that Class 2 had a difficult time with moduli 360.

Q3: Do you have any Hispanic or Latino heritage?



(a) Class 2 (b) Class 1

Figure 14: Student Population

From Class 2, 10 out of 25 students identified with Latino or Hispanic heritage. In contrast, Class 1, had 17 out of 26 students that identified with the same heritage. From the data we can interpret that students of Latino and Hispanic backgrounds are inadequately prepared for college-level math and science courses because they represent a significantly large portion of the overall population in the classrooms.

Q4: Choose a statement that best applies to you from below.

1: I don't know any Mayan achievements, I was never interested.

2: I have heard something about Mayan science, but do not know details.

3: I have listed examples of Mayan science above, but I cannot describe them. I have never make an effort to find out more.

4: I know of several Mayan achievements described above and I am very proud of them.

The statistical results for Class 2 for Q4 was as follows,

$\mu = 2.1$, $\sigma = 0.8$, $M = 2$ and mode $= 2$. Here, the distribution shape is symmetric at 2 with a small standard deviation. The questionnaire for Class 1 asked, "Did you know that the Mayans are American ancestors whom had a sophisticated calendar system? (Please describe what you know).” About 84% of students in Class 1, were able to describe and or relate to the “end of the world.” Here, the majority of students knew something about Maya science but did not know any details. The social media publicizes that the Mayans predicted an apocalypse end in December 2012. Students have been introduced to this ancient civilization, but lacked the motivation to verify facts about the Mayans.

Q5: Changing from a non-ten base to base ten: What does 2103_{four} represent in base ten ? (Hint: the first four place values in base four are written below.) 4^3, 4^2, 4^1, 4^0.

The Figure below describes the data being skewed to the right. Using a scale of $R[1,5]$, for quantitative data the results were as follows, $\mu = 1.48$, $\sigma = 0.59$, $M = 1$ and mode 1.

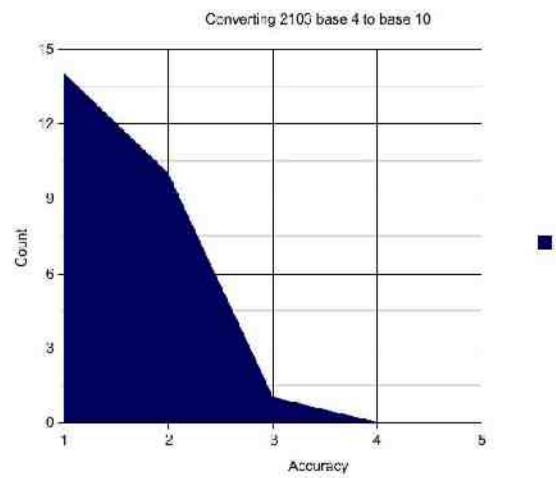


Figure 15: Base Four conversion to Base Ten (Class 2)

The student highest score was 3 out of 5 points, indicates that college students enrolled in remedial math courses don’t conceptualize numerical systems that are different than base 10. Q5 was presented in an arithmetical manner and the average student left this blank and no effort was put forth to attempt to solve this problem. Data suggests that students are capable of modular arithmetic when presented with visual aids such as in question 2, but they lack the confidence to attempt problems with an algebraic approach.

Q6: Do you know about any mathematical or scientific achievements of Mayans?

Students received a score according to their ability to list scientific achievements. Here the most frequent answer was Maya calendar. Using a scale of $R[1,5]$, for quantitative data the results were as follows, μ 2.79, σ 1.56, M 4 and mode 4. The majority of the scores were either 1 or 4. Only one student received a score of 5 for the following answer, " I only know that they were good astronomers and could predict movements of stars and planets. They had sight lines in their cities for certain dates."

Q7: On a scale of 1 to 5, (with being the highest), how likely are you to discuss math with family and friends outside of the classroom ?

5: likely to have discussions about math with family and friends

4: somewhat discusses math with family and friends

3: neutral

2: not sure if math is discussed with family and friends

1: does not discuss math with family and friends

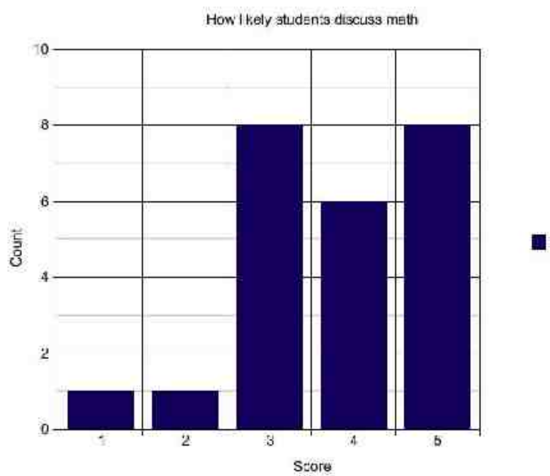
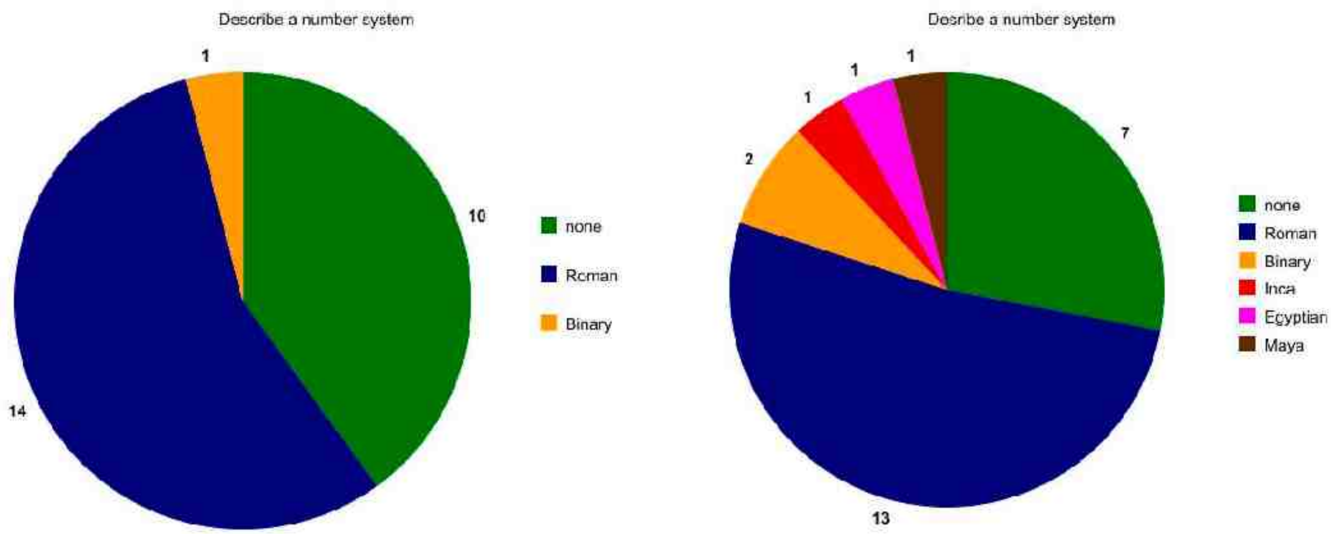


Figure 16: Discussing Math (Class 2)

The Figure above summarizes the following results from excel μ 3.8, σ 1.1, M 4 and there was no mode. Eight students indicated they were neutral about discussing math beyond lecture and eight students indicated they are likely to have discuss math with friends and family. Here, only one student did not answer the question and did not receive a score. Students may have been given too broad of a question as to discussing math. A student discusses math in lecture, tutoring, group studying, and talking about general classes as a student.

Q8: Do you know any other number system besides the decimal system? (for example Mayan, Roman, binary etc.) If yes, write your birthday, using the day and month using the system.



(a) Class 2 (b) Class 1

Figure 17: Identifying a Number System

The most frequent answer referred to the Roman numerals, that has distinct symbols for 1, 5, 10, 50, 100, 500, and 1000. These numerals are commonly found on clocks where the highest number is twelve. This is interesting because most students could not represent large numbers using Roman numerals. Identifying the existence of various number systems is helpful to understand moduli arithmetic. Both classes identified Roman and Binary number systems. Although, Class 1 had more variation for cultural number systems than Class 2.

Q9: Using the number system from above, add the numbers in your birthday, using the day and month.

Here, we obtained $\mu = 2.5$, $\sigma = 1.9$, $M = 1$ and mode $= 1$. The majority of the students do not know how to add roman numerals despite knowing how to write the correct symbols for numbers less than 50. Students may struggle with such representation due to the confusion arising from increasing a number and decreasing a number. For example iv is the subtract one from 5, and vi is add one to five.

Q10 can be found in appendix.

4.2 Maya Lecture and Activity

A lesson was created using powerpoint, worksheets and manipulatives to understand modular arithmetic in an Ethnomathematics approach using the ancient Mayan civilization. The in-class worksheet (see appendix) given to Class 2 had three distinct sections. In section 1, students identified four visual representations of Mayan numbers and provided its corresponding base ten representation. For section 2, a number in base 10 then converted and provided the corresponding Mayan arithmetic notation for each number. An independent observational study came from section 3 as students were very intrigued about the clarification of the Mayan's prediction of the world ending. Section 3 asked students to covert the date December 21, 2012 AD into the corresponding Mayan calendar date, the Long Count.

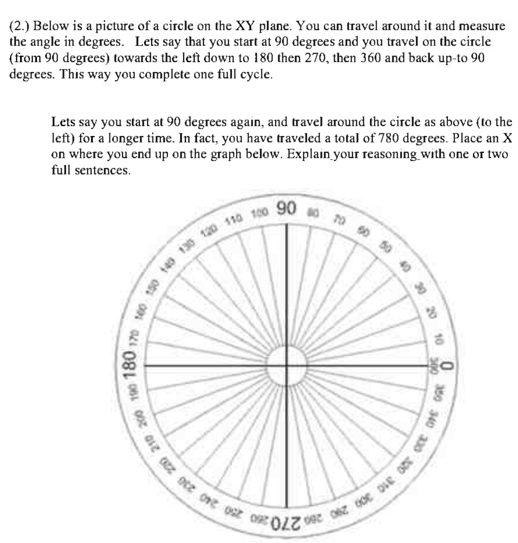
4.3 Parametric Statistics

Class 2 was given a post-test that aimed to measure the effects of the Mayan Lecture and Activity. We tested several hypothesis to determine the quantitative and qualitative value of the lesson plan. We used statistical analysis using a paired t-test for the 13 subjects in Class 2 that completed the pretest and the post-test. The alternative hypothesis are calculated as one-tailed statistical tests with 12 degrees of freedom and let $\alpha = 0.05$. Hence, $H_1 : \mu_1 < \mu_2$. Where μ_1 indicates data from pre-test and μ_2 indicates data from post-test.

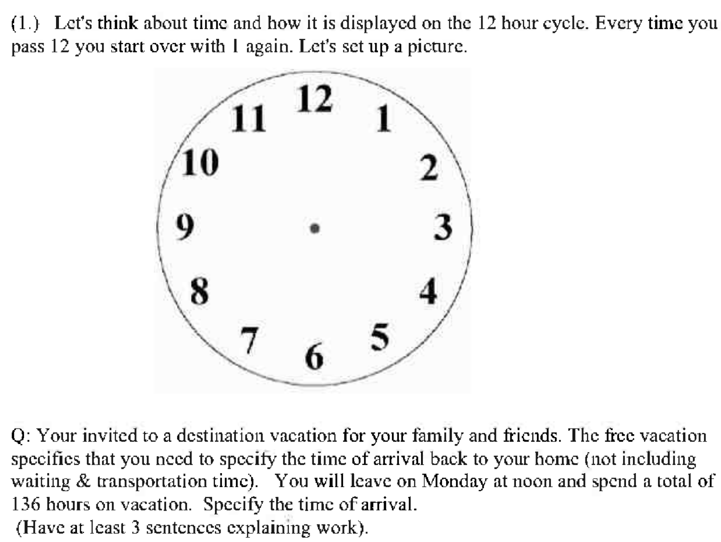
From Q1 in the Pretest and Q2 in the post-test both measure current interest level in math using a range from 1 to 5 (with 5 being the highest).
We test the following hypothesis,
$H_0 : \mu_1 \quad \mu_2$, to determine if the activity had a change on students interest level in math.

The calculated test statistic is $t = -0.2907$ and p-value $= 0.38812$. Since the p-value $> \alpha$, then we fail to reject H_0 . The data suggests that there was not any significant difference between student's level of interest in math. There were five students that indicated the same level of interest in math before and after the powerpoint presentation of modular arithmetic in a Mayan background. A possible confounding factor is having students fill out the post-test during finals week in a math course. During the week of finals, students may be reaching a high level of frustration as the semester comes to an end.

We test the following hypothesis,
$H_0: \mu_1 = \mu_2$, to determine if the activity had a change on students ability to use modular arithmetic as an analytical skill.
The hypothesis measured students ability to arrive at an answer when given a visual aid to compute modular arithmetic.



(a) Pre-Test (Class 2)



(b) Post-Test (Class 2)

Figure 18: Moduli Arithmetic with Visual Representation

The data analysis comes from the pretest Q2, and from post-test Q1 (Figure above). Recall, the answer for pre-test Q2 was 150°, and post-test Q1 is Sunday at 4am. Here we have, $t = -1.72$ and p-value = 0.05541. Here, the p-value is very close to α, therefore having insufficient data to make strong decision.

We test the following hypothesis,

$H_0: \mu_1 - \mu_2$, to determine if Maya Lecture and Activity increased student interest in the Maya civilization.

From Q3 on post-test, On a scale of 1 to 5, (with 5 being the highest), how often did you study the Mayans outside of the classroom?

5: spent more than 4 hours understanding the Mayans

4: searched the internet/books/magazines/TV about the Mayans

3: neutral

2: not sure if furthered my understanding of the Mayans

1: does not make any effort to understand the Mayans

From Pretest Q4, Choose a statement that best applies to you from below.

1: I don't know any Mayan achievements, I was never interested.

2: I have heard something about Mayan science, but do not know details.

3: I have listed examples of Mayan science above, but I cannot describe them. I have never make an effort to find out more.

4: I know of several Mayan achievements described above and I am very proud of them.

We compute the following, $t = -5.196$ and p-value $= 0.00011$. There is significant data to reject the Null Hypothesis, hence, the Maya Lecture and Activity increased students interest in the Mayas. Each student from Class 2 scored higher or the same after the Maya Lesson Plan. Algebra curriculum can benefit from this activity to increase students broader knowledge of the history of Mathematics.

We test the following hypothesis,

$H_0 : \mu_1 = \mu_2$, to determine if the activity had a change on students analytical skills using modular arithmetic.

The hypothesis measured students ability to arrive at an answer when not given a visual aid to compute modular arithmetic. The data analysis comes from the pretest Q5, and the from the post-test Q4.

Pretest Q5, What does 2103_{four} represent in base ten?

Here the correct answer should be expressed and calculated as follows,

$$2 \cdot 4^3 + 1 \cdot 4^2 + 0 \cdot 4^1 + 3 \cdot 4^0 = 147$$

Post-test Q4, We have the number 12,141. We need to make an equation that contains the sum of its multiples in base twenty. Set up the equation so that the sum of the multiples in base twenty is 12,141.

Here the correct answer should be expressed and calculated as follows,

$$1 \cdot 20^3 + 10 \cdot 20^2 + 7 \cdot 20^1 + 1 \cdot 20^0 = 12,141$$

Here we have, $t = -2.497$ and p-value $= 0.014$. Data suggests students benefited from a Maya lecture aimed to introducing modular arithmetic in algebra classrooms. Students are likely to convert from base 10 to base 20 after the Maya activity, whereas they were unlikely to covert from base 4 to base 10 prior to activity.

We test the following hypothesis,

H_0: $\mu_1 = \mu_2$, to determine if activity had improved students ability to visually represent modular arithmetic.

Pretest Q2, recall the XY-plane with moduli 360 from pretest Q2.

Post-test Q5, Write/Draw the number 18,651 in Maya arithmetic notation

The Figure below displays the ideal representation to Q5.

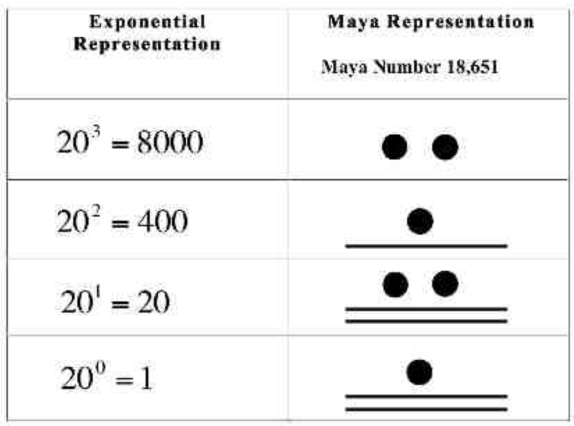


Figure 19: The Maya number 18,651 (Class 2)

Using the vigesimal system, we can calculate 18,651 as follows,

$$11 \cdot 20^0 + 12 \cdot 20^1 + 6 \cdot 20^2 + 2 \cdot 20^3 = 18,651$$

Pretest Q2, determined if students can understand a number in base 4 and convert the same number to base 10. However, post-test Q5 determined if students can convert from a decimal number system to a vigesimal number system.

The statistical results were as follows, $t = -2.497$ and p-value $= 0.014$. Data indicates that we should reject H_0, the activity improved students ability to visually represent modular arithmetic. Recall, earlier in this section we compared PreTest Q2 with Post-test Q1, where the results were inconclusive. Both sets of data measures students improvement of modular arithmetic using visual representations.

We test the following hypothesis,

$H_0 : \mu_1 = \mu_2$, to determine if students are interested in Mayan achievements and/or Maya science.

Pretest Q4 and Post-test Q6 are exactly the same ,

Choose a statement that best applies to you from below

1: I don't know any Mayan achievements, I was never interested

2: I have heard something about Mayan science, but do not know any details

3: I have listed examples of Mayan science above, but I cannot describe them. I have never make an effort to find out more

4: I know of several Mayan achievements described above and I am very proud of them

The results are interesting because, $t = -0.433$ and p-value $= 0.336$, thus failing to reject H_0. Hence, students were less interested in Mayan achievements and/or Maya science after the Maya Lesson activity.

We test the following hypothesis,

H_0 μ_1 μ_2, to determine if activity had improved students knowledge of mathematical or scientific achievements of the Mayans.

Pretest Q6,

Do you know about any mathematical or scientific achievements of Mayans?

Describe them below

Post-test Q7, Write/Draw the number 18,651 in Maya arithmetic notation

The Figure below displays the Pretest Q7 for Class 2.

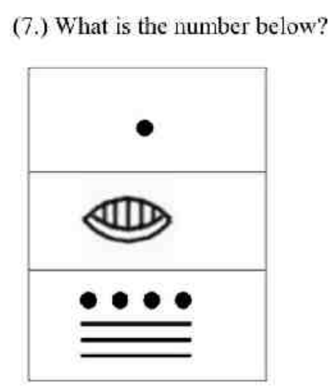


Figure 20: Maya Number Respresentation (Class 2)

We obtain the following, $t = -2.99$ and p-value $= 0.0056$, thus rejecting H_0. Hence, students improved their knowledge of mathematical or scientific achievements of the Mayans. The results are interesting because when students were self-reporting such as in Post-test Q6, they indicated low interest in Mayan achievements. However, Post-test Q7 demonstrated students ability to replicate Maya mathematics.

We test the following hypothesis,

H_0 : $\mu_1 - \mu_2$, to determine if activity had improved students discussion in mathematics.

Pretest Q7,

Q7: On a scale of 1 to 5, (with being the highest), how likely are you to discuss math with family and friends outside of the classroom ?

5: likely to have discussions about math with family and friends

4: somewhat discusses math with family and friends

3: neutral

2: not sure if math is discussed with family and friends

1: does not discuss math with family and friends

Post-test Q8,

Q8: On a scale of 1 to 5 below, how often did you talk about Mayans and mathematics/calendar within the last month?

5: had many discussions about Mayans with family and/or friends

4: had one or two discussions about Mayans with family and/or friends

3: talked about mathematics, but not related to Mayans

2: I only heard others discussing Mayans with family and/or friends

1: did not hear any discussion about Mayans

The results were $t = -0.2$ and p-value = 0.422, thus rejecting H_0. Data suggests that the Maya Lecture and Activity improved students discussion in mathematics in particular to the Mayans. This information leads us to believe that students will get interested in mathematics if they are exposed to different ancient number systems.

5 Artifacts in Guatemala

The Spanish conquest heavily influenced western culture. According to Munoz [7] , "In Mexico, the original mestizaje occured between the colonizing Spanish conquistadores and the indigenous women from various tribes or nations. But over time other races and cultures have modified the original mixing." Although, the Spanish conquest diversified the Mexican identity it almost wiped out the indigenous culture. For example, Catholicism was heavily forced upon the Mayans and rejected many Mayan gods and traditions.

The California Department of Education reported Latino and/or Hispanic students in grades 9-12 had the highest number of dropouts. "Like the Native American peoples, Mexican Americans were subjected to a process of colonization which, in addition to undermining their culture, relegated the majority of them to a permanent pool of cheap labor for US capital " [7]. The undermining of the Maya, mestizaje and the Mexican identity is an underlying factor of the demise of the modern Latino/Hispanic student. It is impossible to determine an isolated reason for Latino students not having high educational goals. The culture of the Latino student emerges from a domino effect from the history of their ancestors. Some of the earliest factors that may have influenced the Latino student include the following: Spanish Colonization, Treaty of Guadalupe Hidalgo, and the Chicano Movement of the 1960's. According to Munoz [7], "Like African Americans, Mexian American working-class children able to attend school were placed in segregated schools. But in contrast, Mexican Americans never had access to even the sort of higher education provided by segregated institutions like the Negro colleges." Mexican Americans were unable to mobilize an influential movement for education, in contrast to educational accomplishments by the African Americans of that era. Currently, Latino students are strongly assimilated into the American culture and rarely exposed to topics that highlight the Latino identity. Mathematics is commonly introduced without any cultural context, hence is not a good place to build ethnic pride for any group. The Chicano Movement of the 1960's paved the way for chicano studies to be available in higher academic institutions in order to provide a vehicle of discovery of the Latino identity. However, the ancient Mayans had a rich culture that is of significance, multidisciplinary and of direct interest to the Latino students should be presented at various educational levels to improve students interest in STEM fields.

The introduction of Maya mathematics with the integration of cultural elements can provide Latino students with a sense of pride. Another component of this research was

Figure 21: MAA 2011

to incorporate the Mesoamerican culture beyond textbooks and articles. I participated in the Mathematical Association of America (MAA) Study Tour 2011 in Guatemala, Mathematics Among the Ancient and Modern Maya. The tour was lead by Dr. Christopher Powell from the Maya Exploration Center.

Studying Maya mathematics in Guatemala gave me an additional perspective of the culture as well as sciences. It is important to note that there is a population of Maya people currently living in Guatemala, they did not all disappear. The mathematical legacy of the ancient Maya is incorporated to everyday living in Guatemala. The careful count of 260-days for the Tzolk'in influence when Guatemalans harvest corn, plant crops, pray or have religious festivities. The creation story found in the Popol Vuh, says that it was the Gods who created people from corn after failing to do so with mud and wood. It is passed down from generations to generations that corn has a count of 260 days from ripe to harvest, and is equal to a women's reproductive cycle. The ancient Maya beliefs about corn remain an integral part of Guatemala's society. Corn is served as a side dish with every meal and can be found in barrels, cloths and woven baskets in the local markets. Traces of the ancient civilization can also be seen in numerous surviving stone monuments and pyramids that have carvings related to Mayan calculations.

The MAA study tour provided the opportunity to understand the cultural framework of the Mayans in Guatemala and Honduras. It also provided me an opportunity to network with a small community of mathematicians that were brought together to share an educational experience of a lifetime. Traveling to Guatemala gave me ability to navigate my graduate studies within the very rich, ancient cultural framework. Creating cultural awareness in mathematical courses will encourage other students to pursue degrees in STEM fields.

6 Conclusion

Our research was motivated by a very dry and abstract mathematical curriculum in the California educational system that fails to address the cultural identity of the students. This study shows that California students benefit from a lesson plan that introduces mathematical problems in cultural context. For example, the vigesimal system used by the Mayans illustrates their society's ability to understand modular arithmetic and how to apply it to the world surrounding them. Students exposed to these topics through Mayan achievements have an opportunity to combine mathematical understanding with a cultural context. This study proved the implementation of ethnomathematics activities in a classroom creates interest in Mayan mathematical discoveries. Students' achievement and their attitudes towards science improved and they appreciated the opportunity to get inspired through an interesting ancient context. Statistical analysis of the pre-test and post-test demonstrates that California students enrolled in introductory algebra courses do benefit from a Maya lesson plan.

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