Impact of story-telling on students' learning of mathematics

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Title of Item

Math Thesis; Storybook; Research Results; Parent Teacher Guide for Math
3 to 5 keywords or phrases to describe the item

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6/25/2014
Date

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Abstract

The two different language based projects were designed to introduce the concepts of parabolas and integrals. Integration was introduced to college students, and eighth grade students worked with parabolas. We incorporated a story and acting exercises to help students be active participants in the learning process. To make it interesting to children, the story was illustrated with colorful pictures in the eighth grade activities. The goals for this research were to evaluate if there was a significant difference in the overall performance of the test groups versus the control groups, and if there was a significant improvement in learning on the conceptual problems. Our data shows that the project was successful and students involved in the story based activities performed better.

Introduction

> *I don't know about you, but when I was in elementary school, math was boring and dry and that was just the way it was. I truly did like math. But in order to feel the thrill of comprehending a difficult concept, you only had one choice: you just had to listen to your teacher's lecture and find your own way to wrap your mind around the new concepts. – Beth Lewis* **[BETH]**

We wanted to involve students in fun language based math activities. We hoped to increase students' interest and the desire to learn math through interactive story based instruction teachers and parents could utilize.

> *Another important concern is the actual language… That is, young children have difficulty in understanding abstract concepts. – Michael L Bernard* **[BERN]**

We believed that when a mathematical concept is taught in a context of an interesting story, it comes alive for the students. Our students were engaged in the plot without realizing that they're participating in a learning process. We wanted to show that if students participate in new teaching methods such as learning mathematics through storytelling, then they improved their performance improves, especially on conceptual understanding.

Jackie Baldwin and Kate Dudding in *Storytelling in Schools* **[BALD]**, advocated having students write math stories. The goal of their study was to have students write a story which contained a math concept covered in class and then test to see how well the students retained the information. They show that the stories helped the students to see links between math and their daily lives.

The article, *In Abstract: Avoid Concrete Examples When Teaching Math*, analyzed two learning environments with the following results:

> *One group was taught using generic symbols such as circles and diamonds. The other groups were taught using practical scenarios such as combining liquids in measuring cups. The researchers then tested their grasp of the concept by seeing how well they could apply it to an unrelated situation, in this case a children's game. The results: students who learned using symbols on average scored 80 percent; the others scored between 40 and 50 percent, according to Kaminski.* **[NIKH]**

This quote shows that often learning in context may cause lack of abstract understanding; therefore, we needed to balance symbolic and practical approaches. The story we introduced took concepts into concrete examples that participants could visualize and draw. Student performance based on the teaching method in the study groups showed higher retention of information and improved level of understanding in concepts versus their peers in the control groups.

Motivation

We wanted to evaluate whether the story based method of teaching really matters. Hence this study was designed to test whether students in control groups and test groups performed differently on their post-tests compared to their respective pre-tests. Students in both test groups participated in reading the story. Furthermore, the college students volunteered to play different characters from the story and put on a play during class session, while the eighth grade students discussed the pictures out loud.

Our story was called “The Mathlin Kids.” The plot walked participants through a series of hurdles which required mathematical know-how to overcome so that the characters may recover a missing book. This missing book contained fundamental math concepts essential to our world, and their task was to find and return the book to its proper place.

Education literature shows that storytelling is successful in teaching concept and abstract ideas associated with math and science. Literature additionally shows that students are more engaged participants in the learning process when storytelling and illustrations are added, as well as, retain the information better than students who weren’t exposed to this style of teaching. The length of information retention needs to be tested further.

Methodology and Experiment Design
Experiment 1: Integrals

The study was done in two Calculus I sections at California State University Channel Islands. Class duration was 90 minutes, and students were starting on the integration chapter.

At the start of the class, both groups were given five minutes to complete a short survey consisting of four questions. The survey is presented in Appendix A. The survey was evaluating students learning style and math background.

The survey was followed by a pre-test consisting of three questions and students in both groups were given ten minutes to complete it. We had labeled the pre-test "Activity Prior to Lecture," and it's presented in Appendix B. The first question on the pre-test was testing the understanding of the definition of integration. The second question was a word problem involving integration, and the third one was to solve a given integral.

The control group studied the concept of using traditional lecture-based method. The lecture was based on their textbook: *Calculus, Early Transcendental Functions by Larson and Edwards, Fifth Edition*. The lecture started with the definition of Riemann sums, incorporated the previously studied concept of limits along with covering the fundamental theorem of Calculus, and then concluded by presenting the integral.

The test group covered the concept of integration through a short story, *The Mathlin Kids,* presented in Appendix C. *The Mathlin Kids* was a four page long story, with four main characters. These characters were on a mission to find the lost book of *Hisaab*. *Hisaab* contained the fundamental mathematical concepts essential for the Human world. Through the tasks during their search for the book, the characters learnt the concept behind integration. The students in the test group acted out the story and discussed the plot and mathematical ideas.

The time allotted to cover the definition of integration was 40 minutes for both groups.

After the lecture for the control group, and *The Mathlin Kids* activity for the test group, both groups followed three integration practice problems done by the instructor on the board for the class. These common integration functions and practice problems are presented in Appendix D. The allotted time for this portion was twenty minutes.

A post-test consisting of three questions was administered to both groups. The students were again given ten minutes to complete it. We labeled the post-test "Activity After Lecture." The post-test is presented in Appendix E.

Methodology and Experiment Design
Experiment 2: Parabolas

This part of the study was done using two eighth grade Honors Algebra sections in one of the local schools in Oxnard. Class duration was 53 minutes.

At the start of the class, students were given "Quiz 1," the pre-test consisting of four questions, presented in Appendix F. The students in both groups were given an equal amount of time to complete it. The first and fourth questions on the pre-test were checking their understanding of the graphs of parabolas. The second question was purely computational, and the third question involved graphing.

The control group covered the concept of parabolas by a mixture of lecturing and examples which their teacher had prepared.

The test group covered the concept of parabolas through an excerpt taken from the short story, *The Mathlin Kids,* presented in Appendix G. The story was covered in a group environment by reading aloud, illustrated by pictures of the characters and concepts, followed by an active group discussion.

After the lecture for the control group, and the story time based on *The Mathlin Kids* activity for the test group, both groups were given a post-test consisting of four questions. The layout of the post-test was similar to the pre-test where the first and fourth questions on the post-test were conceptual. The second question was purely computational, and the third question involved graphing. The post-test was called "Quiz 2" and is presented in Appendix H. The students were again allotted an equal amount of time to complete the post-tests.

Methodology and Experiment Design
Hypothesis Testing

For each experiment we computed results to unpaired t-tests to compare the samples at the beginning of each study. We were disappointed to find out that the test and control groups in both studies were not similar. Both the Calculus and Eight grade test groups were significantly weaker than their peers in the control groups. At this point, we had hoped that both groups would improve significantly.

After collecting the post-tests, paired t-tests were done between the pre-tests and post-tests in each class, as well as, t-tests of the test group's performance versus the control group's performance on the post-tests. We checked for improvement of understanding the concept by testing the following hypotheses:

Test Group	Control Group
H_o: $\mu_{XPre} = \mu_{XPost}$	H_o: $\mu_{YPre} = \mu_{YPost}$
H_a: $\mu_{XPre} < \mu_{XPost}$	H_a: $\mu_{YPre} < \mu_{YPost}$
$H_{a'}$: $\mu_{XPost1} > \mu_{YPost1}$	$H_{a'}$: $\mu_{YPost1} < \mu_{XPost1}$

The scoring schemes to represent the collected data for the survey, pre-tests and post-tests are presented in Appendix K.

Hypothesis

Our overall hypothesis was the following:

If students participate in the new teaching methods of learning mathematics through storytelling, then their understanding and interest in mathematics increases.

We did test the following statistical hypotheses to evaluate students' understanding and performance:

H_c: Learning mathematics through storytelling has no effect on students' performance.

H_a: Learning mathematics through storytelling has a positive effect on students' performance.

$H_{a'}$: Learning mathematics through storytelling has a positive effect on students' performance on conceptual questions.

We tried to measure the performance of each group and compared their results.

Data Collection

Data was collected from the control and test groups for both experiments by administering pre-tests before learning the concept, and post-tests after the activities. Additionally, a survey from the college students prior to the pre-test evaluating their math background (Appendix A) was administered; and, the eighth grade students were simply asked to comment on the story at the end of the post-test (Appendix I).

We calculated the following statistical parameters to describe students' performance:

- Average score for the pre-test in the control groups
- Average score for the post-test in the control groups
- Average score for the pre-test in the test groups
- Average score for the post-test in the test groups
- Improvements on individual questions from the pre-tests were also compared to the corresponding post-test questions
- Control groups' pre-test and post-test questions were compared to the test groups' pre-test and post-test questions.
- Statistical parameters, such as, mode, median, standard deviations, various correlation coefficients and t-test results representing data were also calculated.

The raw data and calculated parameters are included in Appendix L. All data analysis was done using MS Excel.

Data and Analysis
Experiment 1

In this chapter we provide descriptive analysis of our data collected in Experiment 1, our Calculus students.

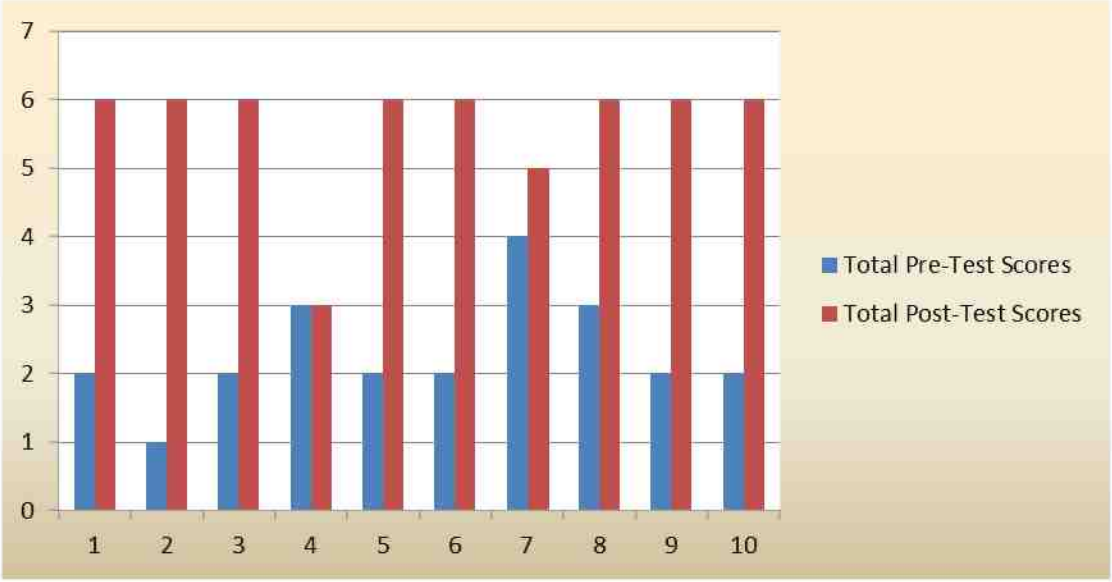


Figure 1: Calculus Test Group – Pre-Test and Post-Test Results by Student

Figure 1 shows the pre-test and post-test scores of the 10 students in the test group. Notice how all the students made great improvements, with the exception of one student. Eight out of the ten students received a perfect score on the post-test.

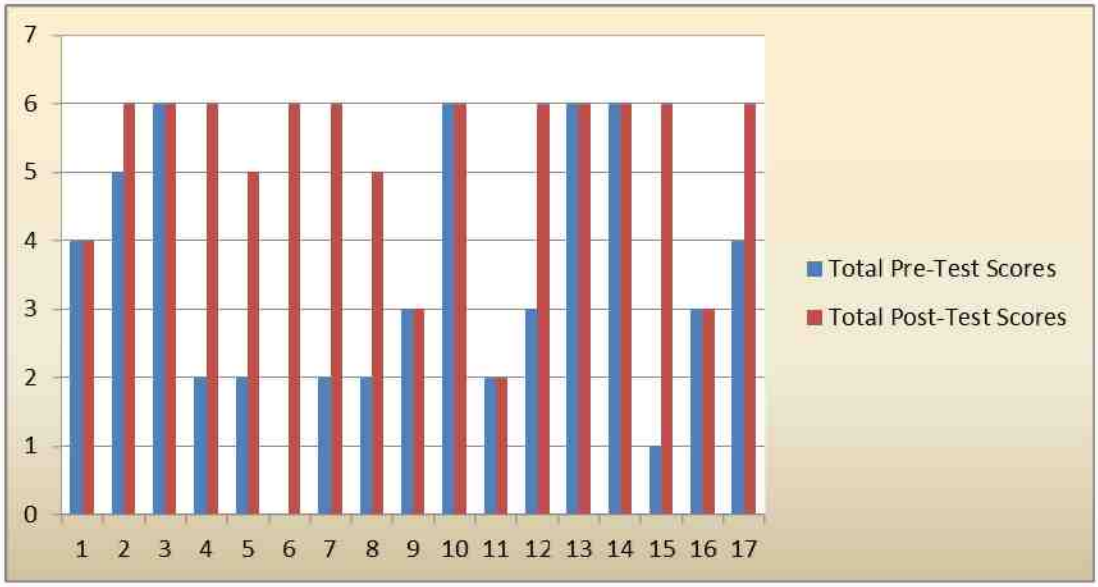
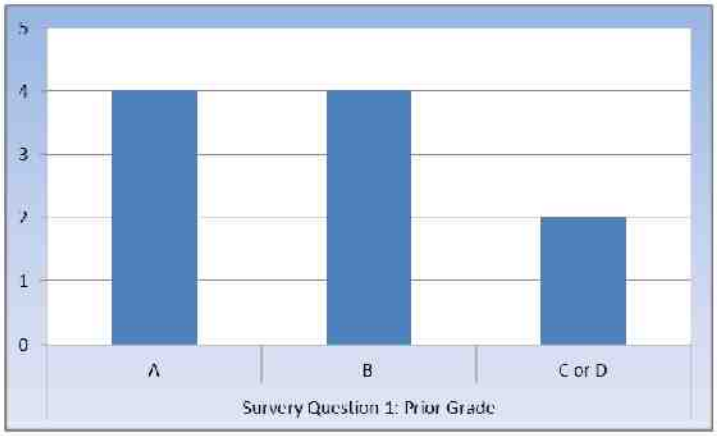
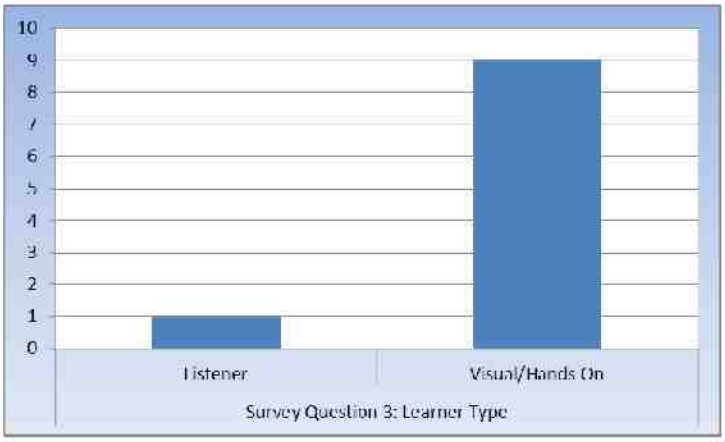


Figure 2: Calculus Control Group – Pre-Test and Post-Test Results by Student

Figure 2 shows the comparison of pre-test and post-test scores for the 17 control group students. Four students scored a perfect score on the pre-test and the post-test. Then we see another four students whose scores remained unchanged. Therefore, out of 17 students, if we ignore the four that scored a perfect scores on the pre-test and post-test, we see that 69% of the control group students improved, compared with 90% of the test group students.





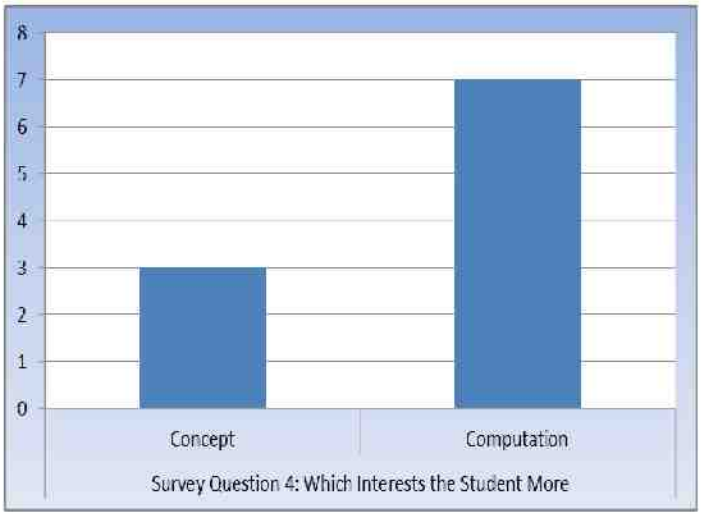


Figure 3: Calculus Students Survey Results

The results of the student survey are displayed in Figure 3. Note that 80% of students got at least a B in their prior math course. Survey Question 2 was disregarded since every single student in both groups liked stories. Almost all students declared themselves as visual/hands on learners; hence visuals and acting should be considered as a part of student response testing. Note that only three students from this test group were more interested in learning concepts, rather than computations. It was interesting to see how each student performed on the concept based question from this group compared to their peers in the control group.

Pearson Correlations: Test Group		
Q1 & Pre-Test	-0.273	Little to no correlation
Q1 & Post-Test	0.116	Not a strong correlation
Q3 & Post-Test	0.218	Little to no correlation
Q4 & Pre-Test	-0.285	Little to no correlation
Q4 & Post-Test	-0.285	Little to no correlation

Figure 4: Pearson Correlation Coefficients for Calculus Test Group

We then ran the Pearson correlation on the survey question responses to the pre-test and post-test questions presented in Figure 4 and found that there was no correlation between the survey responses and how the test group students performed on the tests.

	Test: Pre-Test	Control: Pre-Test	Test: Post-Test	Control: Post-Test
Mean	2.3	3.352	5.6	5.176
Mode	2	2	6	6
Median	2	3	6	6
Standard Deviation	0.823	1.902	0.966	1.333

Figure 5: Calculus Test Group versus Control Group

In Figure 5 note the difference in mean scores for the Test Group versus the Control Group. The test group had a mean 1.052 below the mean score of the control group. This means that the preparation of the test group was much weaker than that of the control group. However, the test group scored 0.424 units above the mean score of the control group on the post-test. The mean scores from the pre-test to the post-tests were 3.3 units apart for the test group versus 1.824 units for the control group. Our test group improved more than the control group.

Unpaired T-Test Results between the test group and the control group:			
For the Pre-Tests:	0.0555	Since 0.0555 > 0.05 there is no significant difference between the scores; although there is some slight difference.	
For the Post-Tests:	0.1949	Since 0.1949 > 0.05 there is no significant difference between the scores	
Paired T-Test Results:			
Pre-Test vs. Post Test	Test Group	9.27E-05	Since 0.0000927 < 0.05 there was significant improvement
Pre-Test vs. Post Test	Control Group	0.002	Since 0.002 < 0.05 there was significant improvement

Figure 6: Calculus Test Group versus Control Group T-Test Results

The unpaired t-test results presented in Figure 6 show that there is no significant difference in the pre-test results of the two groups on a 95% confidence interval; hence the groups were compatible at the beginning of the experiment. However, if we expand our confidence interval to 90%, we see that the test group was indeed at a significant disadvantage at the start of the study. Our study group was weaker. To our amazement, the test group displayed no significant difference in the post-tests even if we switch to a 90% confidence interval. Therefore we conclude that our story based teaching method was successful. Students in both groups performed similarly at the end of the study. Paired t-tests display that both groups showed significant improvement from pre-test to post-test.

The values in Figure 7 were used to calculate a t value of 0.87656. The critical value used (1 tail at α = 0.05 (95% confidence level)) is 1.740, which was obtained from tables. Since the calculated t value of 0.87656 < 1.740 we will **fail to reject** the null hypothesis. H_0: Learning mathematics through storytelling has no effect on students' performance. The results obtained show that the students who learned with the story-based method had a positive difference in performance.

Post-Test: Test Group	Post-Test: Control Group	
5.6	5.176	Mean
0.966	1.333	Standard Deviation
10	17	Count

Figure 7: Calculus Students Data Used For Critical Value Comparison

Next we tested the improvement on conceptual question 1 in the two groups. Interesting discovery on question 1, was the mean score on pre-test of the Test Group versus that of the Control Group was 1.9 versus 2.05, respectively. For the conceptual question on the post-test the mean score of the Test Group versus the Control Group was 3.0 versus 2.82, respectively. Every student in the test group had a perfect score of 3 on the conceptual question. Hence, our story telling method helped the weaker students the most.

Average Improvement on the Conceptual Question		
	Test Group	**Control Group**
Mean	1.1	0.76
Mode	1	0
Median	1	0
St Dev	1.1	1.09

Figure 8: Average Improvement on the Conceptual Question

Unpaired t-test on question 1 shows the test group did significantly better at 90% confidence interval.

Data and Analysis
Experiment 2

In this chapter we provide descriptive analysis of our data collected in Experiment 2, our Eighth grade students.

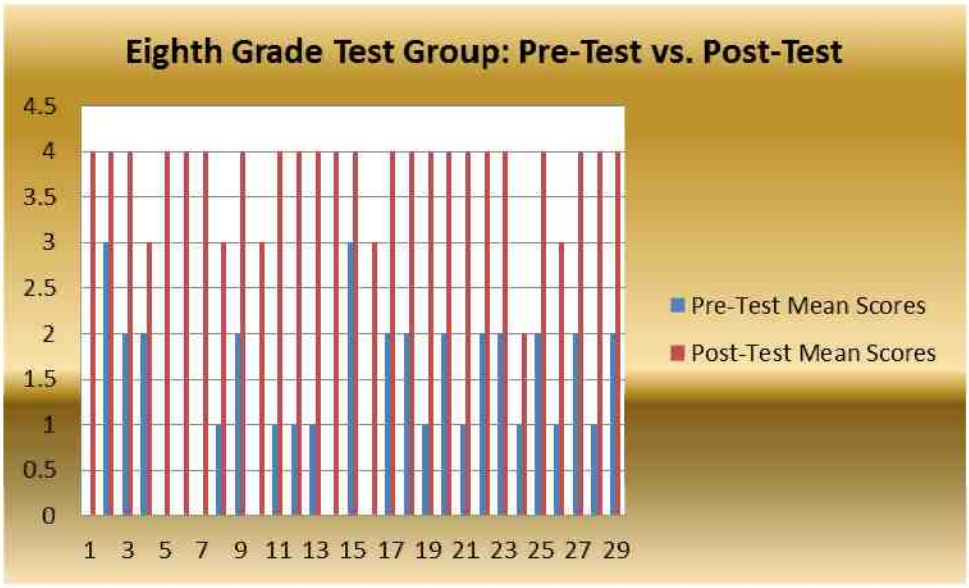


Figure 9: Eighth Grade Test Group – Pre-Test and Post-Test Results by Student

Figure 9 shows the pre-test and post-test scores of the 29 students in the test group. Notice how all the students made great improvements. 23 students received a perfect score on the post-test; 5 of which went from a pre-test result of zero to a perfect post-test score.

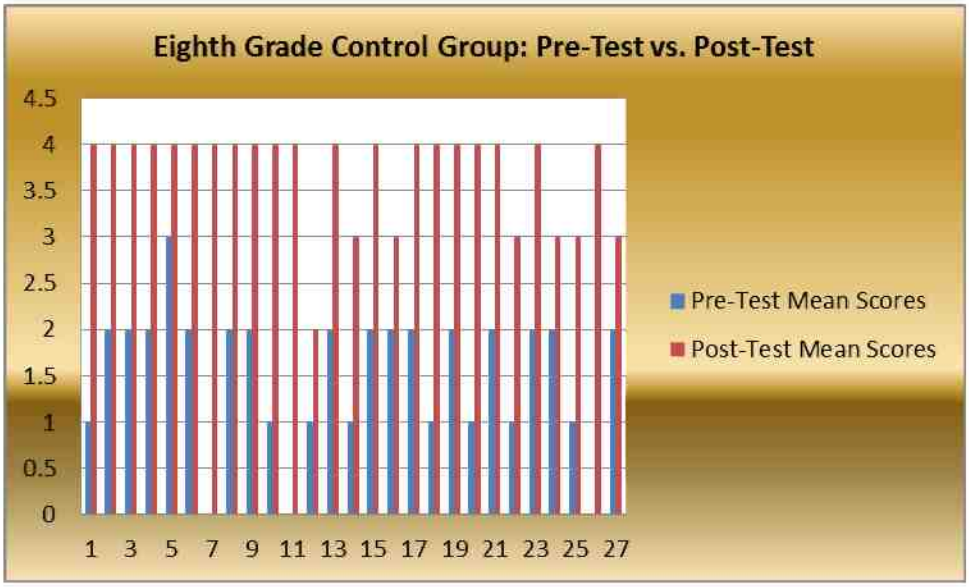


Figure 10: Eighth Grade Control Group – Pre-Test and Post-Test Results by Student

Figure 10 shows that all 27 control students also improved from pre-test to post-test. In this group only three students went from scoring a zero on the pre-test to a perfect score on the post-test. Overall, 20 students in this control group received a perfect score. Therefore the test group had a higher percentage of perfect scores on the post-tests: 79.3% in the test group versus 74.1% in the control group.

Figure 11 shows the results of the Pearson correlations on the individual questions.

Notation for the table above is as follows:

- Lower case "x" represents the pre-test for the control group, while an upper case "X" represents the post-test for the control group
- Lower case "y" represents the pre-test for the test group, while an upper case "Y" represents the post-test for the test group
- The numerical subscripts represent the question number in the pre-tests, and the alphabetical subscripts represent the corresponding questions on the post-test. For example, "x_1, X_a" means we are comparing question one in the pre-test for the control group to question 1 on the post-test for the control group

Analysis - Pearson Correlations		
x_1, X_a	Can't Calculate	Note: Every student had a perfect score of 1 on X_a in the post-test
x_2, X_b	0.1474	There is little to no correlation
x_3, X_c	0.0555	There is little to no correlation
x_4, X_d	0.1673	There is little to no correlation
y_1, Y_a	Can't Calculate	Note: Every student had a perfect score of 1 on Y_a in the post-test
y_2, Y_b	0.2583	There is little to no correlation
y_3, Y_c	Can't Calculate	Note: Every student scored a zero on y_3 in the pre-test
y_4, Y_d	Can't Calculate	Note: Every student had a perfect score of 1 on Y_d in the post-test

Figure 11: Pearson Correlation Coefficients for Eighth Grade Group

Points to note in Figure 11 are that every student in the test group received a perfect score on both of the conceptual questions – questions 1 and 4. Every participant in the control group only received a perfect score on question 1. Also note that all 29 participants from the test group started out by receiving a zero on question 3 in the pre-test, and then 27 of these 29 participants had a perfect score on question 3 on the post-test.

	Test: Pre-Test	Control: Pre-Test	Test: Post-Test	Control: Post-Test
Mean	1.2759	1.5185	3.7586	3.7037
Mode	2.0000	2.0000	4.0000	4.0000
Median	1.0000	2.0000	4.0000	4.0000
Standard Deviation	0.9218	0.7530	0.5110	0.5417

Figure 12: Eighth Grade Test Group versus Control Group

In Figure 12 note the difference in mean scores for the Test Group versus the Control Group. The test group had a mean score of 0.2426 units below the mean score of the control group. This means that the preparation of the test group was much weaker than that of the control group. However, the test group scored 0.0549 units above the mean score of the control group on the post-test. The mean scores from the pre-test to the post-tests were 2.4827 units apart for the test group versus 2.1852 units for the control group. Our test group improved more than the control group.

Unpaired T-Test Results between the test group and the control group:			
For the Pre-Tests:	0.1438	Since value > 0.05 there is no significant difference between the scores	
For the Post-Tests:	0.3489	Since value > 0.05 there is no significant difference between the scores	
Paired T-Test Results:			
Pre-Test vs Post Test	Test Group	7.98E-13	Since value < 0.05 there was significant improvement
Pre-Test vs Post Test	Control Group	3.13E-14	Since value < 0.05 there was significant improvement

Figure 13: Eighth Grade Test Group versus Control Group T-Test Results

The unpaired t-test results presented in Figure 13 show that there is no significant difference in the pre-test results of the two groups on a 95% confidence interval. Paired t-tests display that both groups showed significant improvement from pre-test to post-test.

The values in Figure 14 were used to calculate a t value of 0.3904. The critical value used (1 tail at α = 0.05 (95% confidence level)) is 1.740, which was obtained from tables. Since the calculated t value of 0.3904 < 1.740 we will **fail to reject** the null hypothesis. H_0: Learning mathematics through storytelling has no effect on students' performance. The results obtained show that the students who learned with the story-based method had a positive effect in performance.

Post-Test: Test Group	Post-Test: Control Group	
3.7586	3.7037	Mean
0.5110	0.5417	Standard Deviation
29	27	Count

Figure 14: Eighth Grade Students Data Used For Critical Value Comparison

Interesting discoveries:

i. The mean score on question four, which was a conceptual question, mean pre-test score of the Test Group versus that of the Control Group was 0.068966 versus 0.25925, respectively. On the

post-test the mean score of the Test Group versus the Control Group was a perfect score of 1.0 versus 0.92593, respectively.

ii. Additionally, the test group showed improvement in all four questions versus the control group improving in three out of the four questions.

iii. Every student in the test group scored a perfect score of 1.0 on questions one and four, while every student in the control group scored a perfect score on only question one. Note the mean improvement in the improvement from pre-test to post-test score on question 4 of the test group versus the control group as presented in Figure 15.

Average Improvement on Question Four		
	Test Group	**Control Group**
Mean	0.9310	0.6667
Mode	1.0000	1.0000
Median	1.0000	1.0000
St Dev	0.2579	0.4804

Figure 15: Average Improvement on Question Four

Student feedback revealed that this method of teaching through story-telling and illustrations "is a creative way to show how quadratic problems function." Another student claimed that the student liked the book "because the colorful demonstrations taught me a lot about parabolas." Interestingly enough, the students which commented "No, I did not like the story because it made no sense; I did like the pictures though!" and "No, because it didn't tell me what happened to the book after. I did like the illustrations though" ended up with perfect scores on the post-tests. More student feedback is included in Appendix I.

Results
Experiment 1

Recall that our overall hypothesis was the following: If students participate in the new teaching methods of learning mathematics through storytelling, then their understanding and interest in mathematics increases. Also recall that tested the following statistical hypotheses to evaluate students' understanding and performance:

H_0: Learning mathematics through storytelling has no effect on students' performance.

H_a: Learning mathematics through storytelling has a positive effect on students' performance.

$H_{a'}$: Learning mathematics through storytelling has a positive effect on students' performance on conceptual questions.

Based on the paired p-values of the pre-test and post-test results for the two groups, there was significant improvement in both groups: for the test group, p-value is equal to 9.27293E-05, and for the control group, p-value is equal to 0.002230549. Improvement in the test group was more significant than the improvement in the control group; this was mainly due to the scores on the conceptual problem. However, we will fail to reject H_0 since the calculated t value of 0.87656 was less than 1.740, the critical value of 1 tailed test at $\alpha = 0.05$ (95% confidence level)). The results obtained show that the students who learned with the story-based method had a difference in performance. Based on the p-value of the post test results for the two groups, which was equal to 0.19 which is greater than significance level of 0.05 we can say that both groups – the test and the control groups – learned the material taught to them regardless of the teaching method used in the class. In the control group, the average score on the Post Test was 5.17, and in the Test Group, the average score was 5.6. Point to note, the test group did perform slightly better than the control group. Furthermore, we will accept H_a, learning mathematics through storytelling has a positive effect on students' performance, and since the paired t-test results confirmed a significant difference in the pre-test and post-test results of the test group at a 95% confidence interval.

The mean score on the pre-test for the test group was lower than the mean score on the pre-test for the control group: 2.3 versus 3.35. However, the mean post-test score for the test group was higher than the mean post-test score for the control group: 5.60 versus 5.17. Please note that question 2, a word problem, was thrown out on the pre-tests and post-tests of both Calculus groups since both groups scored a zero on the pre and post-tests. Additionally, based on Pearson values, there was little to no association between the survey question answers and how the students scored on the pre-tests or the post-tests in the Calculus group.

Analyzing the collected data, we will also accept $H_{a'}$, which was that learning mathematics through storytelling has a positive effect on students' performance on conceptual questions. The mean score for the test group on the conceptual question on the pre-test was 1.9 (compared to the control group at 2.05), their post-test mean score was a perfect score of 3 (compared to the control group at 2.8).

In summary, the story and activities done in the Calculus class were as effective as a traditional lecture. Additionally, students' performance with the story based method was significantly better and the students enjoyed the class.

Results
Experiment 2

Recall that our overall hypothesis was the following: If students participate in the new teaching methods of learning mathematics through storytelling, then their understanding and interest in mathematics increases. Also recall that tested the following statistical hypotheses to evaluate students' understanding and performance:

H_C: Learning mathematics through storytelling has no effect on students' performance.

H_a: Learning mathematics through storytelling has a positive effect on students' performance.

$H_{a'}$: Learning mathematics through storytelling has a positive effect on students' performance on conceptual questions.

Based on the paired p-values of the pre-test and post-test results for the two groups, there was significant improvement in both groups: for the test group, p-value is equal to 7.98E-13, and for the control group, p-value is equal to 3.13E-14. Point to note is that the test group showed improvement in all four questions versus the control group improving in three out of the four questions. Based on the results from the hypothesis testing against a critical value we will fail to reject H_C since the calculated t value of 0.3904 was less than 1.740, the critical value of 1 tailed test at $\alpha = 0.05$ (95% confidence level)). The results obtained show that the students who learned with the story-based method had a difference in performance. Based on the p-value of the post test results for the two groups which was equal to 0.3489 which is greater than significance level of 0.05 we can say that both groups – the test and the control groups – learned the material taught to them regardless of the teaching method used in the class. In the control group, the average score on the Post Test was 3.7037, and in the Test Group, the average score was 3.7586. Point to note, the test group did perform slightly better than the control group. Furthermore, we will accept H_a, learning mathematics through storytelling has a positive effect on students' performance, since the paired t-test results confirmed a significant difference in the pre-test and post-test results of the test group at a 95% confidence interval. The mean score on the pre-test for the test group was lower than the mean score on the pre-test for the control group: 1.27 versus 1.5. However, the mean post-test score for the test group was higher than the mean post-test score for the control group: 3.76 versus 3.70.

Drilling down even further into the collected data, we will also accept $H_{a'}$, which was that learning mathematics through storytelling has a positive effect on students' performance on conceptual questions. The mean score on question four, which was a conceptual question, mean pre-test score of the Test Group versus that of the Control Group was 0.068966 versus 0.25925, respectively. On the post-test the mean score for the conceptual questions of the Test Group versus the Control Group was a perfect score of 1.0 versus 0.92593, respectively.

In summary, the story and illustrations covered in the Eighth grade class were as effective as a traditional lecture. Additionally, students' performance with the story based method was significantly better on the conceptual questions. Furthermore, student feedback revealed that they thoroughly enjoyed the illustrations and different approach to the introduction of parabolas.

My Contributions to Math Education

1. Developed a total of three children's math story books, along with creating the characters, which yielded successful results when tested.

2. This thesis tests the third book in the series. The book was based on feedback and results achieved on the first two books. The study was conducted in Fillmore Unified School District.

3. Incorporated common core math material in the story book.

4. Designed a parent / teacher guide to accompany the book as an added resource. The guide includes additional discussions, as well as, quizzes and tests which allow any instructor to test the progress of their student. The quizzes and tests mimic the format of the standardized tests administered in the State of California.

Conclusions and Future plans

It was a real eye opener to see how students improved their learning of math concepts through an interesting story based teaching method. The results are very encouraging and we hope to do future research in the field. We hope that the practitioners in the field of teaching Mathematics find this study useful, and encourage their students to use math in stories. Over time, we hope there will be a series of math textbooks with story sections.

Our initial recommendation is to include short stories while introducing new math concepts that illustrate the underlying ideas. More introductory story telling should be done to students at all ages. Future research plans include testing this entire story for a longer time period at elementary schools and evaluating students' conceptual understanding. The hope is that teachers themselves will generate context stories that may improve educational experiences and learners performance in the classrooms.

Appendix A – Student Survey

This survey was given to the college students prior to administering the pre-test.

1. Circle the most current math class you took. Write the letter grade you received in it.
- Pre-Calculus. Grade: __
- Calculus . Grade: __
- Other. Please Specify: _____. Grade: __
2. I am more interested in: (Circle one of the choices below)
- "Computational" Math
- "Conceptual" Math
3. Do you like stories? (Circle one of the choices below)
- Yes
- No
4. Do you like Art? (Circle one of the choices below)
- Yes
- No
5. Do you like Acting? (Circle one of the choices below)
- Yes
- No
6. How do you feel you learn math best? (Circle one of the choices below)
- Listening
- Reading
- Hands On Learning/Visual
7. How many other classes are you currently taking? (Circle one of the choices below)
- 1 more class other than this one
- 2 more classes other than this one
- 3 more classes other than this one
- 4 or more classes other than this one

Appendix B – Activity Prior to Lecture

1 – Draw an x-y grid. Then draw a random continuous non-zero function. Now shade in the integral from x = 0 to x = 1.

2 – The number of number of firms registered in the U.S. increased significantly in the early 1990s, at a rate of approximately 1,400 per year. Further, a total of 21,600 firms were registered by September 1994. Use a definite integral to estimate the number registered in September 1991.

3 – Solve $\int x^3 + 2x^1 \, dx$ from x=0 to x=1. Please write down the formula/integration function you will be using.

Appendix C – "The Mathlin Kids"

We're at the Calcu-concept town's bakery, and the door of the bakery flings open blowing in a gush of wind and lots of snow. In the midst of all that snow, there stands the town's bookkeeper.

Bookkeeper: "I have some terrible news Mathlin. It seems that someone has stolen Hisaab from the town's library."

Mathlin: "Oh no!"

Child 1: "What's Hisaab?"

Mathlin: "It's the ancient book that contains fundamental mathematical concepts. Mathematical concepts are essential for the growth and development of human society."

Mathlin: "Are there any clues as to who took the book?"

Bookkeeper: "I didn't see any Mathlin."

Matheman: "We should go to the library to investigate."

Mathlin: "Good Idea Matheman. Bookkeeper, we are on the case."

Bookkeeper: "Thank you Mathlin."

Mathlin: "There's room for just two children in the carriage with us. The rest will have to stay back."

Two children, Mathlin and Matheman all got into the carriage and went to the library.

Mathlin: "Did anyone find any clues outside the building?"

Child Two: "There is a sequence of numbers on the side wall of the building with a missing number."

They all walk over to the sequence of numbers.

The sequence of numbers reads as follows: "4, 6, 10, 12, 16, 18, , 24, 28, 30"

Child 1:"I know the pattern. It is +2 then +4. If we follow the pattern of +2, +4, then the missing number in the sequence is 22!"

The wizard waved his wand and inserted "22" in the number sequence.

All of a sudden, there was thunder and lightening.

Children: "Wow!"

Mathlin: "Everyone stay behind me."

They entered the doorway. Mathlin entered first. Matheman went in second. The children followed.

It was dark. Mathlin used his wand as a flashlight. They were at the top of a giant slide.

A loud voice said *"Slope please."*

Mathlin: "The rise is -10. The run is 5. So we divide -10 by 5. So our slope is negative 2."

The Voice:"Absolutely right! You may slide."

Everyone slid down the giant slide and landed on a giant mat with two intersecting curves. There was a big sign that read: "Jump at the equilibrium point"

Matheman: "Children, an equilibrium point is a point is where two curves intersect. Let's all walk over to that point and jump."

Upon jumping, they were transported to a mathemagical land and were set down under a giant talking tree.

There were mathematical symbols and shapes everywhere. This land was extremely colorful.

Tree: "Hello Mathlin. I've been expecting you."

Mathlin:"Hello tree. Do you know what happened to Hisaab?"

Tree: "Ah yes, the book of mathematical concepts. You must find the book and return it to the Calcu-concept town's library for future humans to use. Follow that alphabet road and you'll see a house on your right. That is where Mr. Quadratic lives. He will point you in the right direction. Good luck!"

Mathlin, Matheman, and the children followed the alphabet road until they finally saw Mr. Quadratic's house on their right.

They went up to the door and the door opened.

A Voice: "Come on in!"

Mathlin, Matheman, and children went inside.

They saw the Mr. Quadratic and Mr Cubic sitting at a round table having tea and pastries.

Mr. Quadratic: "Hello, I'm Mr Quadratic. This is my cousin, Mr Cubic. Would you like to join us for some tea and pastries?"

Mathlin: "No, thank you. We're investigating the disappearance of Hisaab from the human world. Do you have any information we can use?"

Mr Quadratic: "We sure do! We saw this ghost like creature running down the alphabet road with Hisaab. He went till the end of the alphabet road where there is a giant wall."

Mr Quadratic: "The wall has two parabolas on it. One is a positive parabola, and one is a negative parabola."

Child 1: "What's a parabola?"

Mr Quadratic: "Parabola is the name of the shape that a quadratic function makes. I'm Quadratic."

Child 1: "What's a quadratic function?"

Mr Quadratic: "A quadratic function is where an unknown quantity is squared. Let's say the unknown quantity is "x". x^2 is the unknown quantity squared. x^2 is a quadratic function."

Child 1: "Okay. I'll remember that."

Mathlin: "What do we do when we get to the giant wall?"

Mr Quadratic: "Connect the maximum point of one of the parabola's to the minimum point of the other parabola. The wall will disappear for a few minutes and you can pass through. That is where the creature has hidden Hisaab."

Mathlin, Matheman and the children went till the end of the alphabet road. They got to the giant wall that had the two parabolas.

Child 1: "Let's connect the maximum point of one parabola, to the minimum point of the other parabola."

Child 2: "What's a maximum and what's a minimum?"

Matheman: "A maximum point is the highest point in a curve. A minimum point is the lowest point in the curve."

Child 2: "I see. So we connect this point to that point – right?"

Mathlin: "Absolutely!"

Mathlin connected the two points with his magic wand and the giant wall disappeared.

Right in front of them was a curve. The area under the curve was colored.

Mathlin: "Where do you suppose Hisaab is? We have no more leads, and we must find that book!"

Matheman: "What if we find the area under the curve?"

Mathlin: "True. "

Child 2: "How do we calculate the area under the curve?"

Matheman: "What if we use rectangles?"

Mathlin: "Rectangles?"

Matheman: "Yes, we will try and fit as many rectangles under the curve as possible. Since the area of a rectangle equals its width times its height, we calculate the area of each rectangle and then add up all the different areas to get the total area under the curve."

Child 1: "What about the space left over under the curve that isn't part of the rectangles?"

Mathlin: "We'll make the rectangles as small as possible."

Mathlin magically created many infinitely small rectangles and put them under the curve.

A Voice: "Good job Mathlin! That is exactly the concept behind calculating area under a curve."

A Voice: "I was hoping you would show up soon. The creature that stole Hisaab plans to destroy the book tonight so no human can ever learn from it again. You must return Hisaab to its proper place. Hurry! The creature will be coming back soon."

Mathlin: "Where is Hisaab?"

A Voice: "In one of the infinitely small rectangles. Cast a spell on the rectangles to throw the book out to you before the creature returns."

Mathlin:"Goaabiloo-ak-kaana!"

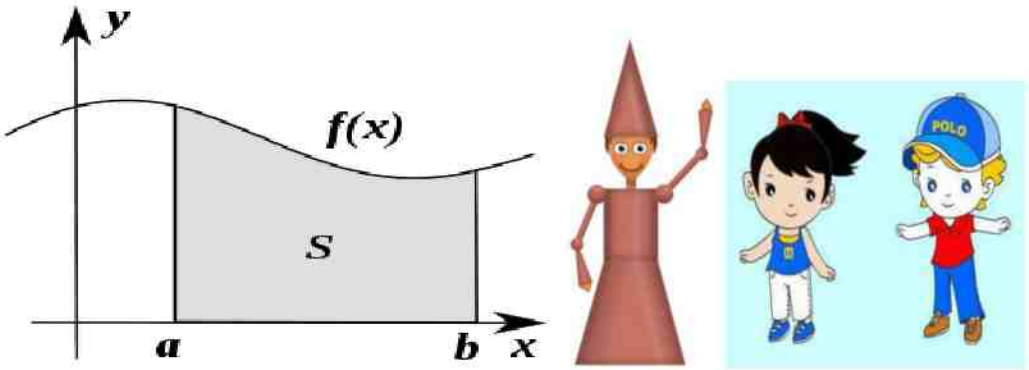
Hisaab came flying out magically into Mathlin's hands.

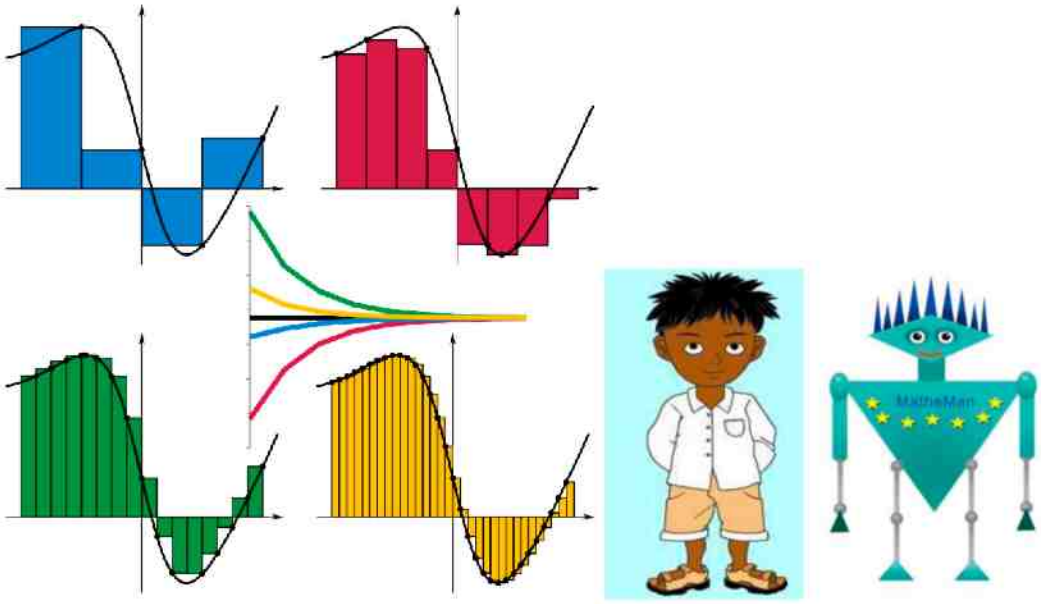
Mathlin:"Thank you."

A Voice:"You're welcome. Now hurry!"

Mathlin waved his wand and the whole team was transported back to the human world in Calcu-concept's town library, where *Hisaab* was returned to its proper place.

Mathlin cast a spell so the ghost like creature could never steal *Hisaab* again.





Appendix D – Tools and Practice Problems covered in both groups

Tools: Integration of common functions

- $\int x^n\,dx = x^{n+1} / (n+1)$
- $\int e^x\,dx = e^x$
- $\int e^{ax}\,dx = (1/a)(e^{ax})$
- $\int (1/x)\,dx = \log x$
- $\int \sin x\,dx = -\cos x$
- $\int \cos x\,dx = \sin x$

Integration Problems:

- $\int x^4\,dx = x^{(4+1)} / (4+1) + C = X^5 / 5 + C$
- $\int 3x^5 - 4x^3\,dx = \int 3x^5\,dx - \int 4x^3\,dx = 3\ x^5 / 5 \quad 4\ x^4 / 4] + C = (3/5)\ x^5 - x^4 + C$
- $\int x^2\ e^{3x} + 2\,dx = x^3 / 3 - (1/3\ e^{3x} + 2x + C$

Appendix E – Activity after Lecture

1. Draw an x-y grid. Then draw a random continuous non-zero function. Now shade in the integral from x = a to x = b.
2. Tortillas restaurant discovered that CSUCI students loved churros to snack on. The restaurant discovered that the kitchen had to increase churro production at a rate of 10,000 per year. By October 2011, Tortillas restaurant had made 100,000 churros. Use a definite integral to estimate the number of churros Tortillas restaurant had made until October 2009.
3. Solve $\int 4x^4 + x^3\,dx$ from x=0 to x=1. Please write down the formula/integration function you will be using.

Appendix F – Quiz 1

1. What do you call the shape below? (A picture of a parabola was displayed on the quiz)
2. Calculate:
   a. $3^2 - 3$
   b. $2^2 - 3$
   c. $1^2 - 3$
   d. $0^2 - 3$
   e. $(-1)^2 - 3$
   f. $(-2)^2 - 3$
   g. $(-3)^2 - 3$
3. Graph a parabola $y = x^2 - 3$
4. What point is the minimum of the above graph?

Appendix G – Portion of the book covered for eighth grade

(Starting at Page 12 of the actual story) … There was a big sign that read: "Jump on equilibrium point"

MatheMan: Children, an equilibrium point is a point is where two curves intersect. Let's all walk over to that point and jump.

Upon jumping, they were transported to another land and set down under a giant talking tree.

There were mathematical symbols and shapes everywhere.

This land was extremely colorful.

Tree: Hello Mathlin. I've been expecting you.

Mathlin: Hello tree. Do you know what happened to our book called Hisaab?

Tree: Ah yes, the book of mathematical concepts. You must find the book and return it to the Calcu-concept town's library for future humans to use. Follow that road filled with numbers and you'll see a house on your right. That is where Mr Quadratic lives. He will point you in the right direction. Good luck!

Mathlin, MatheMan, and the children followed the road filled with numbers till they finally see Mr. Quadratic's house on their right.

They went up to the door and the door opened.

A Voice: Come on in!

Mathlin, MatheMan, and children go inside.

They see Mr Quadratic and Mr Cubic sitting at a circular table having tea and pastries.

Mr Quadratic: Hello, I'm Mr Quadratic. This is my cousin, Mr Cubic. Would you like to join us for some tea and pastries?

Mathlin: No, thank you. We're investigating the disappearance of Hisaab from the human world. Do you have any information we can use?

Mr Quadratic: We sure do! We saw this ghost-like creature running down the road filled with numbers with Hisaab in his hands. He went till the end of the road where there is a giant wall. The wall has two parabolas on it. One is a positive parabola, and one is a negative parabola. Positive parabolas open up and have a minimum point. Negative parabolas open down, and have a maximum point.

Mark: What's a parabola?

Mr Quadratic: Parabola is the name of the shape that a quadratic function makes. I'm Quadratic.

Mark: What's a quadratic function?

Mr Quadratic: A quadratic function is where an unknown quantity is squared. Let's say the unknown quantity is "x." x^2 is the unknown quantity squared. x^2 is a quadratic function.

Mark: I still don't get it.

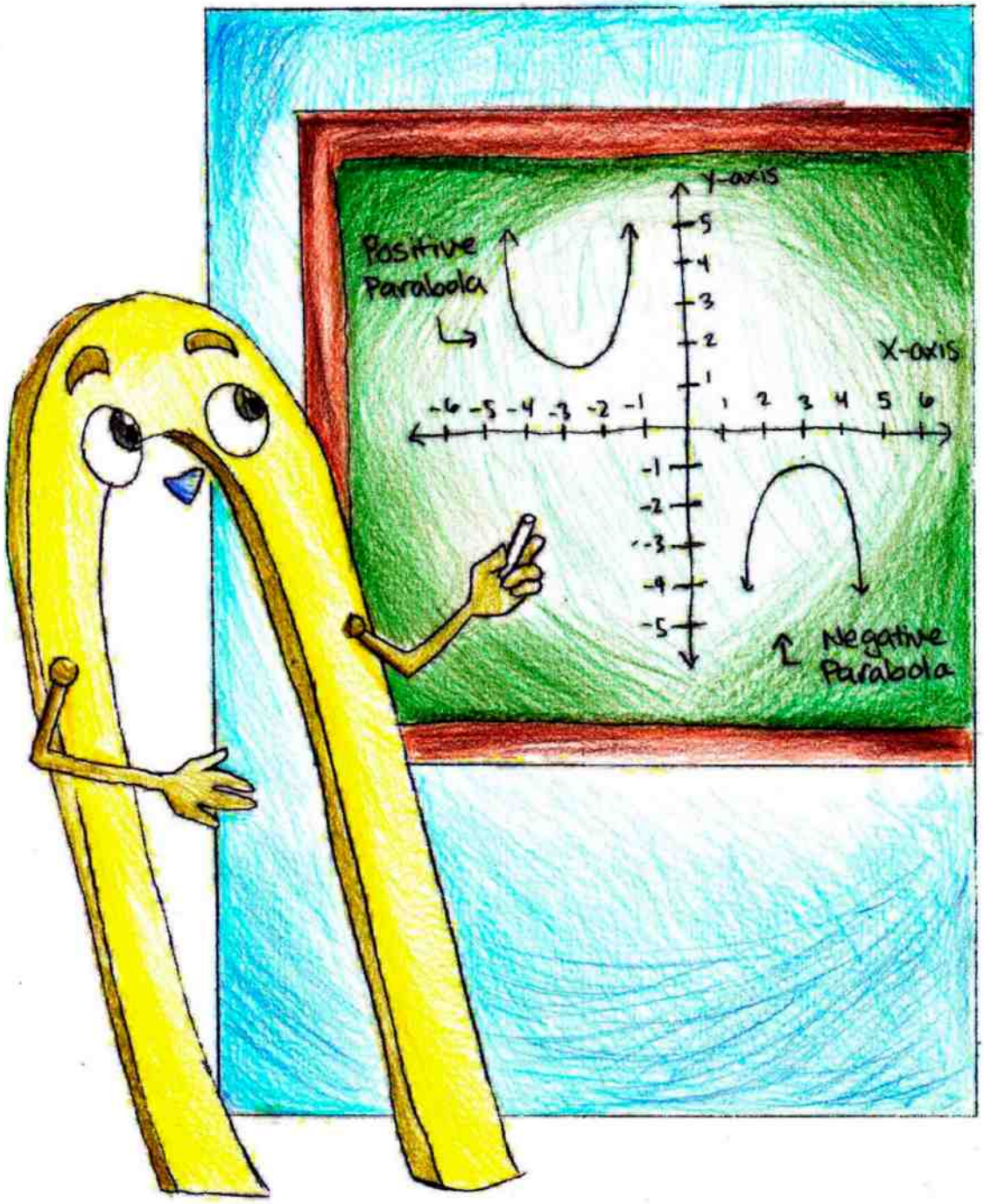
Mathlin: That's okay. Do you remember what a square of a number is?

Mark: Yes. It is multiplying a number by itself. For example, 2 squared would be 2 times 2 which will equal 4.

Mathlin: That's great! Now do you know what a plotting grid looks like?

Mark: No.

Mr Quadratic: Let me draw out the parabolas on a plotting grid so you can visualize it. Remember, a plotting grid has an x-axis, which is the horizontal line, and a y-axis, which is the vertical line. Notice the directions in which the numbers increase and decrease on each axis.


Positive Paraboola
Y-axis
X-axis
Negative Parabola

Mathlin: What do we do when we get to the giant wall?

Mr. Quadratic: Connect the maximum point of one parabola to the minimum point of the other parabola. The wall will disappear for a few minutes and you can pass through. That is where the creature has hidden Hisaab.

Mathlin, MatheMan and the children went till the end of the road filled with numbers where they see the giant wall with the two parabolas.

Abby: Let's connect the maximum point of one parabola, to the minimum point of the other parabola.

Mark: What's a maximum point and what's a minimum point?

MatheMan: A maximum point is the highest point on a curve. A minimum point is the lowest point on the curve.

Mathlin and MatheMan connected the two points by sliding the positive parabola over the negative parabola till the maximum and minimum points connected.

Appendix H – Quiz 2

1. What do you call the shape below? Explain how to recognize this shape. (A picture of a parabola was displayed in the quiz)
2. Calculate:
    a. $3^2 - 1$
    b. $2^2 - 1$
    c. $1^2 - 1$
    d. $0^2 - 1$
    e. $(-1)^2 - 1$
    f. $(-2)^2 - 1$
    g. $(-3)^2 - 1$
3. Graph a parabola $y = x^2 - 1$
4. What point is the minimum of the above parabola?
5. (Optional) Did you like the story about the missing book and the parabolas? What did you like about it?

Appendix – Eighth Grade Test Group Comments

Note: The students did not read the complete story from start to end. The middle section of the story which dealt with parabolas was the only portion covered in class.

- I did like the story because it showed me a little more about negative and positive parabolas.
- I liked the pictures and the book was cool.
- No but I like where it said what parabolas are.
- Yes it is a creative way to show how quadratic problems function.
- Yes I did because the colorful demonstrations taught me a lot about parabolas.
- I like most of the book except for the ending.
- No I did not like the story because it made no sense. I did like the pictures though!
- Yes, I liked that they explained what a quadratic function is clearly.
- I really liked the cute pictures and colored ink! Very refreshing! Please don't leave us hanging though!!
- I did like the story because it gave much detail.
- I liked the story because it tells you about parabolas.
- I don't think the story was geared towards kids our age, but if it was shown to little kids they would like it more.
- This story was made or written for an elementary student, but I do like the animations.
- Yes, I liked it. The story showed great illustrations and it explained parabolas in a simple way.
- The story wasn't so bad but a better ending might work. Overall good story.
- I think this story is not good for 3^{th} graders, but I liked the drawings. I kind of understood better of the maximum and the minimum.
- No, because it didn't tell me what happened to the book after. I did like the illustrations though.
- I like how the book explains what a parabola and a quadratic function are.
- Yes I did because it was informative and I learned a lot.
- I didn't like how the story didn't end with the finding of the book. I did like the animated pictures.
- I did like it. I liked it because it had a lot of good information about the parabolas.
- The story was informative and had great illustrations but I didn't like the ending of the story.
- It helped me remember the name of that type of equation.
- I love the story! But I think it's a bit childish.
- I liked the character names.
- I liked the story. The pictures helped as well.
- Yes, I liked the pictures. It is not high school level.
- Yes, because it was very creative.
- Yes. The story was interesting but you never told us if they found the book or not.

Appendix J: The Complete Story

The Mathlin Kids: Case of the Missing Book

Written by Nausheen Ahmed

Illustrated by Therese Larkin

Author's Note

I would like to thank Dr. Ivona Grzegorczyk, Professor of Mathematics, and California State University Channel Islands for helping make this book possible.

I wanted to give my children a head start in math by teaching them higher level mathematical concepts at an early stage. I started looking for children's storybooks that taught higher level mathematical concepts visually. An easy to follow storybook format and colorful illustrations were important to keep my child's attention. I also wanted a book where there was teamwork and friendship. When I couldn't find such a product on the market, I designed "The Mathlin Kids." This book incorporates common core areas for the grade 3 level. I hope you enjoy using the book as much as I enjoyed creating it.

-Nausheen Ahmed

Illustrator's Note

I am pleased and honored to have helped complete this book for the students to help them learn in a more visual and fun way. I know how challenging understanding mathematics can be at a young age. I thank my 6th grade teacher, Mr. Garcia-Prats, for helping me understand math and realize my own potential in that subject. So I wish to pass it on to other kids and students through these illustrations and help them realize their own potential.

-Therese Larkin

The Mathlin Kids: Case of the Missing Book

We're at the Calcu-concept town's bakery where Mathlin, an individual who loves to teach children new and exciting material, and MatheMan, Mathlin's teaching assistant, have just finished teaching the multiplication tables and the concept of division.

The door of the bakery flings open blowing in a gush of wind and lots of snow.

In the midst of all that snow, there stands the town's bookkeeper.

Bookkeeper: I have some terrible news Mathlin. It seems that someone has stolen Hisaab from the town's library.

Mark: What's Hisaab?

Mathlin: It's the ancient book that contains secrets of mathematics. Mathematical concepts are essential for the growth and development of human society.

Mathlin: Are there any clues as to who took the book?

Bookkeeper: I didn't see any Mathlin.

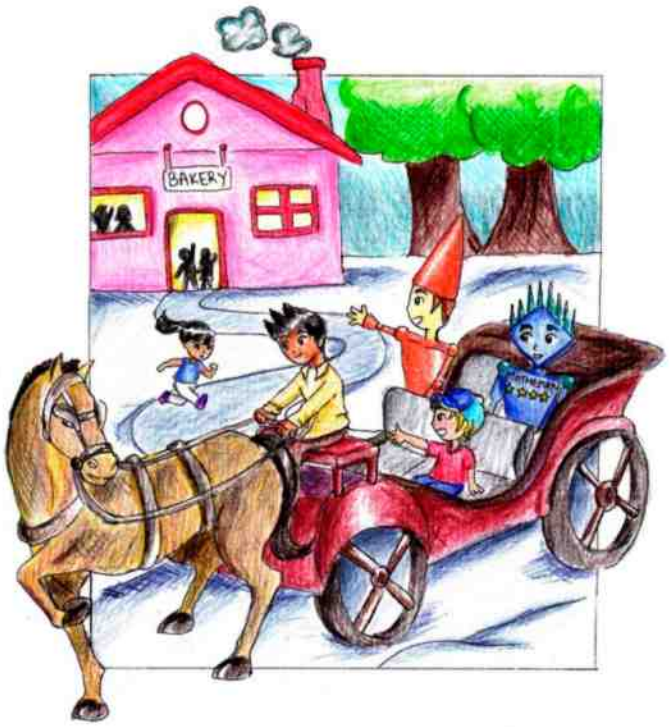
MatheMan: We should go to the library to investigate.

Mathlin: Good idea, MatheMan. Bookkeeper, we are on the case.

Bookkeeper: Thank you, Mathlin.

Mathlin: There's room for just two children in the carriage with us. The rest will have to stay back.

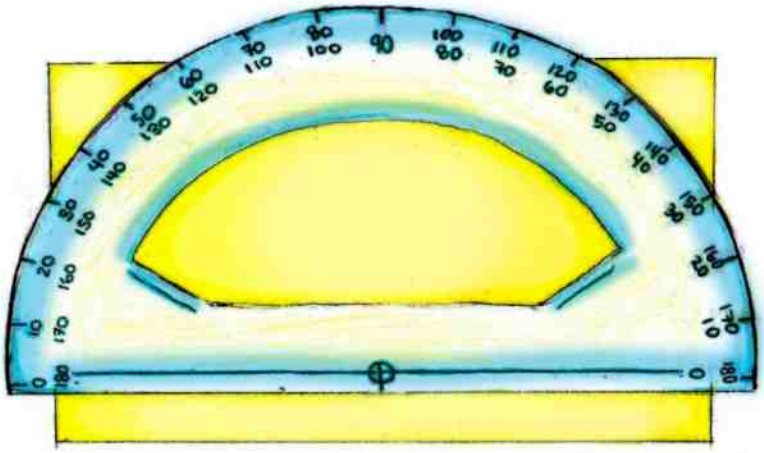
Mathlin, MatheMan, Mark and Abby all get into the carriage.



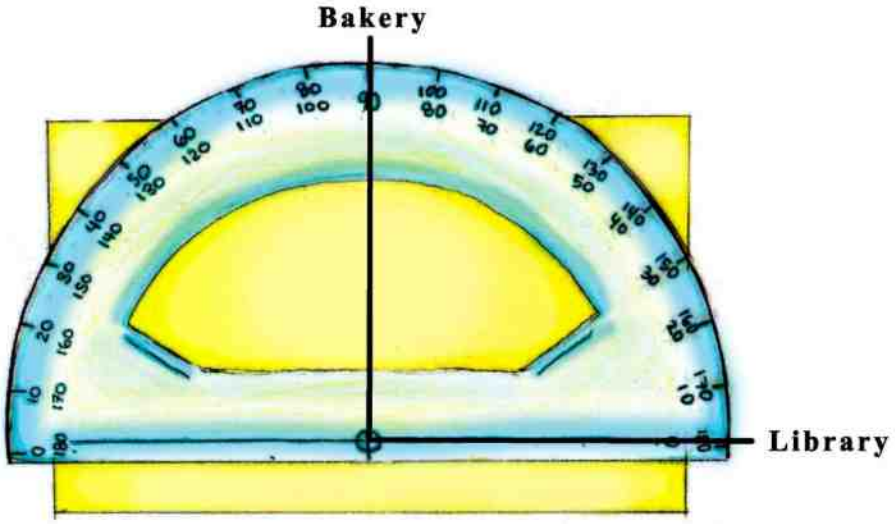
Mathlin: We have to go three miles south on Straight Road, and four miles east on Linear Road to get to the library. The roads make a 90 degree angle.

Mark: What do you mean by “measures 90 degrees” or “measures 180 degrees”?

Mathlin: We normally use a protractor to measure angles. This is what a protractor looks like:



Mathlin: Let’s measure the angle between Straight Road and Linear Road using a protractor. You see the angle between the roads measures 90 degrees.



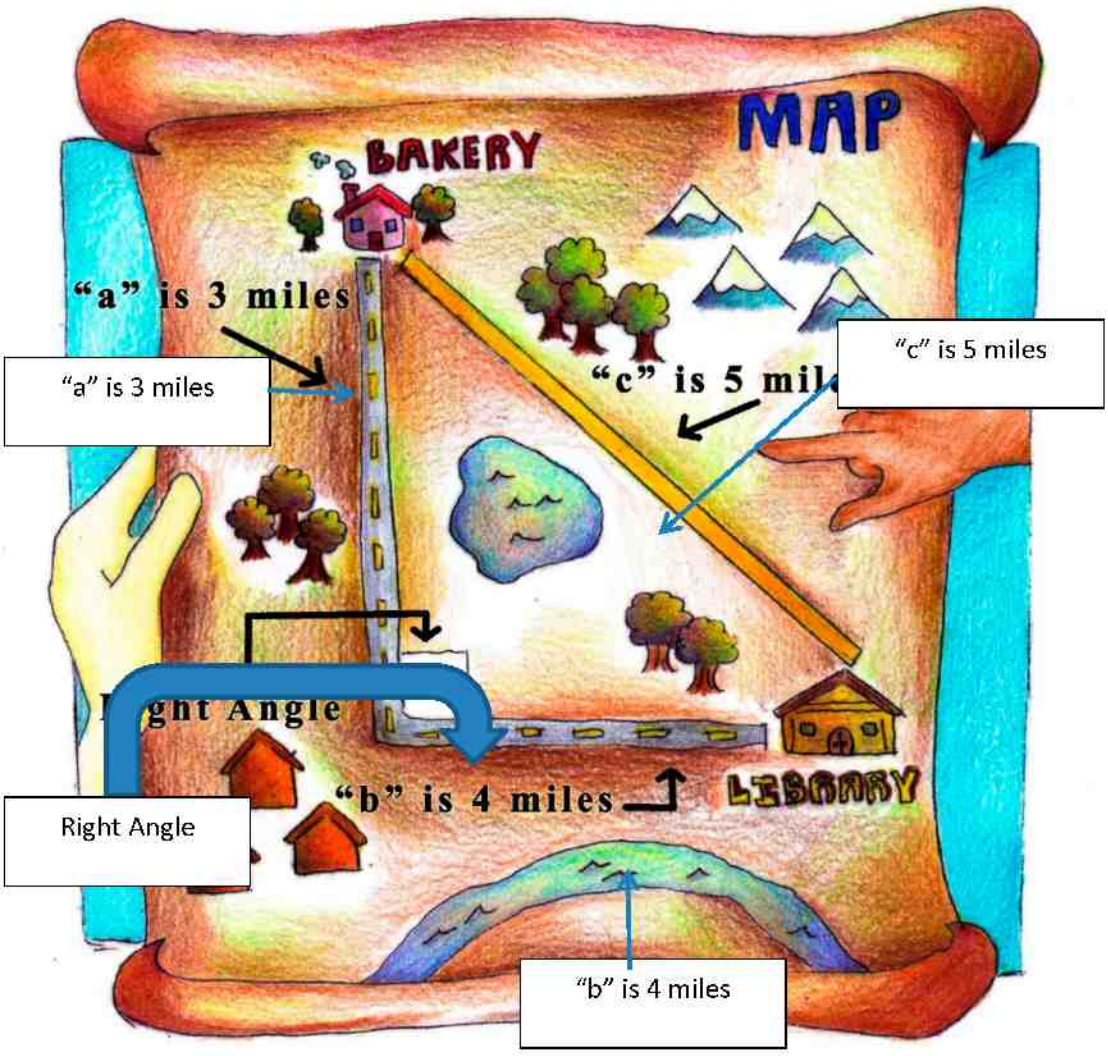
Mathlin: If two roads intersect, meaning cross each other, and make a ninety degree angle, then they are perpendicular. A 90 degree angle is also called a right angle.

Mark: So we have to travel three miles plus four miles. We have to travel seven miles total.

Abby: Why don't we just take Hypotenuse Road? It starts here and goes diagonally to the library. We would cover a lesser distance.

Mathlin: Do you know what shape this is? It's a triangle. Also note, that one of the interior angles of this triangle is 90 degrees, therefore, this triangle is called a right triangle. If two lines, or in this case, if two roads intersect, meaning cross each other at a ninety degree angle, they're said to be perpendicular.

MatheMan: Let's use the Pythagorean Theorem. Pythagorean Theorem is for right triangles. In this case, the roads make a right triangle. A right triangle is a triangle where one of the interior angles is equal to 90 degrees. In this right triangle, let's say Straight Road is side "a" of the triangle, Linear Road is side "b" of the triangle, and Hypotenuse Road is side "c" of the right triangle. The Pythagorean Theorem for right triangles states that $a^2 + b^2 = c^2$. Since side "a" is three miles, and side "b" is four miles, we can say $3^2 + 4^2 = c^2$.



Mark: But I don't know what squared means, MatheMan.

MatheMan: Now, we all learned the multiplication tables recently. A number squared, meaning a number that is raised to the second power, is just a simple way of saying that we should multiply that number by itself. For example: five squared means five times five, which equals twenty-five. Three squared is three times three, which equals nine. Four squared is four times four, which equals sixteen.

MatheMan: $a^2 + b^2 = c^2$ will be $3^2 + 4^2 = c^2$. We get $9 + 16 = c^2$ $9 + 16 = 25$. $c^2 = 25$. How long is side "c"?

Mathlin: Remember, squaring is just multiplying a number by itself.

Abby: Well, $c^2 = 25$. And squaring is just multiplying a number by itself. So we have to find a number that equals 25 when squared. That number is five. Five times five is twenty-five. Therefore, side "c" is five miles. Hypotenuse Road is five miles long!

Mathlin: Absolutely correct!

Mark: I'm curious. If triangles are three-sided polygons, what are four-sided polygons called?

Mathlin: They are called quadrilaterals.

Mathlin: Let's take Hypotenuse road to get to the library.

The children, Mathlin, and MatheMan take Hypotenuse Road to arrive at the library.

Mathlin: Let's walk around the perimeter of the library to see if there are any clues.

Mark: What's a perimeter?

MatheMan: Perimeter is the distance around a shape. The top view of the library reveals that the shape of the library is a rectangle. To find the perimeter of this rectangle, we will find out how long each of the four sides are, and then add the lengths of all these sides together.

The children and MatheMan walk around the library and take measurements.

Abby: The long sides are fifteen feet, and the two short sides are seven feet. The perimeter is $15 + 7 + 15 + 7$.

Mark: I know what the sum is! Hey you all out there, do you want to count with me? $15 + 7 + 15 + 7 = 44$ feet.


LIBRARY
Perimeter
7ft
15ft
15ft
7ft
7+15+7+15

Mathlin: Did anyone find any clues outside the building?

Abby: There is a sequence of numbers on the side wall of the building with a missing number.

They all walk over to the sequence of numbers.

The sequence of numbers reads as follows:

4, 6, 10, 12, 16, 18, , 24, 28, 30

Mathlin: The sequence reads 4, 6, 10, 12, 16, 18, then there is a missing number, then the sequence continues to 24, 28, and 30. Maybe there is a way to find the missing number. Are there any rules? Or some pattern we can recognize?

MatheMan: What's the pattern in this sequence?

Abby: The pattern is +2 then +4. If we follow the pattern of +2, +4, then the missing number in the sequence is 22!

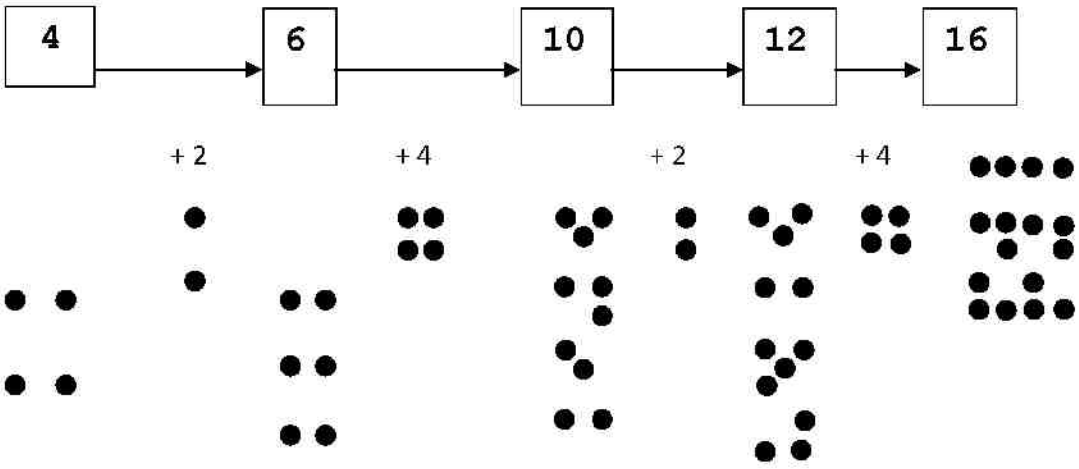
Mark: How did you find the pattern?

Abby: Well, the first number is 4 and the second number is 6. I know that 4 + 2 is equal to 6.

Abby: Then to get from 6 to 10, we have to add 4 because 6 + 4 is equal to 10.

Mark: I see. Then 10 + 2 is 12. And 12 + 4 is 16.

Abby: That's how I got the pattern. First add 2, and then add 4.



Mark: 16 plus 2 is equal to 18, and then we have to add 4 to 18. 18 plus 4 is equal to 22. And that's why you said the missing number is 22.

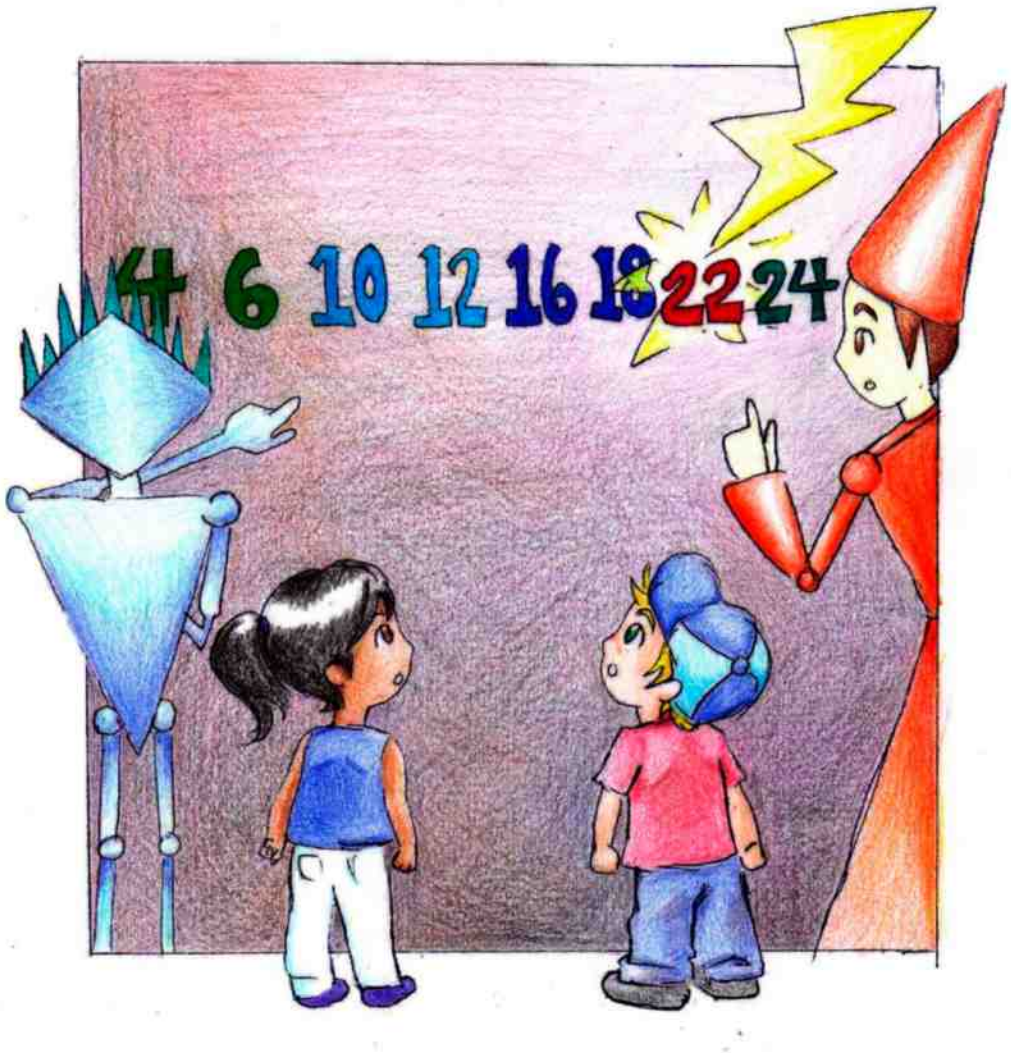
Abby: Correct.

Mathlin: Good job!

Mathlin walks over to the wall and writes "22" in the space left for the missing number in the number sequence.

All of a sudden, there was thunder and lightning.

The lightning hits the number 22 and a doorway opens up.



Mathlin: Everyone stay behind me.

They entered the doorway. Mathlin enters first. MatheMan and the children follow. It's dark. Mathlin uses his flashlight so everyone can see where they are going.

They arrive at the top of a giant slide.

Loud Voice: Slope, please.

MatheMan: The slide has the rise and the run labeled.

Mark: Rise? Run?

MatheMan: The slope is the measure of how steep a line is. The slope is the "rise" divided by the "run." Rise is the change in height. Run is the distance in which that change in height occurs.

Mathlin: The rise is -10. Keep in mind that the rise is a negative number because the slide is going down. The run is 5. So we divide -10 by 5. 10 divided by 5 is 2. And since 10 is negative, because the slide is going down, our slope is negative 2.

Loud Voice: Absolutely right! You may slide.

Everyone slides down the giant slide and lands on a giant mat with two intersecting curves.

There was a big sign that read: “Jump on equilibrium point”

MatheMan: Children, an equilibrium point is a point is where two curves intersect. Let’s all walk over to that point and jump.

Upon jumping, they were transported to another land and set down under a giant talking tree.

There were mathematical symbols and shapes everywhere.

This land was extremely colorful.

Tree: Hello Mathlin. I’ve been expecting you.

Mathlin: Hello tree. Do you know what happened to our book called Hisaab?

Tree: Ah yes, the book of mathematical concepts. You must find the book and return it to the Calcu-concept town’s library for future humans to use. Follow that road filled with numbers and you’ll see a house on your right. That is where Mr Quadratic lives. He will point you in the right direction. Good luck!

Mathlin, MatheMan, and the children followed the road filled with numbers till they finally see Mr. Quadratic's house on their right.

They went up to the door and the door opened.

A Voice: Come on in!

Mathlin, MatheMan, and children go inside.

They see Mr Quadratic and Mr Cubic sitting at a circular table having tea and pastries.

Mr Quadratic: Hello, I'm Mr Quadratic. This is my cousin, Mr. Cubic. Would you like to join us for some tea and pastries?

Mathlin: No, thank you. We're investigating the disappearance of Hisaab from the human world. Do you have any information we can use?

Mr. Quadratic: We sure do! We saw this ghost-like creature running down the road filled with numbers with Hisaab in his hands. He went till the end of the road where there is a giant wall. The wall has two parabolas on it. One is a positive parabola, and one is a negative parabola. Positive parabolas open up and have a minimum point. Negative parabolas open down, and have a maximum point.

Mark: What's a parabola?

Mr. Quadratic: Parabola is the name of the shape that a quadratic function makes. I'm Quadratic.

Mark: What's a quadratic function?

Mr. Quadratic: A quadratic function is where an unknown quantity is squared. Let's say the unknown quantity is "x." x^2 is the unknown quantity squared. x^2 is a quadratic function.

Mark: I still don't get it.

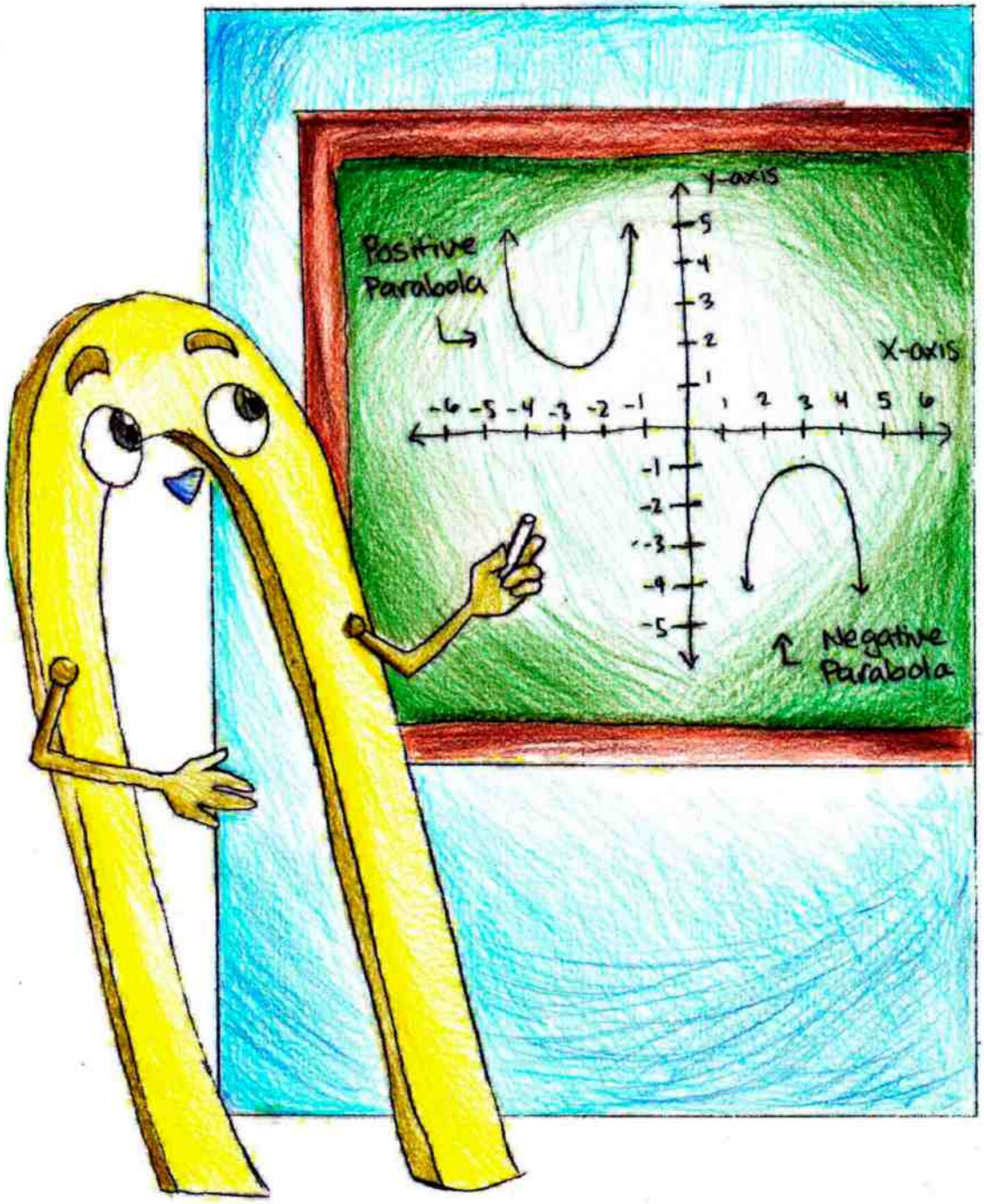
Mathlin: That's okay. Do you remember what a square of a number is?

Mark: Yes. It is multiplying a number by itself. For example, 2 squared would be 2 times 2 which will equal 4.

Mathlin: That's great! Now do you know what a plotting grid looks like?

Mark: No.

Mr. Quadratic: Let me draw out the parabolas on a plotting grid so you can visualize it. Remember, a plotting grid has an x-axis, which is the horizontal line, and a y-axis, which is the vertical line. Notice the directions in which the numbers increase and decrease on each axis.


Positive Paraboola
Y-axis
X-axis
Negative Parabola

Mathlin: What do we do when we get to the giant wall?

Mr Quadratic: Connect the maximum point of one parabola to the minimum point of the other parabola. The wall will disappear for a few minutes and you can pass through. That is where the creature has hidden Hisaab.

Mathlin, MatheMan and the children went till the end of the road filled with numbers where they see the giant wall with the two parabolas.

Abby: Let's connect the maximum point of one parabola, to the minimum point of the other parabola.

Mark: What's a maximum point and what's a minimum point?

MatheMan: A maximum point is the highest point on a curve. A minimum point is the lowest point on the curve.

Mathlin and MatheMan connected the two points by sliding the positive parabola over the negative parabola till the maximum and minimum points connected.

Right in front of them was a curve. The area under the curve was colored.

Mathlin: Where do you suppose Hisaab is? We have no more leads, and we must find that book!

MatheMan: What if we find the area under the curve?

Mark: How do we calculate the area under the curve?

Abby: What if we use rectangles?

MatheMan: Yes, we will try and fit as many rectangles under the curve as possible. Since the area of a rectangle equals its width times its height, we calculate the area of each rectangle and then add up all the different areas to get the total area under the curve.

Abby: What about the space left over under the curve that isn't part of the rectangles?

Mathlin: There shouldn't be much since we can make the rectangles very thin to fill all the area.

Loud Voice: Good job children! That is exactly the concept behind calculating area under a curve. Let me help you by showing you all the rectangles!

Loud Voice: I was hoping you would show up soon. The creature that stole Hisaab plans to destroy the book tonight so no human can ever learn from it again. You must return Hisaab to its proper place. Hurry! The creature will be coming back soon.

Mathlin: Where is Hisaab?

Loud Voice: In one of the infinitely small rectangles.

Mathlin, MatheMan, and the children start looking inside the rectangles. They find the book.

Mathlin, MatheMan and the children hurry back to the wall and return to the human world in Calcu-concept's town library. Hisaab is returned to its proper place. The ghost-like creature is put behind bars in the other land for stealing Hisaab. There, behind bars, the ghost-like creature learns a valuable lesson: Good always wins over bad in the long run!


JAIL
MATHE MAN
HISAAB

Appendix K: Grading Scales

Grading scale for the short survey for the Calculus Groups (Experiment 1):

1 – Grades were assigned either a 0 for C or D, 1 for B and 2 for A
2 – 1 for No, 2 for Yes
3 – 0 for Listener, 1 for Reader, or 2 for Hands On/Visual
4 – 1 for Concepts and 2 for Calculations

Grading scale for Pre-Test / "Activity Prior to Lecture" AND Post-Test / "Activity After Lecture" – Experiment 1

For #1

- 1 point for drawing the x-y grid AND a function
- 1 point for marking the interval
- 1 point for shading in the correct area

For # 2

- 1 point for setting up the correct integral including labeling the correct interval
- 1 point for solving the integration
- 1 point for correct answer/ subtraction done correctly

For # 3

- 1 point for writing down the integration formula correctly
- 1 point for plugging in the right numbers in the formula
- 1 point for solving the integration

Grading scale for Pre-Test / "Quiz 1" AND Post-Test / "Quiz 2" – Eight Grade Group (Experiment 2)

For #1

- 1 point for getting the correct name

For # 2

- 1 point total for computing the correct answer on all 7 equations

For # 3

- 1 point for correctly graphing the equation

For # 4

- 1 point for correctly stating the minimum point

Appendix L: Raw Data

Experiment 1

	Class I: Test Group								Improv.
Student	Pre-Test				Post-Test				x_1, X_a
ID #	x_1	x_2	x_3	Tot $x_{1,2,3}$	X_a	X_b	X_c	Tot $X_{a\,b\,c}$	
1	2	0	0	2	3	0	3	6	1
2	0	0	1	1	3	0	3	6	3
3	0	0	2	2	3	0	3	6	3
4	3	0	0	3	3	0	0	3	0
5	2	0	0	2	3	0	3	6	1
6	2	0	0	2	3	0	3	6	1
7	3	0	1	4	3	0	2	5	0
8	3	0	0	3	3	0	3	6	0
9	2	0	0	2	3	0	3	6	1
10	2	0	0	2	3	0	3	6	1
Mean	1.9	0	0 4	2.3	3	0	2.6	5.60	1.1
Mode	2	0	0	2	3	0	3	6	1
Median	2	0	0	2	3	0	3	6	1
St Dev	1.100505	0	0.699206	0.823273	0	0	0.966092	0.966092	1.100505

Survey: Test Group				
Previous Grade	Like Stories?	Learner Type?	Interested In?	Student
Q1	Q2	Q3	Q4	ID #
1	2	2	2	1
2	2	2	2	2
2	2	2	2	3
1	2	2	2	4
2	2	2	2	5
2	2	2	1	6
1	2	0	2	7
1	2	2	1	8
0	2	2	2	9
0	2	2	1	10
0 = C or D	1 = No	0 = Listener	1 = Concepts	
1 = B	2 = Yes	1 = Reader	2 = Computation	
2 = A		2 = Visual/Hands On		

	Class II: Control Group								
Student	Pre-Test				Post-Test				Improv.
ID #	y_1	y_2	y_3	Tot $y_{1,2,3}$	Y_a	Y_b	Y_c	Tot $Y_{a,b,c}$	y_1, Y_a
11	1	0	3	4	3	0	1	4	2
12	2	0	3	5	3	0	3	6	1
13	3	0	3	6	3	0	3	6	0
14	2	0	0	2	3	0	3	6	1
15	2	0	0	2	2	0	3	5	0
16	0	0	0	0	3	0	3	6	3
17	2	0	0	2	3	0	3	6	1
18	2	0	0	2	2	0	3	5	0
19	3	0	0	3	3	0	0	3	0
20	3	0	3	6	3	0	3	6	0
21	2	0	0	2	2	0	0	2	0
22	3	0	0	3	3	0	3	6	0
23	3	0	3	6	3	0	3	6	0
24	3	0	3	6	3	0	3	6	0
25	0	0	1	1	3	0	3	6	3
26	3	0	0	3	3	0	0	3	0
27	1	0	3	4	3	0	3	6	2
Mean	2.058824	0	1.294118	3.352941	2.823529	0	2.352941	5.176471	0.764706
Mode	3	0	0	2	3	0	3	6	0
Median	2	0	0	3	3	0	3	6	0
St Dev	1.028992	0	1.490164	1.902011	0.392953	0	1.221739	1.333946	1.09141

Survey: Control Group				
Previous Class	Like Stories?	Learner Type?	Interested In?	Student
Q1	Q2	Q3	Q4	ID #
2	2	2	2	11
0	2	2	1	12
2	2	2	2	13
1	2	0	2	14
1	2	2	2	15
1	2	2	2	16
2	2	2	2	17
0	2	2	2	18
0	2	2	2	19
1	2	2	1	20
1	2	2	1	21
0	2	2	2	22
0	2	2	1	23
0	2	2	1	24
0	2	2	2	25
1	2	2	1	26
2	2	2	2	27
0 = C or D	1 = No	0 = Listener	1 = Concepts	
1 = B	2 = Yes	1 = Reader	2 = Computation	
2 = A		2 = Visual/Hands On		

Appendix L: Raw Data

Experiment 2

Control Group	Grade 8, Algebra Honors													
Student	Pre-Test					Post-Test					Improv.	Improv.	Improv.	Improv.
ID #	x_1	x_2	x_3	x_4	Tot $x_{1,2,3,4}$	X_a	X_b	X_c	X_d	Tot $X_{a,b,c,d}$	x_1, X_a	x_2, X_b	x_3, X_c	x_4, X_d
1	1	0	0	0	1	1	1	1	1	4	0	1	1	1
2	1	0	0	1	2	1	1	1	1	4	0	1	1	0
3	0	1	0	1	2	1	1	1	1	4	1	0	1	0
4	1	1	0	0	2	1	1	1	1	4	0	0	1	1
5	1	1	0	1	3	1	1	1	1	4	0	0	1	0
6	1	1	0	0	2	1	1	1	1	4	0	0	1	1
7	0	0	0	0	0	1	1	1	1	4	1	1	1	1
8	1	1	0	0	2	1	1	1	1	4	0	0	1	1
9	0	1	0	1	2	1	1	1	1	4	1	0	1	0
10	0	1	0	0	1	1	1	1	1	4	1	0	1	1
11	0	0	0	0	0	1	1	1	1	4	1	1	1	1
12	1	0	0	0	1	1	0	0	1	2	0	0	0	1
13	1	1	0	0	2	1	1	1	1	4	0	0	1	1
14	0	1	0	0	1	1	1	1	0	3	1	0	1	0
15	1	1	0	0	2	1	1	1	1	4	0	0	1	1
16	1	1	0	0	2	1	1	1	0	3	0	0	1	0
17	0	1	1	0	2	1	1	1	1	4	1	0	0	1
18	0	1	0	0	1	1	1	1	1	4	1	0	1	1
19	0	1	0	1	2	1	1	1	1	4	1	0	1	0
20	1	0	0	0	1	1	1	1	1	4	0	1	1	1
21	1	1	0	0	2	1	1	1	1	4	0	0	1	1
22	0	0	0	1	1	1	0	1	1	3	1	0	1	0
23	1	0	0	1	2	1	1	1	1	4	0	1	1	0
24	1	1	0	0	2	1	0	1	1	3	0	-1	1	1
25	0	1	0	0	1	1	0	1	1	3	1	-1	1	1
26	0	0	0	0	0	1	1	1	1	4	1	1	1	1
27	1	1	0	0	2	1	1	0	1	3	0	0	0	1
Mean (μ)	0 555556	0 666667	0 037037	0 259259	1 518519	1	0 851852	0 925926	0 925926	3 703704	0 444444	0 185185	0 888889	0 666667
Mode	1	1	0	0	2	1	1	1	1	4	0	0	1	1
Median	1	1	0	0	2	1	1	1	1	4	0	0	1	1
St Dev	0 50637	0 480384	0 19245	0 446576	0 752962	0	0 362014	0 26688	0 26688	0 541708	0 50637	0 557262	0 320256	0 480384

Test Group	Grade 8, Algebra Honors													
Student	Pre-Test					Post-Test					Improv.	Improv.	Improv.	Improv.
ID #	y_1	y_2	y_3	y_4	Tot $y_{1,2,3,4}$	Y_a	Y_b	Y_c	Y_d	Tot $Y_{a,b,c,d}$	y_1, Y_a	y_2, Y_b	y_3, Y_c	y_4, Y_d
28	0	0	0	0	0	1	1	1	1	4	1	1	1	1
29	1	1	0	1	3	1	1	1	1	4	0	0	1	0
30	1	1	0	0	2	1	1	1	1	4	0	0	1	1
31	1	1	0	0	2	1	0	1	1	3	0	-1	1	1
32	0	0	0	0	0	1	1	1	1	4	1	1	1	1
33	0	0	0	0	0	1	1	1	1	4	1	1	1	1
34	0	0	0	0	0	1	1	1	1	4	1	1	1	1
35	1	0	0	0	1	1	1	0	1	3	0	1	0	1
36	1	1	0	0	2	1	1	1	1	4	0	0	1	1
37	0	0	0	0	0	1	0	1	1	3	1	0	1	1
38	0	1	0	0	1	1	1	1	1	4	1	0	1	1
39	1	0	0	0	1	1	1	1	1	4	0	1	1	1
40	1	0	0	0	1	1	1	1	1	4	0	1	1	1
41	0	0	0	0	0	1	1	1	1	4	1	1	1	1
42	1	1	0	1	3	1	1	1	1	4	0	0	1	0
43	0	0	0	0	0	1	0	1	1	3	1	0	1	1
44	1	1	0	0	2	1	1	1	1	4	0	0	1	1
45	1	1	0	0	2	1	1	1	1	4	0	0	1	1
46	1	0	0	0	1	1	1	1	1	4	0	1	1	1
47	1	1	0	0	2	1	1	1	1	4	0	0	1	1
48	1	0	0	0	1	1	1	1	1	4	0	1	1	1
49	1	1	0	0	2	1	1	1	1	4	0	0	1	1
50	1	1	0	0	2	1	1	1	1	4	0	0	1	1
51	1	0	0	0	1	1	0	0	1	2	0	0	0	1
52	1	1	0	0	2	1	1	1	1	4	0	0	1	1
53	1	0	0	0	1	1	0	1	1	3	0	0	1	1
54	1	1	0	0	2	1	1	1	1	4	0	0	1	1
55	1	0	0	0	1	1	1	1	1	4	0	1	1	1
56	1	1	0	0	2	1	1	1	1	4	0	0	1	1
Mean (μ)	0 724138	0 482759	0	0 068966	1 275862	1	0 827586	0 931034	1	3 758621	0 275862	0 344828	0 931034	0 931034
Mode	1	0	0	0	2	1	1	1	1	4	0	0	1	1
Median	1	0	0	0	1	1	1	1	1	4	0	0	1	1
St Dev	0 454859	0 508548	0	0 257881	0 921821	0	0 384426	0 257881	0	0 510964	0 454859	0 552647	0 257881	0 257881

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PARENT/TEACHER GUIDE

The Mathlin Kids: Case of the Missing Book

By: Nausheen Ahmed

Revised: September, 2013

Table of Contents:

Multiplication, Division, Fractions, Decimals and Percentages

Instructions: Please use the following script to review "Multiplication, Division, Fractions, Decimals and Percentages" with your students.

Mark: Mathlin, what if I have a problem where I'm asked to multiply by 100, 1000, or 10,000, do I have to multiply out the numbers?

Mathlin: Mark, that's an excellent question. There is an easier and faster way to do multiplication when you're multiplying numbers by 10, 100, 1000 or 10,000. Let's say you are asked to do the following problem: 2.25 × 100. First, how many zeros does one hundred, 100, have?

Mark: One hundred, 100, has two zeros.

Mathlin: Excellent! Since 100 has two zeros, we will simply move the decimal point in 2.25 two places to the right; therefore, 2.25 times 100 will equal 225.

Mark: Well, what if I had to do 33.659 × 1000?

Mathlin: First, let's count the number of zeros in 1000. 1000 has three zeros. Since 1000 has three zeros, we are going to move the decimal place in 33.659 three places to the right. Therefore, the solution is 33.659 × 1000 = 33,659.

Mark: Cool! Now what I had to divide? Let's say I was asked to solve 2.25 ÷ 100.

Mathlin: First, let's count the number of zeros in 100. 100 has two zeros. Since we are dividing, we will be moving the decimal point to the left this time. So 2.25 ÷ 100 = 0.0225

Mark: How about 33.659 ÷ 1000?

Mathlin: Why don't you try this one, Mark?

Mark: Okay, so first I count the number of zeros in 1000. 1000 has three zeros. Therefore, I move the decimal point in 33.659 three places to the left. 33.659 ÷ 1000 = 0.033659

Mathlin: Great! Now what will the answer be if I asked you to solve 550 ÷ 100?

Mark: Where's the decimal point?

Mathlin: Even though we don't write a decimal point at the end of whole numbers, there is always a decimal point there. So 550 is actually 550. We can even place zeros after the decimal point. So now you know, 550 = 550. = 550.0 = 550.00

Mark: So then since 100 has two zeros, I will just move the decimal point over two places to the left. So my answer becomes: 550. ÷ 100 = 5.50

Mark: Do I always have to use the division symbol, ÷, when writing a division problem?

Abby: No, Mark. Division problems are often written as fractions.

MatheMan: That's right. Fractions basically tell you what portion of a whole we're talking about. For example, let's assume I bake a pie. Now I slice it into 8 equal pieces. How can I write this as a fraction? I would write 8/8. Now, let's say we all eat 3 pieces out of the 8 total pieces available. How can we write that as a fraction?

Mark: 3 out of 8. So we'll write it as 3/8?

MatheMan: Good job! Here's some math vocabulary for you. If the number on the top is bigger than the number on the bottom, as in the fraction 8/5, the fraction is called an Improper Fraction. If the number on the top is smaller than the number on the bottom, the fraction is called a Proper Fraction. The top number is known as the numerator. The bottom number is known as the denominator.

Mark: What if the number on the top equals the number on the bottom?

Mathlin: Then we really have a whole number in disguise. For example, in the pie I supposedly baked, I had 8 pieces. If I eat all 8 pieces out of the 8 pieces available, 8/8, I have eaten 1 whole pie. 8 ÷ 8 = 1.

Abby: Mark, remember the birthday party we had for Mathlin last year? We had baked two pies. We ended up eating from both pies, rather than finishing 1 pie first before eating from the second pie.

Mark: Oh yeah! We had cut the first pie into 8 equal pieces, and then I wanted a bigger piece for myself so I cut the second pie into 6 equal pieces. I had already eaten 1 piece from the second pie, so I had eaten 1/6 of the pie, before the guests arrived.

Abby: Right! And then our guests ate 4 pieces from the first pie. The first pie had 8 total pieces. They ate 4/8 of the pie.

MatheMan: Children, instead of saying 4/8 of the first pie, why don't you just say that the guests ate ½ of the first pie?

Mark: Why can we say that?

MatheMan: You can actually reduce 4/8 to its simplest form. The number 4 goes into both 4 and 8 4 × 1 = 4, and 4 × 2 = 8. So that makes 4/8 = 4 × 1 / 4 × 2 = 4/4 × ½. And we know that 4/4 = 1. So we can think of it as 1 × ½ = ½.

Mark: Nice! I learn something new every day! So our guests ate ½ of the first pie. Did you know that they had more pie later? They ate 2 pieces from the second pie also.

Abby: Mark, you had 1 piece from the second pie, and the guests had 2 pieces from the second pie. That means a total of 3 pieces out of the 6 available pieces were eaten. We can write that as 3/6.

Mark: Hey I know this! 3/6 can be reduced to its simplest form! 3 × 1 = 3 and 3 × 2 = 6, so that makes 3/6 = 3 × 1 / 3 × 2 = 3/3 × ½ = 1 × ½ = ½.

MatheMan: Hurray! My dear you've got it! Now, can you tell me how much pie was eaten all together?

Mark: You mean how much pie was eaten from both the pies?

MatheMan: Yep!

Mark: Well, 4 out of 8 pieces were eaten from the first pie. And 3 out of 6 pieces were eaten from the second pie. So we ate 7 pieces?

MatheMan: Nope! The size of a piece in the first pie is different from the size of the piece in the second pie. Mathematically, we would write the problem like this: 4/8 + 3/6. You see, the bottom numbers, the denominators, are different. We need to have the same numbers on the bottom, to add or subtract fractions. We cannot add or subtract pieces of different sizes. What number do both 8 and 6 go into?

Abby: I know! 24! I know that because 8 × 3 = 24 and 6 × 4 = 24.

MatheMan: Good Job! So let's multiply.

Mark: Okay, so that makes this fraction 4/ (8×3) = 24.

MatheMan: Well, not exactly. You see, if you multiply the bottom number, the denominator, with a number, you have to multiply the top number, the numerator, with the exact same number.

Mark: So I multiply the denominator and the numerator by 3?

MatheMan: Yes.

Mark: Okay, so that makes 4/8 = (4×3)/ (8×3) = 12/24. Is this right?

MatheMan: Right!

Mark: Cool! So then 3/6 = (3×4)/(6×4) = 12/24 because we have to multiply 6 by 4 to make it 24, and if we multiply the denominator by a number, we absolutely have to multiply numerator by that number also. In our case, that number is 4.

MatheMan: Yes!

Abby: So now we can add 12/24 + 12/24 = 24/24. Hey! 24/24 is 1 whole pie! 1 whole pie was eaten out of the two available pies.

Mark: Hey, I know another way we can do this. We had already reduced 4/8 to ½, and 3/6 to ½. ½ + ½ = 2/2, 2/2 is 1 whole!

MatheMan: You children are so smart!

Mark: Can we write fractions as decimals?

MatheMan: Yes you can. ½ is basically 1 divided by 2, also written as 1 ÷ 2. What is 1 ÷ 2? First, since we can't really multiply 2 by anything that will give us 1 as an answer, let's place a decimal and add a zero. Now, 2 × 5 = 10, so we'll place a 5 on top and complete our division. So our answer is ½ = 0.5

Abby: Are there any common fraction to decimal values we should know?

MatheMan: Yes. Here they are: ¼ = 0.25, ½ = 0.50, and ¾ = 0.75

Abby: Hey, Mark, do you know what percent of the first pie got eaten?

Mark: Hey, that's a really easy question. Go convert any decimal to a percent; I know we just have to multiply the decimal by 100. ½ of the first pie got eaten. ½ is 0.50, so we multiply 0.50 by 100 to go to a percent. 100 has two zeros, so we will move the decimal point two places to the right. 0.50 × 100% = 50% of the first pie got eaten!

Abby: Good job, Mark!

Mark: I would like to add another comment. Since we multiply by 100 go to from decimal to percentage, we will divide to go from percentage to a decimal. For example, 50% is just 50/100, which is 50 ÷ 100. 100 has two zeros and we are dividing, so we move the decimal point two places to the left. 50 ÷ 100 = 0.50

MatheMan: Great job!

Mark: What if I have to multiply fractions?

MatheMan: Well, just multiply the top numbers, and then multiply the bottom numbers. For example, 2/4 × 1/5 = (2 × 1) / (4 × 5) = 2/20. You always want to reduce, or simplify, the fraction. Since 2 is a factor of both 2 and 20, we can reduce 2/20 to 1/10 because 2 × 1 = 2 and 2 × 10 = 20.

Abby: Is dividing fractions as easy as multiplying them?

MatheMan: Yes it is. If our problem was 2/4 ÷ 1/6, the first thing we would do is flip the second fraction upside-down. The fraction we get by flipping the second fraction upside-down is known as a reciprocal. And then we just multiply. Let's begin. The reciprocal of the second fraction 1/6 is 6/1. And then we multiply 2/4 × 6/1 = (2 × 6) / (4 × 1) = 12/4. Now let's simplify: 2 × 6 = 12, and 2 × 2 = 4, so our fraction becomes 6/2. 6/2 is an improper fraction, let's reduce further. Remember, it's a division problem in disguise. Since 2 × 3 = 6, we can simplify 6/2 to 3.

Charts and Graphs

Graphs are useful tools to organize and show information. There are many different types of charts: Tally charts, bar graphs, pie charts and more.

Tally charts: Each tally mark in a tally chart represents one object. For example, to mark 4 bananas, one would make 4 tally marks on the chart.

Tally marks are grouped in sets of five. This just means that every 5th tally mark one would make will be a diagonal line. This practice will help count the tally marks on the tally chart faster.

Let's look at an example:

Number of lines in the tally for bananas: We have 4 bananas.

Item we are counting:	Tally:
Bananas	IIII
Oranges	𝍸 𝍸 I

Notice the diagonal line

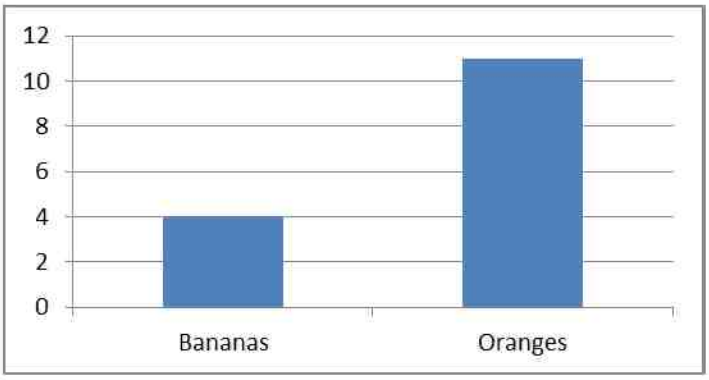
5 + 5 + 1 = 11; we have 11 oranges!

Okay, so what if you wanted to know how many bananas we have? Can you count the number of lines in the tally for bananas? Did you count 4 lines? Very good! You just read a tally chart!

Now let's read the tally chart above to figure out how many oranges we have. Remember, a diagonal line, or a diagonal tally mark, indicates a group of five. Did you count 11 tally marks for the oranges? Excellent!

Bar graphs: Bar graphs are just a different way of showing the information. In bar graphs we use rectangles, or bars, of different length proportional to the value they represent.

Let's represent the information from the tally chart above in a bar graph:



Questions designed for book

Instructions: Please utilize the illustration on page 15 of the book to pose the following questions.

Question 1: In which direction do the numbers on the x-axis increase?

Answer 1: The numbers on the x-axis increase from left to right.

Question 2: In which direction do the numbers on the y-axis increase?

Answer 2: The numbers on the y-axis increase as we move up the y-axis.

Question 3: Why are there arrows at the ends of the parabolas?

Answer 3: The arrows at both ends of the parabolas just means that the lines of the parabolas keep going and going. You will not have to worry about this concept further till you're a little older.

Instructions: Please utilize the blank lines below to create your own questions.

Long Multiplication: Problems and Solutions

Problem 1: 121 × 6 = 726

Long Multiplication: Problem 1		
121	$^{1}121$	$^{1}121$
x 6	x 6	x 6
6	26	726

Step 1: Multiply 1 x 6 = 6

Step 2: Multiply 2 × 6 = 12. We'll write the 2, and carry the 1.

Step 3: Multiply 1 x 6 = 6. But remember we carried the 1. Therefore, 1 + 6 = 7, and we'll write 7.

Problem 2: 298 × 54 = 16,092

Long Multiplication: Problem 2						
$2^{3}98$	$^{3}298$	298	298	$2^{4}98$	$^{4}298$	298
x 54	x 54	x 54	x 54	x 54	x 54	x 54
2	92	1192	1192	1192	1192	1192
			0	0C	90C	14900
						16092

Step 1: Multiply 8 × 4 = 32. Write the 2, and carry the 3.

Step 2: Multiply 9 × 4 = 36. But remember we carried the 3, therefore, 36 + 3 = 39. Write the 9, and carry the 3.

Step 3: Multiply 2 × 4 = 8. But remember we carried the 3, therefore, 8 + 3 = 11. Since we don't have any more integers in the first row to multiply with, we'll write 11.

Step 4: Now we'll start multiplying the top row, 298, by 5, which is in the tens column. Before we start the multiplication, we are going to place a 0 in the units column in the new row under 1192.

Step 5: Multiply 8 × 5 = 40. We write the 0, and carry the 4.

Step 6: Multiply 9 × 5 = 45. But remember, we carried the 4, therefore, 45 + 4 = 49. We write the 9, and carry the 4.

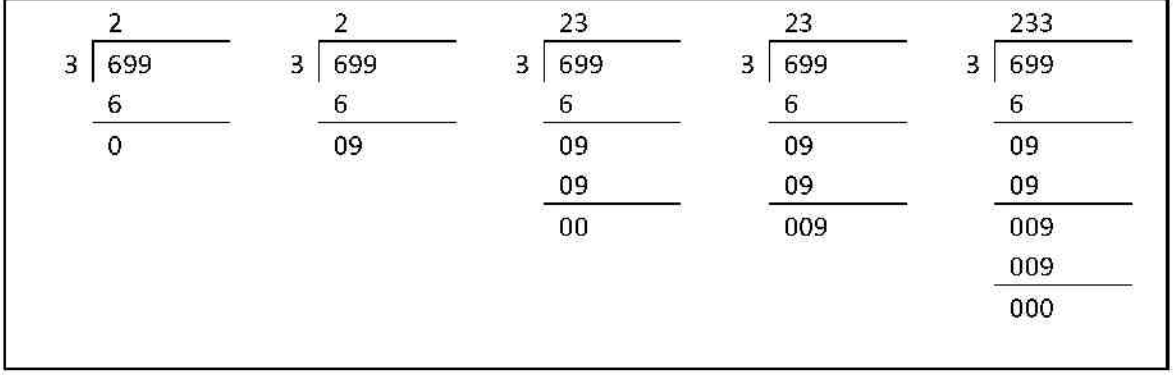
Step 7: Multiply 2 × 5 = 10. But remember, we carried the 4, therefore, 10 + 4 = 14. Since we don't have any more integers in the first row to multiply with, we'll write 14.

Step 8: Now we'll add the two rows. 1192 plus 14900 is equal to 16,092.

Long Division: Problems and Solutions

Problem 1: 699 ÷ 3 = 233

Long Division: Problem 1



Step 1: Set up the problem as shown above. Take a look at the left most digit of the number being divided. In our case, it's 6. 6 is the left most digit in 699. Does 3 divide 6? Yes, 3 × 2 = 6. Therefore write 2 above the 6, 3 × 2 = 6, therefore, write 6 in the left most spot below 699. Now subtract: 6 − 6 = 0. Therefore, write 0 under the 6.

Step 2: Let's bring the second left most digit, in this case 9, down to the level where we wrote 0.

Step 3: Take a look at this digit we brought down. Does 3 divide 9? Yes, 3 × 3 = 9. Therefore write 3 above the 9 (in the very first row) and write 9 in the last row and subtract. 9 minus 9 is 0, hence you'll notice I have written 00 in the very last row.

Step 4: Let's bring the third left most digit (this is the last digit in the number), down to the level where we wrote 00.

Step 5: Take a look at this digit we brought down. Does 3 divide 9? Yes, 3 × 3 = 9. Therefore write 3 above the 9 in the very first row and write 9 in the last row and subtract. 9 minus 9 is 0, hence you'll notice I have written 000 in the very last row.

Therefore, 699 ÷ 3 = 233

Instructions: Cover problems 2 thru 4 on the next page as additional examples for the students.

Problem 2: 725 ÷ 25 = 29

Long Division: Problem 2

0	02	02	029
25 | 725	25 | 725	25 | 725	25 | 725
0	0	0	0
72	72	72	72
	50	50	50
	22	225	225
			225
			0

Problem 3: 456 ÷ 24 = 19

Long Division: Problem 3

0	01	01	019
24 | 456	24 | 456	24 | 456	24 | 456
0	0	0	0
45	45	45	45
	24	24	24
	21	216	216
			216
			0

Problem 4: 352 ÷ 11 = 32

Long Division: Problem 4

0	03	03	032
11 | 352	11 | 352	11 | 352	11 | 352
0	0	0	0
35	35	35	35
	33	33	33
	02	022	022
			022
			0

Division discussion points for teachers, parents and tutors

- Placing a zero in front of the number does not change the number. Examples include: 005 = 5, 010 = 10, 069 = 69, 055 = 55, and 0101 = 101.
- Have your students' double check their answers using multiplication.
- As a challenge, have your students simplify the fraction portion of a long division problem to the simplest form and then convert to a decimal answer.
- Go over decimals in long division with your students. I'm going to show you a couple examples below.

Example 1: 1 ÷ 2 = 0.5

Example 1

```
  0        0.       0.5
2 | 1    2 | 1.0  2 | 1.0
  0        0        0
  1        10       10
                    10
                    0
```

Example 2: 3 ÷ 4 = 0.75

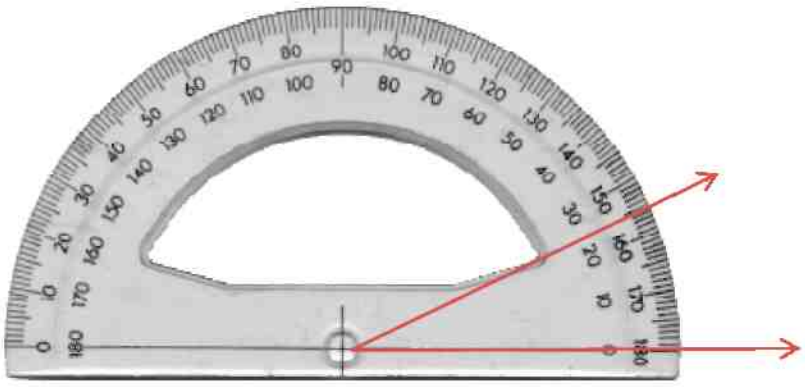
Example 2

```
  0        0.       0.7      0.7      0.75
4 | 3    4 | 3.0  4 | 3.0  4 | 3.0  4 | 3.0
  0        0        0        0        0
  3        30       30       30       30
                    28       28       28
                    02       020      020
                                      020
                                      0
```

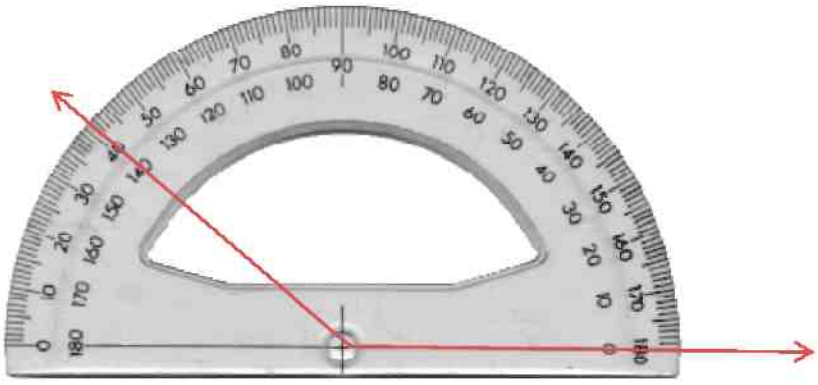
Instructional notes about the examples above:

- First thing to remember is that 6 = 6.0 = 6.00. I can add as many zeros after the decimal point as I want to without changing the actual number.
- For Example 1:
  - I ask myself, how many time does 2 go into 1?
  - The answer is none.
  - So we multiply by 0. I will place a 0 on top, a 0 under the 1, and then subtract 1 - 0 = 1.
  - Now I'm supposed to bring down the next digit, but there is no next digit after 1. Aha! 1 = 1.0 = 1.00. So what I will do is simply add the decimal point to the dividend, making it 1.0
  - I'll then carry the decimal point up to the answer line, and move the 0 down as the next digit.
  - Now, how many times does the number 2 go into 10?
  - The answer is 5 since 2 × 5 = 10.
  - I'll write a 5 on the answer line, and then subtract, 10 - 10 = 0.

Answer Key: Protractor Challenge

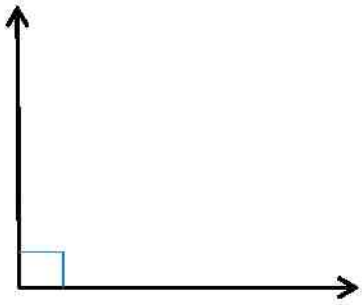
1. 

Angle Measure: 25 degrees. This type of angle is called: Acute Angle

2. 

Angle Measure: 140 degrees. This type of angle is called: Obtuse Angle

Use the picture below to answer Question 2:



Question 1: Do you know why there are arrows at the end of each line above?
Answer 1: Arrows at the end of the lines just means that the lines keep on going.
Question 2: Do you know what the symbol between the two lines means in the picture above?
Answer 2: It indicates that this angle is a right angle. The angle measure for a right angle is 90 degrees.

Place Value Quiz

1. A number has five ones and two tens. What is the number?
   a. 5
   b. 25
   c. 15
2. What is the value of the two in twenty-two?
   a. 20
   b. 2
   c. 200
3. In the number 20, which digit is in the tens place?
   a. 0
   b. 2
   c. None of the above
4. In the number 30, which digit is in the ones place?
   a. 3
   b. 0
   c. None of the above
5. In the number 19, which digit is in the ones place?
   a. 9
   b. 1
   c. None of the above
6. Thirty-six times ten equals three hundred plus sixty. What is another name for three hundred plus sixty?
   a. 3360
   b. 3600
   c. 30060
   d. 360
7. What is another way to write one hundred twenty-one?
   a. 120 + 21 + 0
   b. 100 + 20 + 1
   c. 100 + 2 + 1
8. Which digit is in the hundreds place in 360?
   a. 0
   b. 6
   c. 3

9. 14 + 1 + 4 can be written as:
    a. 20
    b. 19
    c. 17
10. A number has six ones and three tens. What is that number?
    a. 63
    b. 36
    c. 306
11. What is the value of eight in twenty-eight?
    a. 28
    b. 82
    c. 8
    d. 2
12. In the number 31, which digit is in the tens place?
    a. 1
    b. 3
    c. None of the above
13. In the number 34, which digit is in the ones place?
    a. 4
    b. 3
    c. None of the above
14. What is another name for twenty-five plus five?
    a. 255
    b. 25
    c. 30
    d. 29
15. What is another way to write nineteen plus 6 plus zero?
    a. 90 + 6 + 0
    b. 19 + 1 + 6
    c. 19 + 6 + 0
16. What is another name for five plus nine plus one?
    a. 11
    b. 13
    c. 15
17. 29 × 10 = 290. The digit 9 is in the _____place.
    a. Ones

b. Tens
c. Hundreds

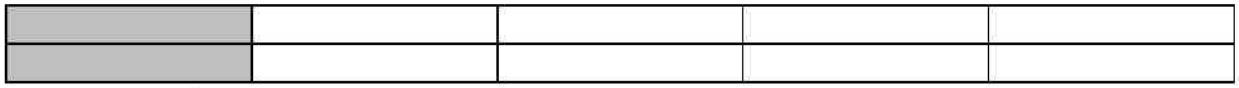
18. 12 × 12 = 144 144 can also be written as:
    a. 100 + 40 + 4
    b. 140 + 40 + 4
    c. 100 + 44 + 4
19. In the number 144, which digit is in the tens place?
    a. 4
    b. 44
    c. 1
    d. 14
20. If we represent digits using letters XYZ, which letter is the ones place?
    a. X
    b. Y
    c. Z
21. If we represent digits using letters XYZ, which letter is the tens place?
    a. X
    b. Y
    c. Z
22. If we represent digits using letters XYZ, which letter is the hundreds place?
    a. X
    b. Y
    c. Z

Fractions Quiz

1. Eighteen divided by nine, also written as 18 ÷ 9 can be represented by which of the following fraction?
    a. 9/18
    b. 18/9

2. What is 18/9?
    a. 1
    b. 3
    c. 5
    d. 2

3. What fractional part of this figure is used?

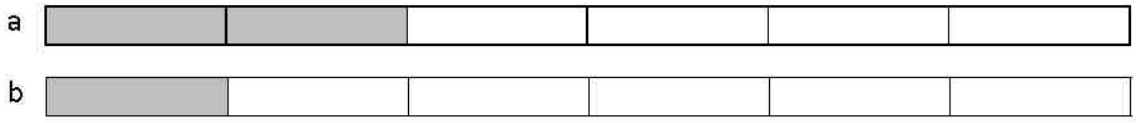


    a. 2/10
    b. 2/8
    c. 1/5
    d. Both a and c
    e. Both b and c

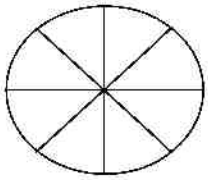
4. If MatheMan cuts an apple pie into six equal pieces and eats one piece. How many of the pie has MatheMan eaten?
    a. 1/6
    b. ½
    c. 1/3
    d. ¼

5) Which of the following fractions is the greatest?
    a. 1/8
    b. 1/6
    c. 1/9

6. Look at the fraction bars. Which fraction bar shows 1/3 used?



7. If I took two pieces of this pie, what fraction of the pie would I be taking?



a. 1/8
b. 2/8
c. 1/4
d. Both b and c

8. What fraction of the images are stars?

a. 3/8
b. 5/8
c. 7/8

9. Which fraction is equal to one whole?

a. 5/5
b. 3/5
c. 4/5
d. 1/5

10. Mathlin divides the whole class into groups to work on different math problems. Each group has one-fourth of all the children in the class. How many groups are there?

a. 2
b. 4
c. 6
d. 8

Equal, Less or Greater Than? Quiz

1. Which number sentence is true?
   a. 3 < 2
   b. 2 < 3
   c. 2 < 2
   d. 3 > 3
2. Which number goes in the blank? **2 < ___ < 5**
   a. 3
   b. 1
   c. 7
3. Which sign when inserted in the blank makes the number sentence true? **3 + 2 __ 5**
   a. +
   b. >
   c. =
   d. <
4. Which number goes in the blank? **5 > __**
   a. 10
   b. 7
   c. 3
5. Which sign makes a true number sentence? **5 – 2 __ 2**
   a. >
   b. <
   c. =
   d. –
6. Which number sentence is true?
   a. 9 = 14
   b. 9 < 14
   c. 14 < 9
7. Mathlin and MatheMan did this addition problem when putting all the cookies in one tray. Which subtraction problem shows that they got the right answer? **5 + 9 = 14**
   a. 14 - 9
   b. 9 - 5
   c. 14 - 10
8. Which of the answer choices can be used to check the following problem? **14 + 1 = 15**
   a. 15 + 1 = 16
   b. 15 - 1 = 14
   c. 12 + 3 = 15
   d. 20 - 5 = 15
9. Which number sentence is an opposite number sentence for fifteen plus four equals nineteen? **15 + 4 = 19**
   a. 16 + 3 = 19
   b. 10 + 9 = 19
   c. 19 – 15 = 4
   d. 6 – 2 = 4
10. What is the solution to this problem? **25 - 6 = __**
    a. 18
    b. 21
    c. 19

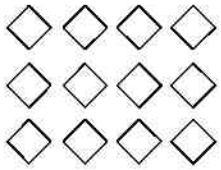
d. 17

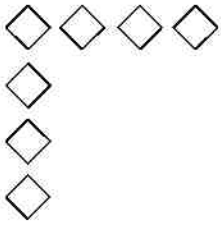
11. What is the solution to this problem? **19 + 6 = __**
    a. 21
    b. 25
    c. 23
    d. 24
12. Mathlin had eight cookies in a tray. The children ate six cookies. How many cookies are left in his tray?
    a. 2
    b. 3
    c. 4
    d. 5
13. What is seven minus three?
    a. 8
    b. 9
    c. 3
    d. 4
14. Which sign makes a true number sentence? **10 - 9 __ 5 - 1**
    a. >
    b. <
    c. =
15. The children ate two cookies out of a total of six cookies. How many cookies are left?
    a. 1
    b. 2
    c. 3
    d. 4
16. There are nine cookies on a tray. The children only eat three cookies. How many are left?
    a. 4
    b. 5
    c. 6
    d. 7
17. Which sign makes a true number sentence? **7 - 5 __ 2**
    a. >
    b. <
    c. =
18. What is the difference in 36 and 30?
    a. 2
    b. 4
    c. 6
    d. 8

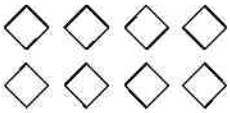
19. Which number sentence is an opposite number sentence for twenty-five plus five equals thirty? **25 + 5 = 30**
    a. 10 + 20 = 30
    b. 15 + 15 = 30
    c. 30 – 25 = 5
    d. 7 – 2 = 5
20. Mathlin had thirty cookies in a tray. He added six more cookies. How many cookies does he have on the tray?
    a. 36
    b. 40
    c. 33
    d. 39

Multiplication and Division Quiz

1. Which drawing shows **4 × 4**?

a. 

b. 

c. 

2. Mathlin bakes three croissants every four minutes. How many croissants will Mathlin have baked after sixteen minutes?

Minutes	4	8	12	16
Croissants	3	6	9	

a. 10
b. 11
c. 12
d. 13

3. Which choice is the same as the number sentence shown below?

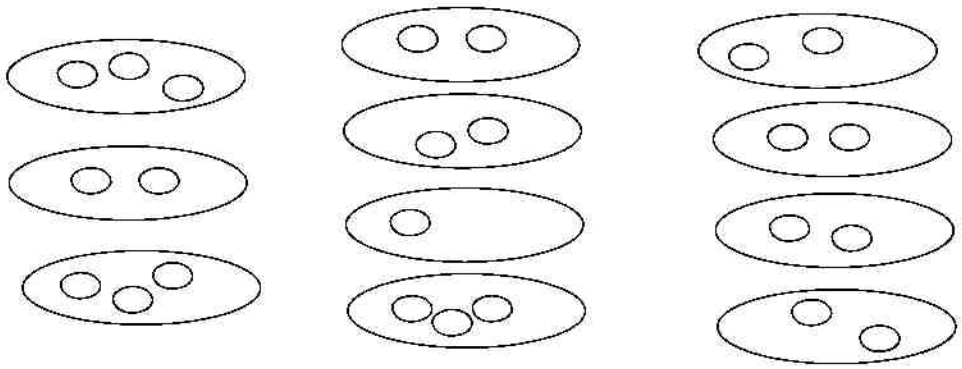
2 × 2 × 2 × 2 × 2 × 2 × 2 = __

a. 2 × 6
b. 8 × 2
c. 2 × 7
d. 9 × 2

4. Mathlin has these star stickers. He will give two star stickers to each of the three top achievers in his learning group. How many stickers will he have left?

a. 1
b. 2
c. 3
d. 4

5. Which picture shows how four children should share eight cookies equally?



A B C

6. There are five tables in the bakery. Twenty children are equally divided into five groups to be seated at each table. How many children are there in each group?
   a. 4
   b. 5
   c. 6
   d. 7

7. Mathlin removes one table out of five tables from the bakery. There are still twenty children waiting to sit and eat. We want to sit an equal number of children at each table. How many children are seated at each table?
   a. 3
   b. 5
   c. 7
   d. 9

8. MatheMan brought four apple pies to the bakery. Each pie is cut into six equal pieces. The twenty children get one piece of pie each. Then Mathlin and MatheMan each take a piece of pie. How many pieces of pie are left?
   a. 2
   b. 4
   c. 6
   d. 8

9. Which number shows the answer to seven times four?
   a. 35
   b. 18
   c. 28
   d. 48

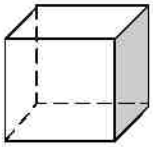
10. One cap costs three dollars. How much will five caps cost?
    a. 5
    b. 10
    c. 15
    d. 20
11. True or False: **3 × 8 = 8 × 3**
    a. True
    b. False

12. Which number when multiplied by nine equals a product of twenty-seven?
    a. 2
    b. 3
    c. 4
    d. 5

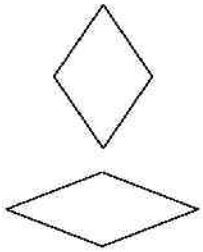
13. Mathlin wants to pass out thirty-six crayons to nine children. He wants every one of the nine children to get an equal number of crayons. How many crayons each do the children have?
    a. Two
    b. Three
    c. Four
    d. Ten

Shapes Quiz

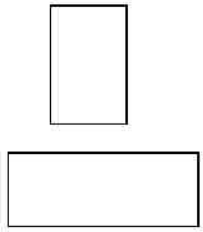
1. How many faces does a cube have?
   a. 4
   b. 7
   c. 6
   d. 8



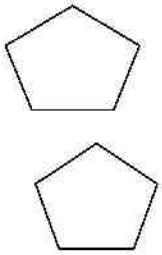
2. Look at the pairs of shapes. Which is not a pair of quadrilaterals?



A



B



C

3. A tetrahedron is a:
   a. Cube
   b. Pyramid
   c. Cone
   d. Sphere

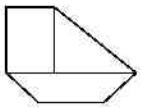
4. What shape are the faces of a tetrahedron?
   a. Triangles
   b. Squares
   c. Circles
   d. Stars

5. What shape is a base of a cylinder?
   a. Triangle
   b. Square
   c. Circle
   d. Rectangle

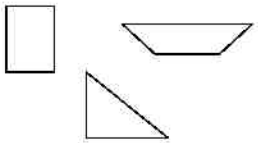
6. A diamond can be made by joining a pair of:
   a. Triangles
   b. Squares
   c. Circles
   d. Rectangles

7. True or False: I can make a hexagon by joining have six equilateral triangles.
    a. True
    b. False

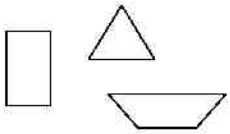
8. What shapes can be joined to make the following shape?



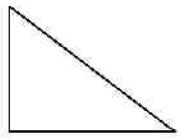
    a. These shapes:



    b. These shapes:

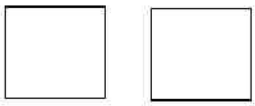


9. The following triangle is known as a(n):



    a. Equilateral Triangle
    b. Right Triangle
    c. Isosceles Triangle

10. What shape can I make by joining the following pair of shapes?



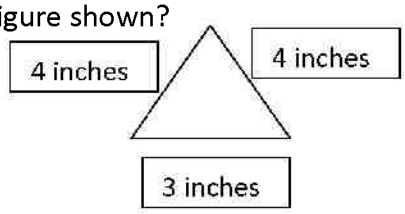
    a. Diamond
    b. Square
    c. Rectangle
    d. Trapezoid

Comprehensive Test One

1. Henrietta is buying 6 board games as gifts. Each board game costs $11.45. Which is the total amount that Henrietta will spend on the board games?
   a. $56.70
   b. $58.60
   c. $66.60
   d. $68.70

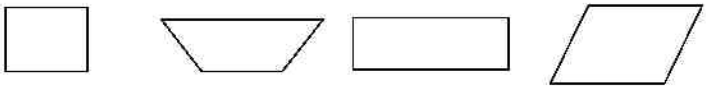
2. Kelly is selling her stuffed animals. There are 56 stuffed animals on a shelf. She has 2 shelves of stuffed animals. Which shows how many stuffed animals Kelly is selling?
   a. 92
   b. 112
   c. 122
   d. 152

3. Which is the perimeter of the figure shown?
   a. 4 inches
   b. 8 inches
   c. 11 inches
   d. 16 inches



4. Jerry buys a glass of juice and a burger at a diner. They cost $9.46 together. He pays with a $20 bill. Which shows how much change Jerry receives?
   a. $10.54
   b. $10.65
   c. $11.45
   d. $11.64

5. Which quadrilaterals appear to have NO right angles?
   a. 1 and 2
   b. 1 and 3
   c. 2 and 3
   d. 2 and 4



6. Which number completes the fact family?

9 × 8 = 72
72 ÷ 9 = 8
8 × □ = 72
72 ÷ □ = 8

a. 3
b. 6
c. 9
d. 21

7. Which shows the correct number sequence?
a. 40 ÷ 8 = 40 ÷ 5
b. 35 ÷ 7 = 28 ÷ 4
c. 16 ÷ 4 = 12 ÷ 2
d. 36 ÷ 9 = 8 ÷ 2

8. The local sidewalk was 695 feet long. It was extended to 965 feet. Which shows how many feet were added to the sidewalk?
a. 270 feet
b. 380 feet
c. 490 feet
d. 595 feet

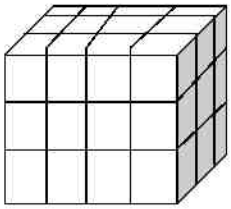
9. Which is the missing value in the table?

Dollars	$1	$2	$3	$4	$5
Quarters	4	8		16	20

a. 10
b. 12
c. 20
d. 14

10. There are 4,715 people attending a conference. Which shows 4,715 written in word form?
a. Forty hundred fifteen
b. Four thousand seven hundred fifteen
c. Four hundred fifteen
d. Four thousand seventy-five

11) Which is the volume for the figure shown?



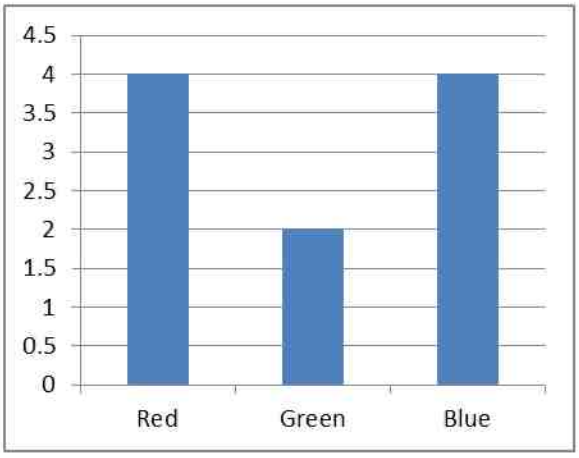
a. 36 cubic units
b. 40 cubic units
c. 45 cubic units
d. 65 cubic units

12) The pirate ship is 1,200 feet long. The government ship is 3,324 feet long. Which is the total length of the two ships?

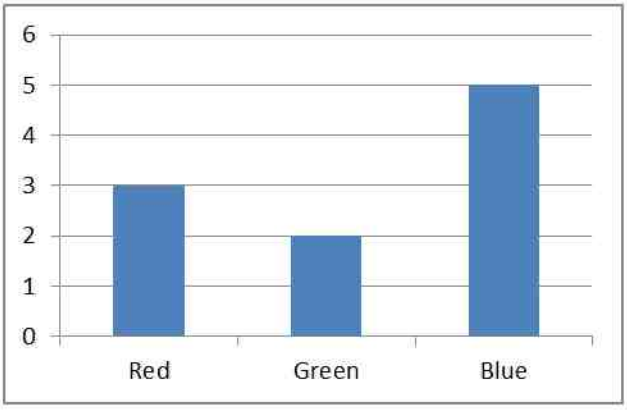
a. 2,345
b. 3,524
c. 4,524
d. 5,624

13) Terry pulled colored candies from a bag. She pulled blue candies 5 times, green candies 2 times and red candies 3 times. Which bar graph displays the results?

a. This graph:



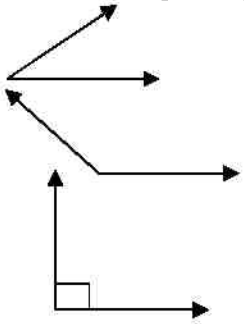
b. This graph:



14. Which angle is greater than a right angle?

a. This angle:

b. This angle:

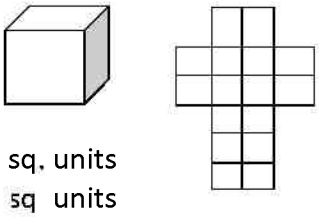


c. This angle:

15. Which is the missing number? □ × 9 = 0

a. 0
b. 1
c. 10
d. 11

16. Which is the total area that covers the solid figure?

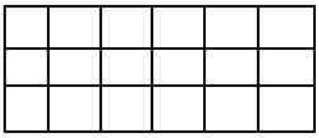


a. 12 sq. units
b. 16 sq units
c. 24 sq. units
d. 36 sq. units

17. A stream is 3,145 feet long. Which shows the value of the 1 in 3,145?

a. 10
b. 100
c. 1,000
d. 10,000

18. Which is the perimeter of the figure?



a. 10 units
b. 12 units
c. 18 units
d. 24 units

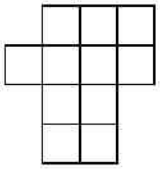
19. Which symbol correctly completes the equation? **56 □ 7 = 4 ▮ 2**

a. ÷

b. –
c. x
d. +

20. Which number sentence is in the fact family for the following numbers? **3, 5, 15**
    a. 3 x 5 = 15
    b. 3 + 5 = 8
    c. 20 ÷ 5 = 4
    d. 15 - 5 = 10

21. Which is the area of the figure shown?



    a. 9 sq units
    b. 10 sq units
    c. 11 sq units
    d. 12 sq units

22. Georgia has 12 cupcakes. She will eat 2 of them, and give the rest to her 5 friends. How many cupcakes will each friend have?
    a. 1
    b. 2
    c. 5
    d. 10

23. Which shows the value of the underlined digit? **1,9<u>7</u>3**
    a. 7
    b. 70
    c. 700
    d. 7,000

24. Which number sentence is NOT correct?
    a. 56 ÷ 7 = 4 x 2
    b. 81 ÷ 9 = 3 x 3
    c. 80 ÷ 10 = 2 x 4
    d. 18 ÷ 3 = 2 x 6

25. Which shows the decimal written in expanded form?

ONES	.	TENTHS
9	.	2

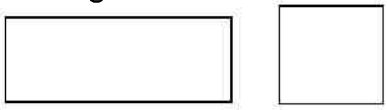
a. 0.09 + 0.002
b. 0.9 + 0.02
c. 9 + 0.02
d. 9 + 0.2

26. The cereal factory produced 9,753 boxes of cereal each year. Which shows how many boxes of cereal the factory produced in 5 years?
a. 48,658
b. 48,765
c. 57,658
d. 57,765

27. Compare the numbers. Which statement is correct?
a. 122 = 221
b. 565 > 575
c. 765 < 767
d. 109 > 190

28. Which shows 90 minutes written in hours and minutes?
a. 2 hours 10 minutes
b. 1 hour 20 minutes
c. 1 hour 30 minutes
d. 2 hours 20 minutes

29. How are the figures alike?



a. Both have right angles
b. Neither have equal angles
c. Both have equal sides
d. Both are pentagons

30. Which is a reasonable estimate of the quotient? 519 ÷ 3
    a. About 170
    b. About 1,700
    c. About 17,000
31. Dina pulled coins from a coin bag. She pulled a quarter 9 times, a dime 6 times and a nickel 3 times. Which tally chart shows these results?
    a. This tally chart:

Coin Results	
Quarters	III
Dimes	𝍸 I
Nickels	𝍸 IIII

    b. This tally chart:

Coin Results	
Quarters	𝍸 IIII
Dimes	𝍸 I
Nickels	III

32. Which type of triangle has NO equal sides?
    a. Equilateral
    b. Right
    c. Scalene
    d. Isosceles

33. Which is the missing value in the table?

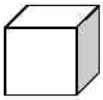
Dollars	$1	$2	$3	$4
Nickels	20		60	80

    a. 20
    b. 40
    c. 60

34. Building A is 909 feet high. Building B is 503 feet high. How much taller is Building A than Building B?
    a. 306 feet
    b. 406 feet
    c. 609 feet
    d. 509 feet

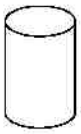
35. Which is the name of the solid figure shown here?

a. Cylinder
b. Square Prism
c. Square Pyramid
d. Cube



36. Which is the name of the solid figure shown here?

a. Cylinder
b. Square Pyramid
c. Cone
d. Sphere



37. Sam bought ½ pounds of walnuts and 2/4 pounds of almonds. How many total pounds of nuts did Sam buy?

a. ¼
b. 2/4
c. ¾

38. Which shows the value of the underlined digit? **9,5<u>1</u>3**

a. 1
b. 10
c. 12
d. 120

39. Which is the missing addend that makes the number sentence correct? 98 − □ = 11

a. 57
b. 62
c. 71
d. 87

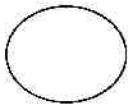
40. Which polygon is shown here?

a. Hexagon
b. Octagon
c. Quadrilateral
d. Trapezoid

41. Jerry has 4/6 cups of cake frosting. She used 2/6 cups of the cake frosting on her cake. Which shows how much cake frosting Jerry has left written in simplest form?
    a. 2/6
    b. 1/3
    c. ¼
    d. 2/4

42. Which shows the following number rounded to the nearest thousand? **3,513**
    a. 3,510
    b. 3,000
    c. 3,500
    d. 3,513

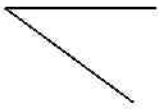
43. Which figure appears to be a polygon?
    a. Circle



    b. Star

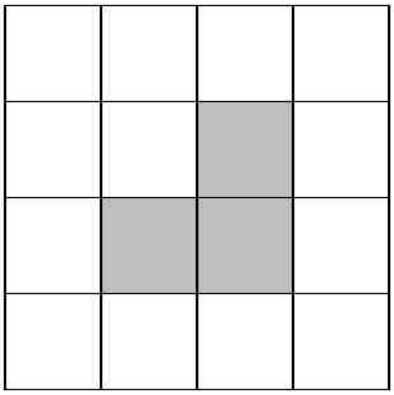
    c. Line

    d. Angle



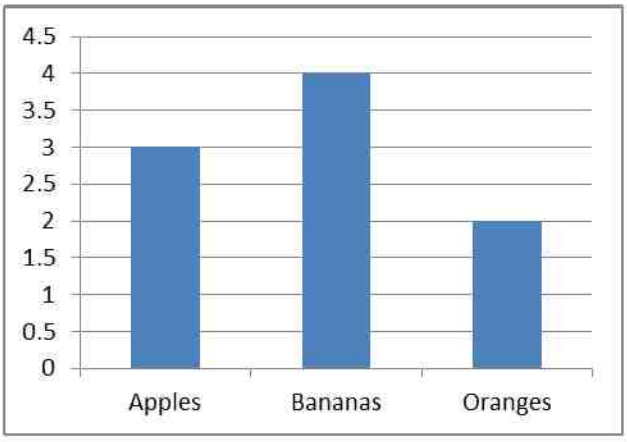
44. Which is the perimeter of the figure show?

    a. 8 units
    b. 10 units
    c. 14 units
    d. 16 units

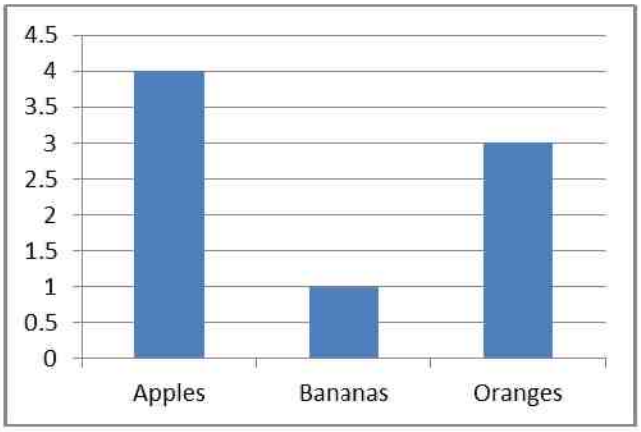


45. Mary packed fruits in a bag for her trip. She packed 3 apples, 4 bananas, and 2 oranges. Which graph displays these results?

a. This graph:



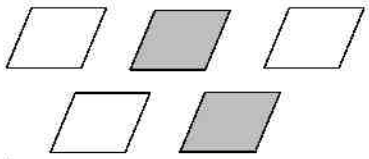
b. This graph:



46. Which is the sum? **596 + 57 = __**

a. 673
b. 695
c. 653
d. 1054

47. Which fraction shows the shaded part of the group?

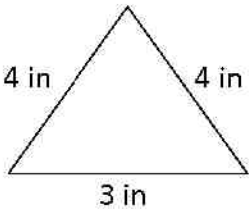


a. ½
b. 2/5
c. 3/5
d. 4/5

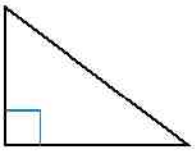
48. Noah is reading a book with 750 pages. He reads 10 pages a day. Which shows how many days it will take Noah to finish his book?
    a. 10
    b. 750
    c. 75
    d. 57

49. Which is the sum? **$8.30 + $0.25 = __**
    a. $6.55
    b. $7.45
    c. $8.55
    d. $9.35

50. Which type of triangle is this?
    a. Isosceles
    b. Equilateral
    c. Scalene
    d. Right



51. Which type of triangle is this?
    a. Isosceles
    b. Equilateral
    c. Scalene
    d. Right



52. Kayden found 1/10 of a dollar on the school playground. Which shows 1/10 written as a money amount?
    a. $0.01
    b. $0.10
    c. $1.10
    d. $1.11

53. Which number completes the conversion? **36 inches = __ feet**
    a. 3
    b. 4
    c. 5
    d. 6

54. Which is the missing number? **__ + 300 + 70 + 7 = 9,377**
    a. 90
    b. 900
    c. 9,000
    d. 90,000

55. Which number sentence is NOT correct?
    a. 54 ÷ 6 = 63 ÷ 7
    b. 36 ÷ 9 = 8 ÷ 2
    c. 40 ÷ 8 = 40 ÷ 5
    d. 90 ÷ 10 = 3 × 3

56. Which shows the numbers ordered from least to greatest?
    a. 6,357; 2,577; 8,905; 3,444
    b. 9,727; 8,365; 5,452; 4,668
    c. 2,446; 3,897; 5,788; 6,323
    d. 8,634; 6,495; 7,747; 5,412

57. Which is the sum? **$5.25 + $1.50 = __**
    a. $5.85
    b. $6.50
    c. $5.75
    d. $6.75

58. Which is the missing factor? **(4 × □) × 5 = 1 × (4 × 5)**
    a. 1
    b. 2
    c. 3
    d. 4

59. Which unit best measures the length of your fingers?
    a. Inches
    b. Feet
    c. Yard
    d. Mile

60. Niko bought 2 DVDs for a total of $24. Which shows how much each DVD costs?
    a. $8
    b. $10
    c. $12
    d. $14

61. Which shows 3,453 written in expanded form?
    a. 3 + 3 + 4 + 5 + 3
    b. 30,000 + 3,000 + 400 + 50
    c. 3,000 + 300 + 50 + 3
    d. 3,000 + 400 + 50 + 3

62. Which is the value of the missing factor? **4 × □ = 8 + 4**

a. 2
b. 3
c. 4
d. 5

63. Jackson sold 3 books for a total of $21.00 for a fundraiser event. Which shows how much each book costs?

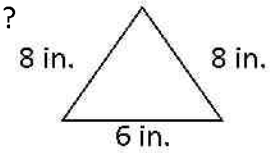
a. $6.00
b. $8.00
c. $7.00
d. $5.00

Comprehensive Test Two

1. Abby is buying 5 stuffed animals for her classmates. Each stuffed animal costs $5.95. Which is the total amount that Abby spent on the stuffed animals?
   a. $28.70
   b. $29.75
   c. $30.75
   d. $29.95

2. Sabrine is selling her clips. There are 47 clips in each box. She has 3 boxes of clips. Which shows how many clips Sabrine is selling?
   a. 137
   b. 145
   c. 122
   d. 141

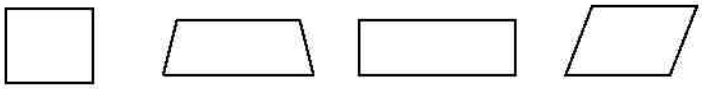
3. What is the perimeter for the figure shown?



   a. 20 inches
   b. 12 inches
   c. 22 inches
   d. 18 inches

4. Tyler buys three books at a book fair. They cost $12.85 together. He pays with a $20 bill. Which shows how much change Tyler receives?
   a. $7.15
   b. $8.15
   c. $11.45
   d. $7.64

5. Which quadrilaterals appear to have 4 right angles?



   a. 1 and 2
   b. 1 and 3
   c. 2 and 3
   d. 2 and 4

6) Which number completes the fact family?

9 × 5 = 45	45 ÷ 5 = 9	5 × □ = 45	45 ÷ □ = 5

a. 3
b. 6
c. 9
d. 21

7) Which shows the correct number sequence?

a. 12 ÷ 4 = 18 ÷ 6
b. 35 ÷ 7 = 28 ÷ 4
c. 16 ÷ 4 = 12 ÷ 2
d. 36 ÷ 2 = 2 ÷ 2

8) A wall was 245 feet long. It was extended to 505 feet. How many feet were added to the wall?

a. 280 feet
b. 375 feet
c. 260 feet
d. 295 feet

9) Which is the missing value in the table?

Dollars	$1	$2	$3	$4	$5
Dimes	10	20		40	50

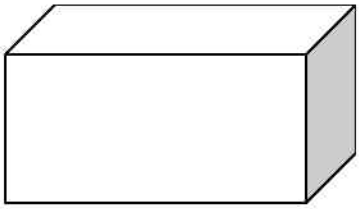
a. 20
b. 30
c. 40
d. 50

10) The carnival had 2,343 people over the weekend. Which shows 2,343 written in a word form?

a. Two hundred forty-three
b. Two thousand three hundred forty-three
c. Two hundred three hundred
d. Twenty three thousand forty-three

11) Which is the volume for the figure shown?

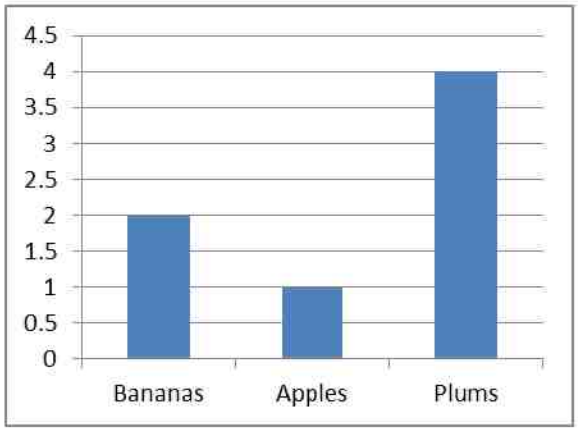
Length = 7 units; Width = 3 units; Height = 4 units



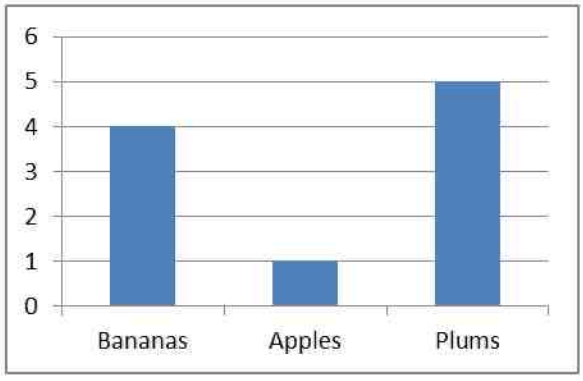
a. 76 cubic units
b. 82 cubic units
c. 84 cubic units
d. 65 cubic units

12. Bella hiked 1,800 feet up a mountain. Sophia hiked 1,544 feet up a mountain. What is the total number of feet they hiked?
    a. 3,344
    b. 3,524
    c. 4,245
    d. 5,462

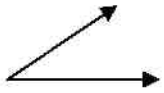
13. Dee pulled fruits from a bag. She pulled 2 bananas, 1 apple and 4 plums. Which bar graph displays the results?
    a. This graph:



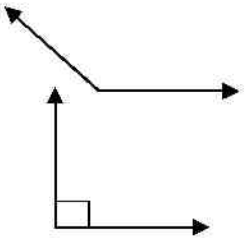
    b. This graph:



14. Which angle is equal to a right angel?
    a.



b.

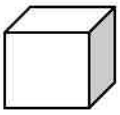


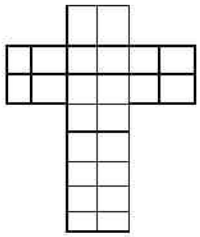
c.

15. Which is the missing number? **□ × 12 = 12**

a. 0
b. 1
c. 10
d. 11

16. Which is the total area that covers that solid figure?



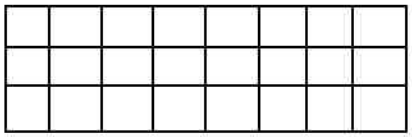


a. 22
b. 36
c. 24

17. A train is 2,145 feet long. Which shows the value of the 4 in 2,145?

a. 40
b. 400
c. 4,000
d. 40,000

18. Which is the perimeter of the figure?



a. 20
b. 16
c. 18
d. 22

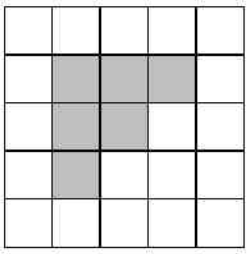
19. Which symbol correctly completes the equation? **16 □ 8 = 4 ÷ 2**

a. ÷
b. −
c. ×

d. +

20. Which number sentence is in the fact family for the following numbers? **4, 6, 24**
    a. 4 × 6 = 24
    b. 4 + 6 = 10
    c. 12 ÷ 6 = 2
    d. 24 − 4 = 20

21. Which is the area of the figure shown?
    a. 5 sq units
    b. 6 sq units
    c. 7 sq units
    d. 8 sq units



22. Nelly baked 5 apple pies. She will keep 1 of them for her family, and give the rest to her 4 friends. How many pies will each friend have?
    a. 1
    b. 2
    c. 5
    d. 10

23. Which shows the value of the underlined digit? **2,<u>5</u>33**
    a. 5
    b. 50
    c. 500
    d. 5,000

24. Which number sentence is NOT correct?
    a. 10 − 2 = 4 × 2
    b. 9 ÷ 9 = 1 × 1
    c. 80 ÷ 10 = 2 × 4
    d. 18 ÷ 3 = 2 × 6

25. Which shows the decimal written in expanded form?

ONES	.	TENTHS

3	.	7

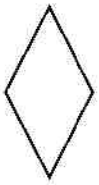
a. 0.03 + 0.007
b. 0.3 + 0.07
c. 3 + 0.07
d. 3 + 0.7

26. The furniture factory produces 3,194 chairs each year. Which shows how many chairs the factory produced in 4 years?
a. 9,786
b. 12,776
c. 13,899
d. 12,682

27. Compare the numbers. Which statement is correct?
a. 246 = 426
b. 975 > 957
c. 768 < 767
d. 209 > 390

28. Which shows 80 minutes written in hours and minutes?
a. 2 hours 10 minutes
b. 1 hours 20 minutes
c. 1 hours 30 minutes
d. 2 hours 20 minutes

29. How are the figures different?



a. One has right angles
b. Neither have right angles
c. Neither are polygons
d. Both are pentagons

30. Which is a reasonable estimate of the product? **3,805 × 10 = __**
a. About 380
b. About 3,800
c. About 38,000

31. Jill rolled a dice. She rolled "2" 6 times, "4" 4 times and "6" 3 times. Which tally chart shows these results?

a.

Dice Results	
2	III
4	𝍸 I
6	𝍸 I

b.

Dice Results	
2	𝍸 I
4	IIII
6	III

32. Which type of triangle has two equal sides and no right angles?
    a. Equilateral
    b. Right
    c. Scalene
    d. Isosceles

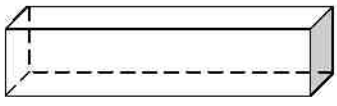
33. Which is the missing value in the table?

Dollars	$1	$2	$3	$4
Pennies	100		300	400

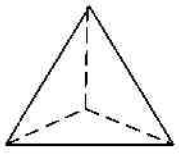
    a. 200
    b. 400
    c. 600

34. Red Lake is 300 feet wide. Clear Lake is 135 feet wide. How much wider is Red Lake than Clear Lake?
    a. 150 feet
    b. 155 feet
    c. 165 feet
    d. 275 feet

35. Which is the name of the solid figure shown?
    a. Cylinder
    b. Square Prism
    c. Square Pyramid
    d. Cube



36. Which is the name of the solid figure shown?
    a. Cylinder



b. Tetrahedron
c. Cone
d. Sphere

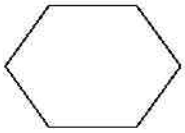
37. Jody brought 1/3 pounds of lean ground beef and 2/3 pounds of roast beef. How many total pounds of meat did Jody buy?
a. 1/3
b. 2/3
c. 3/3

38. Amy bought 1/3 pounds of lean ground beef and 2/3 pounds of roast beef. How many total pounds of meat did Amy buy in the simplest form?
a. 1 pound
b. 2 pounds
c. 3 pounds

39. Which shows the value of the underlined digit? **7,<u>5</u>17**
a. 5
b. 50
c. 500

40. Which is the missing addend that makes the number sentence correct? **34 + ☐ = 17**
a. 17
b. 18
c. 71
d. 27

41. Which polygon is shown here?
a. Hexagon
b. Octagon
c. Quadrilateral
d. Trapezoid



42. Jacob has 3/6 of a sandwich. He eats 1/6 of the sandwich as a snack. Which shows how much sandwich Jacob has left written in simplest form?
a. 2/6
b. 1/3
c. 1/6
d. 2/4

43. Which shows the number rounded to the nearest thousand? **7,701**

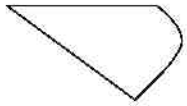
a. 7,700
b. 7,000
c. 7,701
d. 7,770

44. Which figure appears to be a polygon?

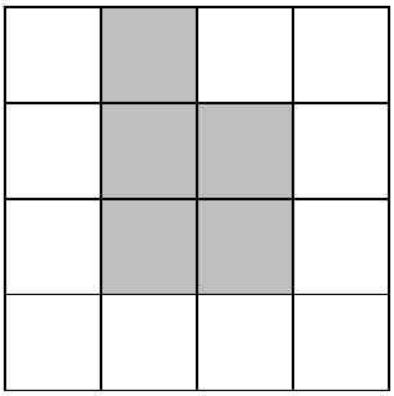
a. 

b. 

c.

d. 

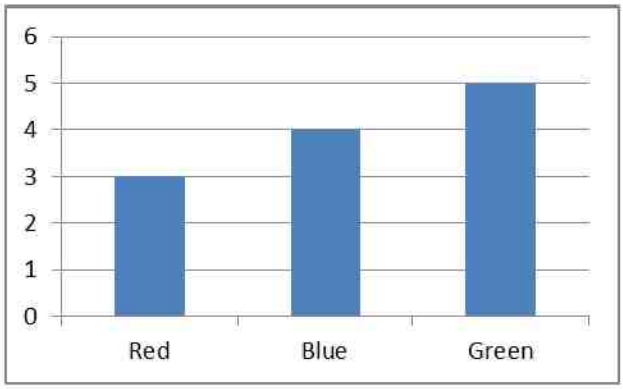
45. Which is the perimeter of the figure shown?



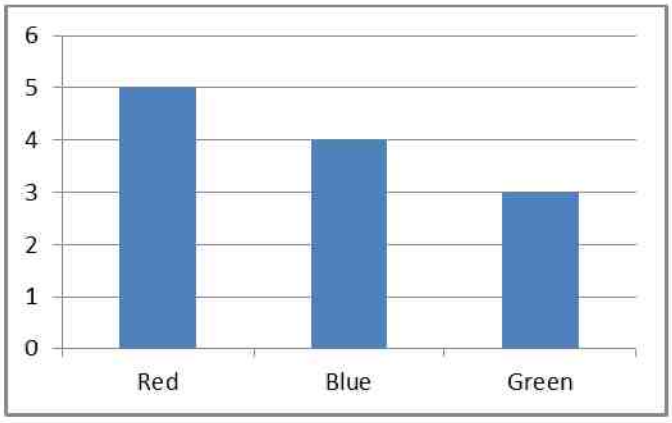
a. 8 units
b. 10 units
c. 14 units
d. 16 units

46) Marina pulled colored balls from a bag. She pulled out a red ball 3 times, a blue ball 4 times and a green ball 5 times. Which graph displays these results?

a.



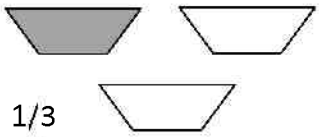
b.



47) Which is the sum? **484 + 82 = __**

a. 376
b. 496
c. 566
d. 576

48) Which fraction shows the shaded part of the group?

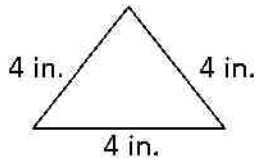


a. 1/3
b. 2/3
c. 3/3
d. 4/3

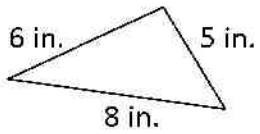
49. Simone is writing a book with 48 chapters. She writes 4 chapters in a week. Which shows how many weeks it will take Simone to finish writing her book?
    a. 10
    b. 12
    c. 16
    d. 20

50. Which is the sum? **$2.50 + $0.32 = __**
    a. $2.55
    b. $2.82
    c. $3.55
    d. $4.35

51. Which type of triangle is this?
    a. Isosceles
    b. Equilateral
    c. Scalene
    d. Right



52. Which type of triangle is this?
    a. Isosceles
    b. Equilateral
    c. Scalene
    d. Right



53. Lynetta needed 4/100 of a dollar more to buy chocolates. Which shows 4/100 written as a money amount?
    a. $0.04
    b. $0.44
    c. $4.40
    d. $4.44

54. Which number completes the conversion? **1 Foot = __ Inches**
    a. 6
    b. 12
    c. 18
    d. 24

55. Which is the missing number? **__ + 600 + 70 + 8 = 5,678**
    a. 50
    b. 500
    c. 5,000
    d. 50,000

56) Which number sentence is correct?

a. 54 ÷ 6 = 49 ÷ 7
b. 45 ÷ 9 = 18 ÷ 2
c. 48 ÷ 8 = 30 ÷ 5
d. 100 ÷ 10 = 3 × 3

57) Which shows the numbers ordered from greatest to smallest?

a. 6,357; 2,577; 8,905; 3,444
b. 9,727; 8,365; 5,452; 4,668
c. 2,446; 3,897; 5,788; 6,323
d. 8,634; 6,495; 7,747; 5,412

58) Which is the sum? **$5.85 + $1.35 = __**

a. $5.85
b. $7.20
c. $5.75
d. $6.75

59) Which is the missing factor? **[6 × □] × 3 = 6 × (6 × 1)**

a. 1
b. 2
c. 3
d. 4

60) Which unit best measures the length of fabric to be used for curtains?

a. Inches
b. Feet
c. Yard
d. Mile

61) John bought 2 posters for a total of $10. Which shows how much each poster costs?

a. $2
b. $3
c. $4
d. $5

62) Which shows 4,561 written in expanded form?

a. 4 + 4 + 5 + 6 + 1
b. 40,000 + 4,000 + 500 + 60
c. 4,000 + 500 + 60 + 1
d. 4,000 + 500 + 60 + 3

63) Which is the value of the missing factor? $6 \times \square = 6 + 6$

a. 2
b. 3
c. 4
d. 5

64) Mark sold 5 fish for a total of $45.00. Which shows how much each fish costs?

a. $7.00
b. $8.00
c. $9.00
d. $10.00

Answer Keys

Place Value Quiz		Fractions Quiz		=, < or > ? Quiz		x and ÷ Quiz	
1	B	1	B	1	B	1	A
2	A	2	D	2	C	2	C
3	B	3	D	3	C	3	B
4	B	4	A	4	A	4	B
5	A	5	B	5	B	5	C
6	D	6	A	6	A	6	A
7	B	7	D	7	B	7	B
8	C	8	C	8	C	8	A
9	B	9	A	9	C	9	C
10	B	10	B	10	B	10	C
11	C	Shapes Quiz		11	A	11	A
12	B	1	C	12	D	12	B
13	A	2	C	13	B	13	C
14	C	3	B	14	D		
15	C	4	A	15	C		
16	C	5	C	16	C		
17	B	6	A	17	C		
18	B	7	A	18	C		
19	A	8	A	19	A		
20	C	9	B				
21	B	10	C				
22	A						

Answer Keys

Comprehensive Test One				Comprehensive Test Two			
1	D	36	A	1	B	36	B
2	B	37	C	2	D	37	C
3	C	38	B	3	C	38	A
4	A	39	D	4	A	39	C
5	D	40	D	5	B	40	A
6	C	41	B	6	C	41	A
7	D	42	B	7	A	42	B
8	A	43	B	8	C	43	B
9	B	44	A	9	B	44	B
10	B	45	A	10	B	45	B
11	A	46	C	11	C	46	A
12	C	47	B	12	A	47	C
13	B	48	C	13	A	48	A
14	B	49	C	14	C	49	B
15	A	50	A	15	B	50	B
16	B	51	D	16	C	51	B
17	B	52	B	17	A	52	C
18	C	53	A	18	D	53	A
19	A	54	C	19	A	54	B
20	A	55	C	20	A	55	C
21	C	56	C	21	B	56	C
22	B	57	D	22	A	57	B
23	B	58	A	23	C	58	B
24	D	59	A	24	D	59	B
25	D	60	C	25	D	60	C
26	B	61	D	26	B	61	D
27	C	62	B	27	B	62	C
28	C	63	C	28	B	63	A
29	A			29	A	64	C
30	A			30	C		
31	B			31	B		
32	C			32	D		
33	B			33	A		
34	B			34	C		
35	D			35	B		

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