Effectiveness of Ladder Method of finding LCD for learning operations on fractions

A Thesis Presented to

The Faculty of the Mathematics Program

California State University Channel Islands

In (Partial) Fulfillment

of the Requirements for the Degree

Masters of Science

by

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December 2012

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Effectiveness of Ladder Method of finding LCD for learning operations on fractions. (MS Thesis)

Title of Item

Fractions, LCD, LCM, GCF, Ladder

3 to 5 keywords or phrases to describe the item

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ACKNOWLEDGEMENTS

I would like to express my appreciation to Professor Ivona Grzegorczyk, who directed my research. Her thoughtfulness, patience, understanding, and insight were invaluable during this endeavor. She has been a mentor, motivator, sources of ideas, and friend through my graduate studies.

I am grateful to my committee member, Professor Jorge Garcia, for reading my thesis and for his support during my graduate studies at CSUCI.

Finally, I would like to express my gratitude to my wonderful wife Shahnaz Mirzaei and my children Omid, Sara, and Ida. A special thanks to Sara for introducing me to the Ladder Method. They supported me throughout my studies and encouraged me to undertake such a monumental endeavor. Their insistence on hard work and discipline combined with their unyielding love was the perfect formula to help me achieve my goals. Without their kindness, understanding, support, generosity, and love I never could have completed my research and finalized my master's thesis.

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Abstract

There are many areas of mathematics that are difficult for students to learn. My extensive teaching experience tells me that students have difficulty finding the Least Common Denominator (LCD). For this reason many students don't like fractions and consequently don't like mathematics. In this study, we designed and tested a new method called *the Ladder Method* for finding LCD that can be used for subtracting and adding fractions. To evaluate effectiveness of this method and students' improvement of understanding of fractions, we designed a pre-test and a post-test. We collected and analyzed data for two groups of basic algebra students. The study group used the Ladder Method activities and the control group used the prime factorization method for finding LCD. Finally via hypothesis testing our results show that the new method significantly improves students' performance and understanding of fractions.

1 Introduction

Various studies and test results indicate that most of the students don't like fractions. It follows from my teaching experience that most students don't experience issues with multiplying and dividing fractions. The reason that they have difficulty with adding or subtracting fractions is that they have trouble calculating the least common denominator denoted by the LCD. Since this is a basic skill, our research is very important for students' success in their education. We believe that the Ladder Method that we introduced in this study is easier for students to learn and use when calculating the LCD than the other current methods used in elementary schools.

The least common denominator of two or more positive integer denominators is equal to the smallest natural number that is divisible by each of them. There are two widely used methods for finding the least common denominator:

Method 1: To find the least common denominator, simply list the consecutive multiples of each denominator (i.e. consecutively multiply by 2, 3, 4, etc.) and then look for the smallest number that appears on both lists. This number is the LCD.

Method 2: To find the least common denominator, factor each of the denominators into primes and circle common factors. Then multiply the first number by the factors of the second number that are not in common to obtain the LCD.

Note that Method #1 works well for small integers only. Method #2 is the better way to calculate the LCD for larger numbers, but the complexity increases very quickly and thus calculations become too messy for many students. Additionally, there are many steps in both of the above methods that require good record keeping, and that is one of the reasons why these algorithms seem complicated and hard to use.

The Ladder Method that we introduce in this study keeps the calculations organized and requires finding only one common factor for two numbers at each step. The consecutive steps produce smaller numbers to work with, hence making tasks easier and giving students the feeling of progress towards finding the LCD for fractions.

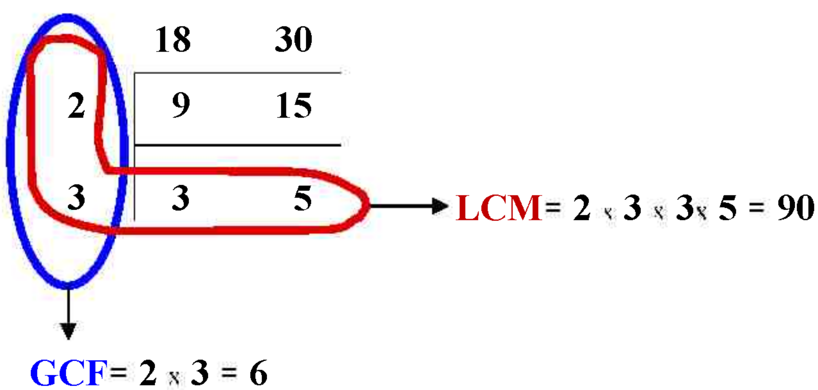
For example let's find the Least Common Multiple (LCM or LCD for fractions) of **18** and **30** using the Ladder Method. We write the numbers on top of a line then we start finding the factor of 18 and 30. The first common factor we can think of is 2, we write the 2 in the left column and divide both 18 and 30 by 2; we get 9 and 15 and we write them respectively under the original numbers 18 and 30. Now we start with 9 and 15. The common factor is 3, we write it in the first column under 2 (the first factor) and we write the quotients respectively under 9 and 15. Now there is no common factor for 3 and 5 except 1. We stop the process. To get the LCM we just need to multiply the circled numbers (L shape circled) which are 2, 3, 3, and 5.

LCM = 2 x 3 x 3 x 5 = 90

Since we cannot find a common factor more than 3 and we already have 2 as a common factor, we multiply them (left column numbers in the Oval) to get the GCF.

GCF = 2 x 3 = 6

Hence with a simple process we calculated LCM and GCF.



You can find more details and examples in the Appendix A.

2 Motivation

As fractions are a basic skill that is required for all mathematics, science, and business courses as well as for other computational fields and everyday life tasks, we have designed and tested new activities for students to learn how to calculate the LCD. Since various studies show that finding the least common multiple of two integers is a problem for many students (even on collage level), we structured this study to improve participants' skills and understanding, especially in the context of fraction operations. Specifically, we have three goals for our research:

1. To show that students will add and subtract fractions better if they know how to find the LCD in an easy way.
2. To show that the performance of students in basic operations with fractions will improve; we will study whether the acquired skills of finding the LCD improves as well.
3. To show that through our learning activities, students' conceptual understanding of the concept of common denominator improves.

Additionally, we hope that our students' fear of fractions will decrease as their skill in finding the LCD increase.

3 Data Collection

We collected data from two groups of algebra students (the Study Group and the Control Group) by administering the pre-test, the post-test, the concept understanding quiz, and an opinion survey asking questions about various parameters that could influence our study. We have started our analysis with the following descriptive statistical parameters:

- Average grade for pre-test in Study Group: μ_x
- Average grade for post-test in Study Group: μ_X
- Average grade for pre-test in Control Group class: μ_y

- Average grade for post-test in Control Group class: μ_Y

We have provided the raw data for each test in the Appendix section. We used MS Excel and MegaStat software for our calculations and data analysis. Both groups of students were enrolled in comparable courses; but to make sure that there were no statistically significant differences in students' fraction-related skills, we compared them before the study. Using unpaired t-test on the pre-test scores we found that there was no statistically significant difference between the two groups, i.e. the students were at the same level regarding their knowledge and fractions-related skills.

3.1 Hypothesis

Our main hypothesis for this study was formulated in the following way:
If: students know how to find the LCD using the Ladder Method, then their performance on problems requiring addition and subtraction of fractions will improve.

There were two groups of students in this study. The first group learned how to find the LCD by using the Ladder Method (Study Group), and the other group used the Prime Factorization Method (Control Group). There were 20 students in each group on the same level of mathematical development.

To start, the following hypothesis was tested to establish that both groups were comparable: we found that the means on their pre-tests were not significantly different when we performed unpaired t –test for the means of two samples as in [Gr]:

- Ho: $\mu_x = \mu_y$
- Ha: $\mu_x \neq \mu_y$ O.C. (original claim), two tails test
- P-value = TTEST(N7:N26,N61:N80,2,2)= 0.715216413 Significant
- Since **0.715216413 > 0.05** we accept Ho and we reject Ha. Hence the scores in the study group for the pre-test are not different from the results in the control group. Hence there was no significant difference between the fraction-related skills of the two groups at the beginning of the study.

Later, to analyze the improvement of students' skills in each group, we tested the following hypothesis using paired *t*-tests between the pre-test and the post-test scores:

Study group:

- **Ho:** $\mu_x \geq \mu_X$
- **Ha:** $\mu_x < \mu_X$

We reported these results in the section 4.5.

Control Group:

- **Ho:** $\mu_y \geq \mu_Y$
- **Ha:** $\mu_y < \mu_Y$

We reported the results of this testing in the Section 5.5.

We also performed unpaired t-tests between the two groups' post-tests to check whether the students using the Ladder Method performed better than the ones using the prime factorization method.

Study group and Control Group:

- **Ho:** $\mu_X \leq \mu_Y$
- **Ha:** $\mu_X > \mu_Y$ **O.C.** (Original Claim)

We reported these results in the Section 6.1.

3.2 Methodology

We started by administering to each participant the pre-test on adding and subtracting fractions that required finding the LCD. Next the Study Group learned the Ladder method with some examples and class activities. Then the Control Group learned how to find the LCD using the prime factorization method (Standard Method) with some examples and class activities. After the learning sessions the post-test and the concept quiz were administered to all students. The grading rubric for the pre-test and the post-test was assessed on a scale from 0 to 6, with partial credit given where the method was correct, but the answer was wrong because the student had made an error in calculation. After the post-test, we asked participants to work on easier but more conceptual problems (Concept Quiz) in order to evaluate their understanding of ideas of numerator, denominator, graphical representations, and operations on fractions. We were expecting high scores on the quiz because all students were enrolled in college algebra courses. The rubric for grading was on the scale 0 to 12. Additionally, we administered a survey immediately after the post-test to analyze possible correlations of various parameters that may have influenced student learning. There were ten questions on the survey given to both groups (Appendix C).

We tested for correlations, between each of the survey questions and the pre-test, the post-test, and the concept quiz scores for both Study Group and Control Group in order to see whether there were some significant relations between them.

We computed the coefficient of correlation by the following Pearson formula known as

Correlation Coefficient (Pearson value), [Gr]:

$$r = \frac{\sum (X - \overline{X})(Y - \overline{Y})}{(n-1)(s_x s_y)}$$

Or the short cut formula:

$$r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}}$$

Summary of Strength and Direction of Coefficient of Correlation

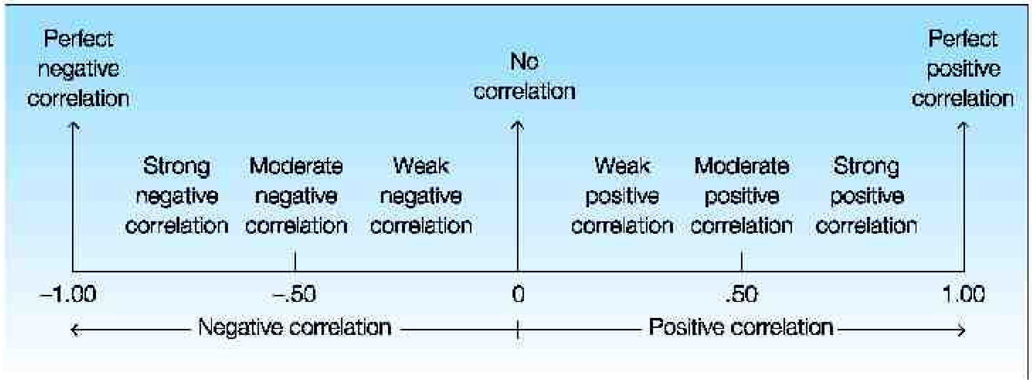


Figure 3.1 Summary of Strength and Direction of Coefficient of Correlation.

We tested effectiveness of the special method (the Ladder method) of finding the LCD in the Study Group and of the standard method (prime factorization) in the Control Group by using statistical methods (Data analysis, Correlation analysis, and Hypothesis testing). We did all of our calculations of correlations, the central tendency values (Mean, Median, Mode, Variance, Standard Deviation) for each data set by using the Data Analysis feature in MS Excel.

We describe the Ladder method of finding the LCD in Appendix A. The Prime Factorization method used by the Control Group is described in Appendix B. The survey questions can be found in Appendix C. The pre-test and post-test questions are included in the Appendix D. The Fraction Concepts Quiz questions are in Appendix E. The analysis of data collected is described in the following chapters.

4 Data Analysis for Study Group

4.1 Description of data

We start this section by presenting the collected data from the survey of the Study Group in the table below. Each question was graded using a predetermined suitable rubric with scores described in the Range row.

	Study Group		n=20 students							
	Survey	Questions								
	Q1 - Visual Learner	Q2 - Gender	Q3 - Hrs Work	Q4 - Units Passed	Q5 - Play Game	Q6 - Study math	Q7 - math level	Q8 - Like math	Q9 - Op. Fracs.	Q10 - Math Methods
RANGE	0-1	F0-M1	0-30	0-50	0-30	0-20	0-4	1-10	1-10	0-3
	1	0	0	0	0	2	2	6	10	2
	1	0	0	0	0	4	2	9	5	2
	1	0	0	0	4	0.5	2	4	5	2
	1	0	0	0	0	3	1	2	6	2
	1	0	0	0	8	4	2	3	8	0
	1	1	5	0	0	2	2	8	5	2
	1	0	0	0	0	0.5	2	4	8	2
	1	0	5	0	0	2	3	5	7	1
	1	1	0	0	5	7	2	6	2	2
	1	1	0	0	5	7	3	8	10	2
	1	0	0	0	0	3	0	3	5	0
	1	0	0	0	0	1	2	8	7	2
	1	1	9	0	0	5	2	7	3	0
	1	1	8	0	0	2	3	7	7	0
	1	0	0	0	0	3	3	8	8	1
	1	0	0	0	0	3	4	1	3	0
	1	0	0	0	5	3	1	1	6	2
	1	0	0	6	0	2	1	7	6	2
	1	1	0	0	0	4	2	4	7	2
	1	1	0	0	3	5	3	7	7	2
Mean	1	0.35	1.35	0.3	1.5	3.15	2.1	5.4	6.25	1.4
Mode	1	0	0	0	0	2	2	8	7	2
Median	1	0	0	0	0	3	2	6	6.5	2
Stdev	0.00	0.49	2.89	1.34	2.50	1.84	0.91	2.50	2.12	0.88

We noted that most of the students' preparation was at the college algebra level and most of the students didn't like to play video games. They typically studied about 3 hours for their math class each week. Interestingly, only four of them worked, and all four worked fewer than 10 hours. Most of them reported at least somewhat liking mathematics; and except for two, they felt that their fraction-related skills were quite strong. In Section 4.2 we calculate correlations between the survey questions and student scores.

The table below displays pre-test and post-test data, as well as data collected on the concept quiz. The grading range for each rubric is displayed in the first numerical row. Students' codes are in the first column.

	PRE-TEST	POST-TEST	Concept Test
	x	X	Cx
	0-6	0-6	0-12
Sy.	5.5	6.0	9.5
Mon.	3.5	6.0	11.0
Ar.	3.5	6.0	10.5
Sh.	5.5	5.5	11.5
Ch.	3.5	5.0	11.0
Fr.	2.5	4.5	10.5
Je.	5.0	5.5	10.5
Fe.	3.0	5.0	11.5
Al.	2.0	3.5	7.5
Ja.	2.5	4.5	7.5
Ka.	4.5	5.0	9.0
Is.	6.0	5.0	11.0
Alx.	4.0	6.0	10.0
ca.	3.5	5.0	8.0
Na.	4.0	5.0	9.0
Ra.	3.0	5.0	8.5
Cas.	4.5	5.5	9.5
Lu.	3.0	4.5	9.0
Ant.	2.5	3.5	9.5
Rog.	5.0	5.0	10.0
Mean	3.83	5.05	9.73
Mode	3.5	5	9.5
Median	3.5	5	9.75
Stdev	1.15	0.72	1.24

Note that the mean of the students' test scores increased from 3.83 to 5.05. Hence, there was an improvement in their performance. We tested the statistical significance of this improvement in Section 4.5.

The two tables below show the descriptive parameters for pre-test and post-test scores for the Study Group.

Pre-Test	
Column1	
Mean	**3.825**
Standard Error	**0.257198981**
Median	**3.5**
Mode	**3.5**
Standard Deviation	**1.15022881**
Sample Variance	**1.323026316**
Kurtosis	**-0.892747769**
Skewness	**0.333533621**
Range	**4**
Minimum	**2**
Maximum	**6**
Sum	**76.5**
Count	**20**

Post-Test	
Column1	
Mean	**5.05**
Standard Error	**0.161815359**
Median	**5**
Mode	**5**
Standard Deviation	**0.723660287**
Sample Variance	**0.523684211**
Kurtosis	**0.464545607**
Skewness	**-0.655054395**
Range	**2.5**
Minimum	**3.5**
Maximum	**6**
Sum	**101**
Count	**20**

Note that the mean increased and the standard deviation decreased, which means that the weaker students improved their scores. Note also that for many good students, there was not much space to improve.

Next we searched for correlation between the test scores and answers on survey questions to evaluate certain variables outside of the study that might influence the collected data.

4.2 Results of Correlation Analysis for Study Group

The table below shows the results of our calculations.

Study group		
Pearson Values between survey questions and pre-test		
PEARSON(Q1, x)	undefined	No correlation was calculated.
PEARSON(Q2, x)	-0.446484756	Weak negative correlation between survey Q2 and pre-test.
PEARSON(Q3, x)	-0.186516282	Almost no correlation between survey Q3 and pre-test.
PEARSON(Q4, x)	-0.168822802	Weak negative correlation between survey Q4 and pre-test.
PEARSON(Q5, x)	-0.233117922	Weak negative correlation between survey Q5 and pre-test.
PEARSON(Q6, x)	-0.41681709	Moderate negative correlation between survey Q6 and pre-test.
PEARSON(Q7, x)	-0.283503786	Weak negative correlation between Q7 and pre-test.
PEARSON(Q8, x)	-0.12077434	Weak negative correlation between Q8 and pre-test.
PEARSON(Q9, x)	0.320389938	Weak positive correlation between Q9 and pre-test.
PEARSON(Q10, x)	0.098505777	Weak positive correlation between Q10 and pre-test.
Study group		
Pearson Values between survey questions and post-test		
PEARSON(Q1, X)	undefined	No correlation was calculated.
PEARSON(Q2, X)	-0.49788331	Moderate negative correlation between survey Q2 and post-test.
PEARSON(Q3, X)	0.129661955	Weak positive correlation between survey Q3 and post-test.
PEARSON(Q4, X)	-0.178891352	Weak negative correlation between survey Q4 and post-test.
PEARSON(Q5, X)	-0.14530648	Weak negative correlation between survey Q5 and post-test.
PEARSON(Q6, X)	-0.411966813	Weak negative correlation between survey Q6 and post-test.
PEARSON(Q7, X)	-0.087730903	Almost no correlation between Q7 and post-test.
PEARSON(Q8, X)	-0.069805796	Almost no correlation between Q8 and post-test.
PEARSON(Q9, X)	0.09414648	Almost no correlation between Q9 and post-test.
PEARSON(Q10, X)	-0.115368073	Weak negative correlation between Q10 and post-test.
Pearson Value between pre-test and post-test		
PEARSON(x, X)	0.595948563	Moderate positive correlation between pre-test and post-test.
Pearson Value between pre-test, post-test, and concept test		
PEARSON(Cx , x)	0.434801012	Moderate positive correlation between concept test and pre-test.
PEARSON(Cx, X)	0.470504069	Moderate positive correlation between concept test and post-test.

Since there are only weak correlations between the survey questions and either the pre-test or the post-test results, we can conclude that learning in our study was not influenced by the variables evaluated by the survey. The results are consistent for both groups (see the results for the control group in Section 5.2). Hence variables like being a visual learner, gender, number of working hours per week, number of units passed, playing video games, number of hours studying math per week, the math level, interest in math, and self-evaluations of fraction skills did not influence students' performance in our study.

4.3 Analysis of raw data for the survey for the Study Group

The chart below displays raw data of students' answers to each survey question. Note that questions were graded on different scales (see figure 4.1).

Results and Findings in Survey (Study Group**)**

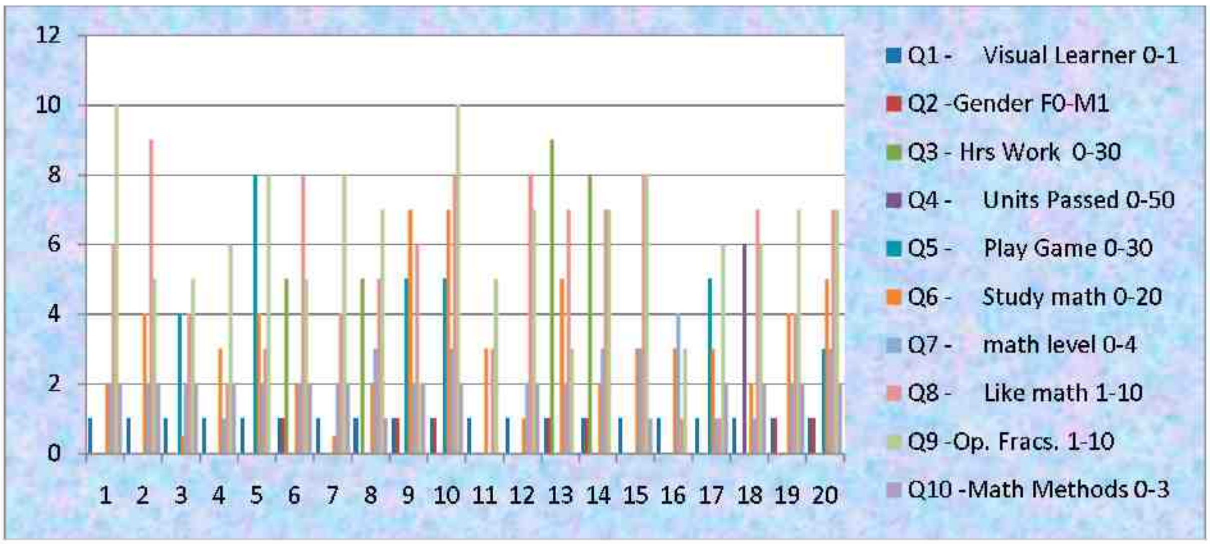


Figure 4.1 Study Group survey results.

In this group most of the students were on college algebra level, and most of the students didn't like to play video games. They studied about 3 hours for their math class each week. Note that majority of the students reported liking mathematics and fractions.

Question 1- Are you a visual learner (Yes=1 and No= 0)?

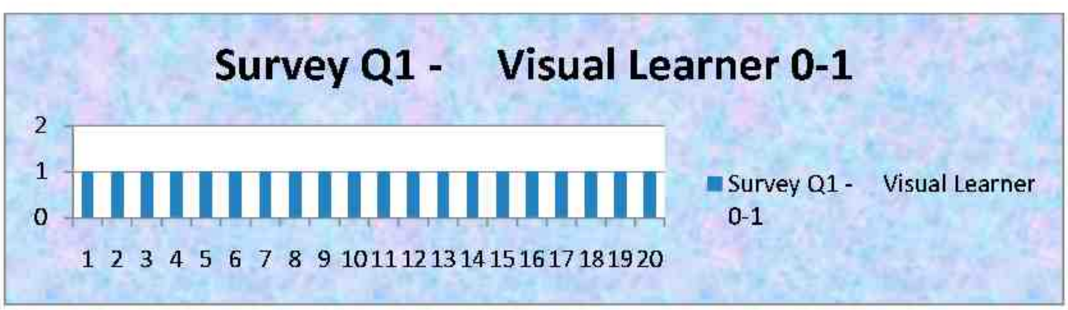


Figure 4.2 All of the students declared themselves as visual learners.

As we see in Figure 4.2, all students were aware that they are visual learners. Since the Ladder method uses visual display of factors in an organized way, it may appeal to this group of learners.

Question 2- What is your gender (Male=1 and **Female= 0)?**

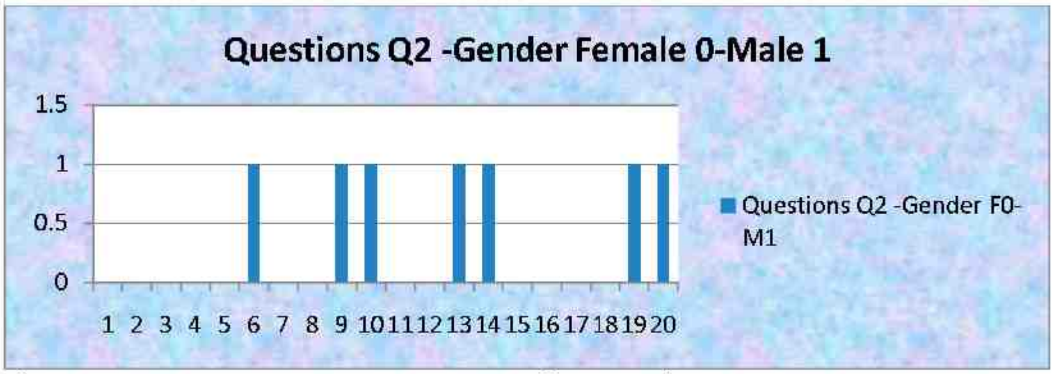


Figure 4.3 There were 7 males and 13 females' students in the study group.

Question 3- How many hours per week do you work?

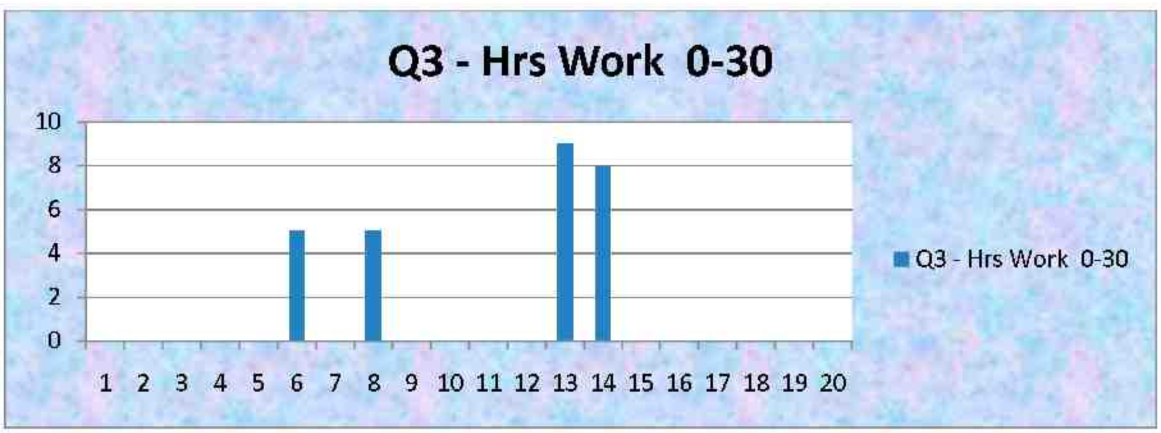


Figure 4.4 Four out of 20 students used to work.

We wanted to see if long working hours influence their performance. The above group shows that majority of students did not work. This is not surprising, as all of them were college freshmen. Two students worked more than 8 hours and two worked about 5 hours. Hence the working hours variable could not influence this group's performance.

Question 4- How many university units have you passed so far?

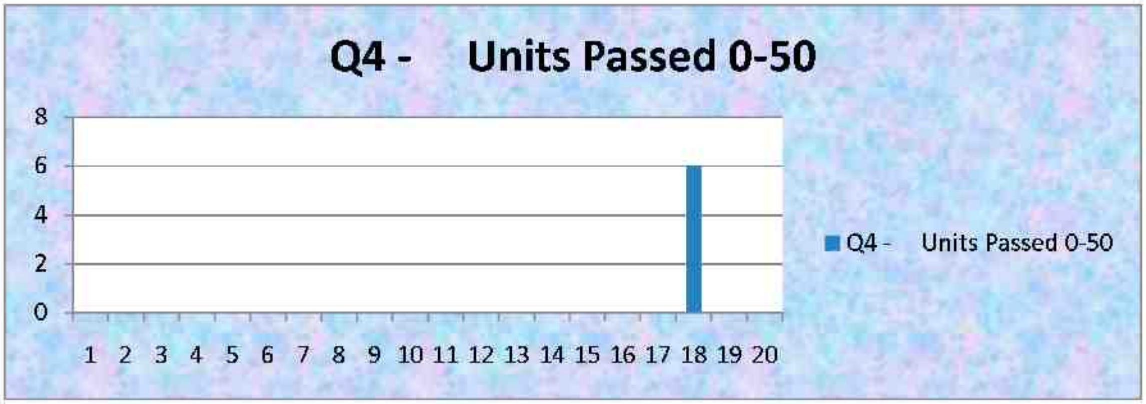


Figure 4.5 Only one student had passed 6 units.

Figure 4.5 shows that the students in the Study Group were entering freshmen without college credits (except for one student who had taken some community college courses counting for 6 university credits). Therefore they were on the same level of their mathematical development.

Question 5- How many hours per week do you play computer games?

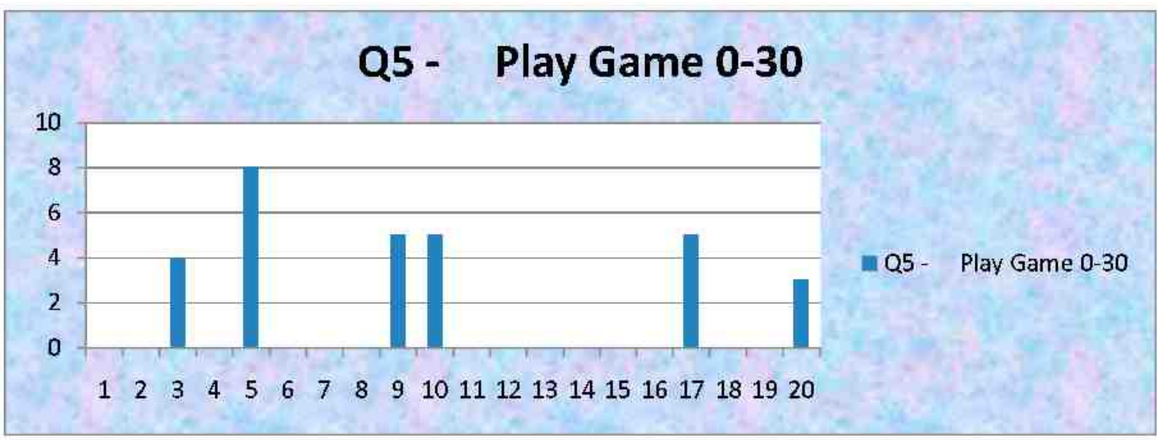


Figure 4.6 Most of the students don't spend time playing computer games.

The above graph shows that in this group the majority of students were not interested in playing computer games. Hence this variable did not influence their performance.

Question 6- How many hours per week do you study for your math class?

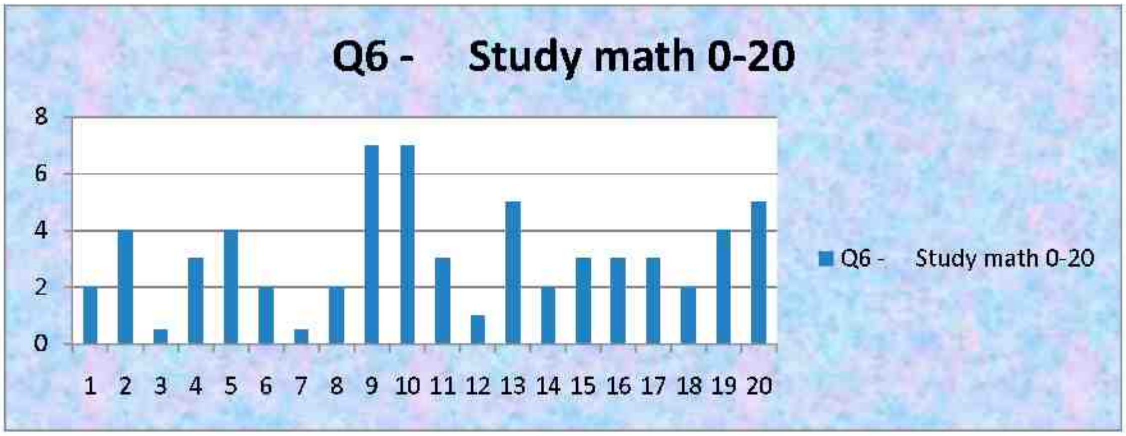


Figure 4.7 Time in hours spent on mathematics homework.

The graph in Figure 4.7 displays self-reported hours spent each week on mathematics homework. Note that most of the students studied between 2 to 4 hours per week. That is the expected time for students to spend working on math assignments at this level. Four students reported more time and three less time spent studying at home. While we would expect some correlation of this variable with pre-test and post-test results, table 4.2 shows no correlation.

Question 7- What is your highest math level class (0-4)?

Pre-algebra=0, Intermediate Algebra=1, College Algebra=2, Statistics=3, and Calculus=4

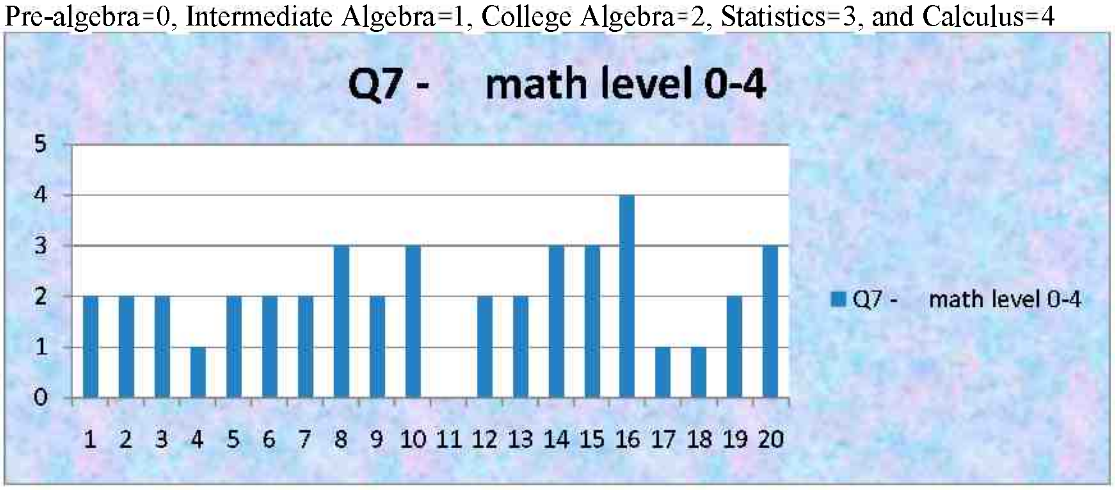


Figure 4.8 Most of the students' math level was college algebra.

Figure 4.8 confirms our expectation that students were more or less on the same level. One student had taken Calculus in high school while five were in statistics courses.

However these students were placed into Algebra courses by the university due to their weak performance on the standard placement tests.

Question 8- On the scale of 1-10 (10 being the highest**), how much do you like math?**

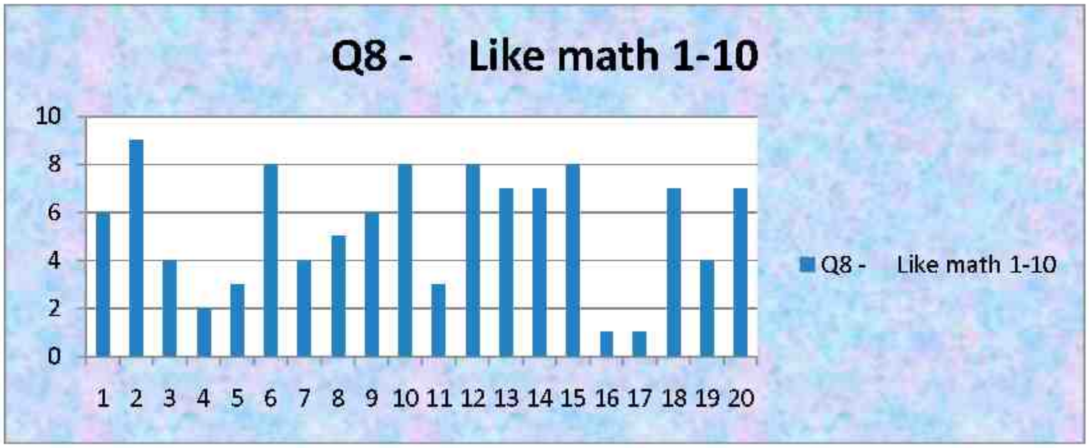


Figure 4.9 About 50% of the students liked math.

The results in Figure 4.9 are quite interesting because according to their scores on the college placement test, students in this study were not ready for college level math courses at the age of 18. But the majority of them reported liking mathematics. Only two students disliked mathematics strongly.

Question 9- On the scale of 1-10 (10 being the highest**), how good are you in adding and subtracting fractions?**

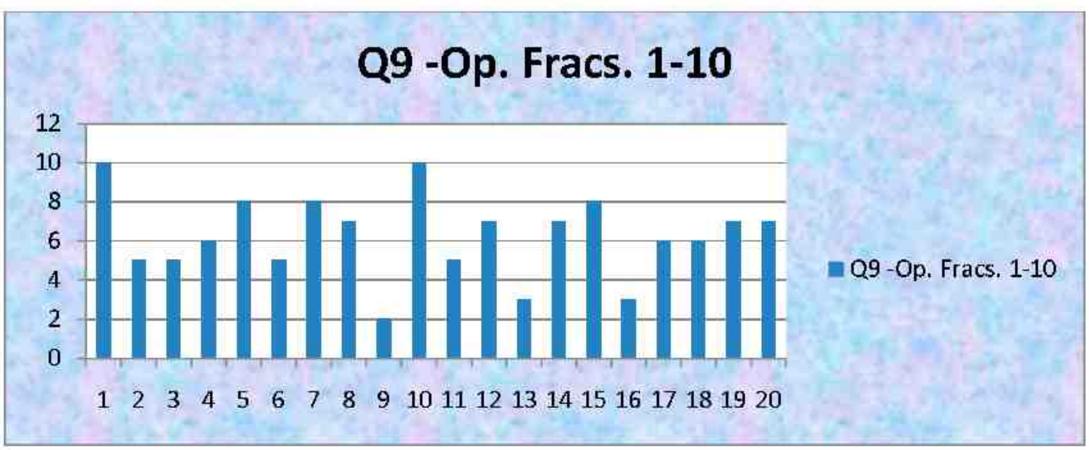


Figure 4.10 Most of the students knew operations on fractions.

The bar graph in Figure 4.10 shows that most of the students reported that they are familiar with operations on fractions. Only three students felt that they had somewhat weak skills in this area.

Question 10- What methods do you use when adding or subtracting fractions? List them and describe.

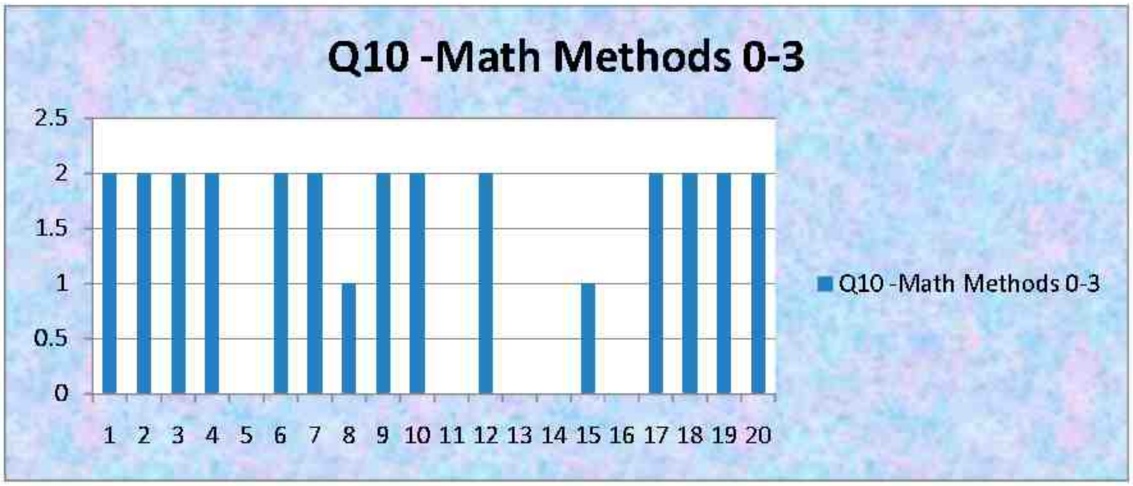


Figure 4.11 Self-reported knowledge of adding and subtracting fractions.

The majority of the students reported knowing various methods of adding and subtracting fractions. The correctness of their descriptions was graded on scale 0-3.

Note that Table 4.2 shows that neither one of the variables described in the above questions included in the survey was correlated with students' performance on the administered the pre-test and the post-test.

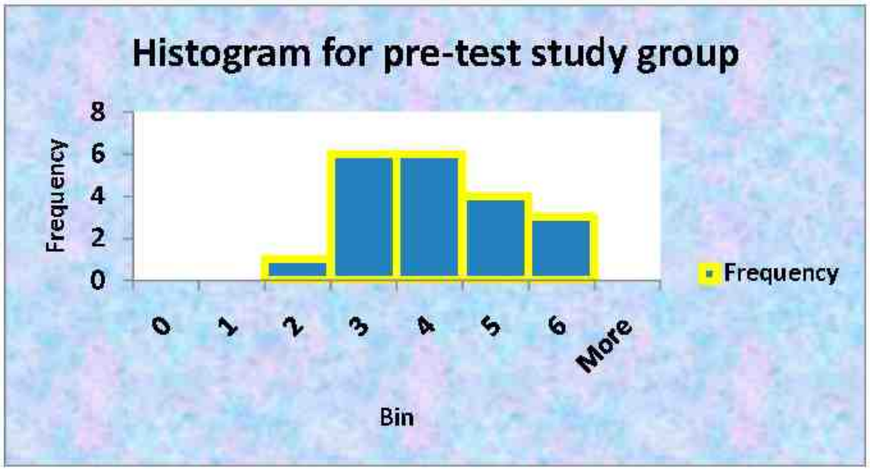


Figure 4.12 Most of the grades for the pre-test are between 3 and 4.

Now we will look at tests results for the Study Group. The histograms in Figures 4.12 and 4.13 show the students' performances on each test. We can conclude that students had some knowledge of finding LCD. However, initially the majority of them were not able to perform on harder problems.

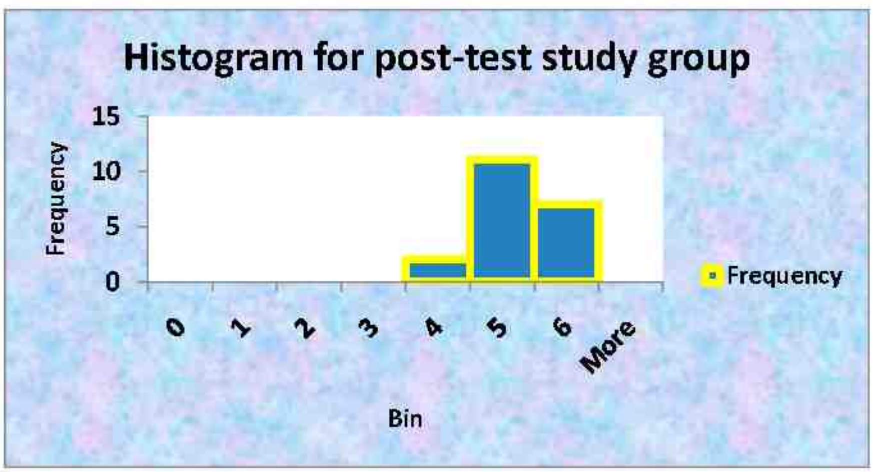


Figure 4.13 Most of the grades for the post-test are between 4 and 5.

We see that the majority of the students were able to do well on the post-test. The biggest improvement was made by students who did not perform well on the pre-test. Most of the students' grades on the pre-test were between 3 to 4 out of 6, but for the post-test the grades were between 4 to 5 out of 6. Hence we see improvement between the pre-test and the post-test in the Study Group. Therefore it looks as if the special method is working.

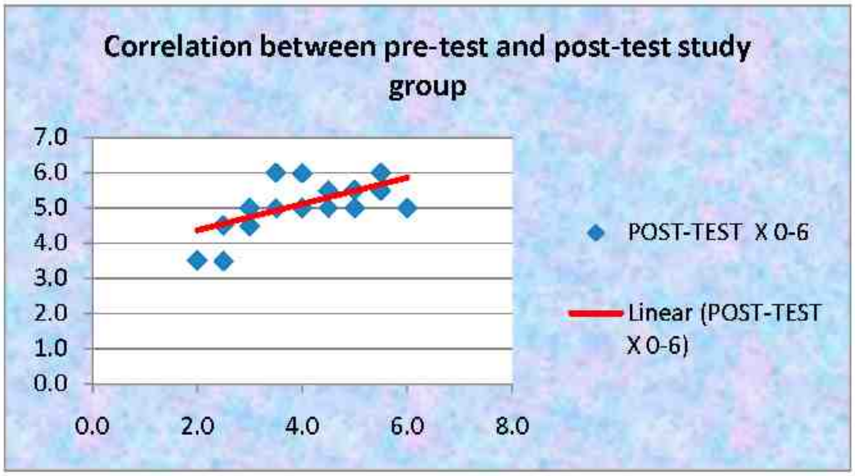


Figure 4.14 Correlation between pre-test and post-test.

There is some positive correlation between pre-test and post-test scores. Since there were several students that achieved perfect scores on the pre-test, the result is not as strong as it could be. But the majority of students improved their scores.

4.4 Concept Quiz

To evaluate whether students in the Study Group had a conceptual understanding of fraction operations, we administered a quiz with conceptual questions. We asked them to graphically represent given fractions and to identify numerators and the denominators. They were also asked to perform operations on simple fractions. In various contexts most of the students did well on the concept quiz, (see Figure 4.15). This showed that they were familiar with fraction-related concepts and simple operations; but when they used larger numbers as denominators on the pre-test, many still had difficulties with addition and subtraction (compare results in Figure 4.15 with pre-test scores from Section 4.1). Since most of the students achieved maximum scores in the concept quiz, we concluded that their difficulties were coming from technical issues related to finding the LCD.

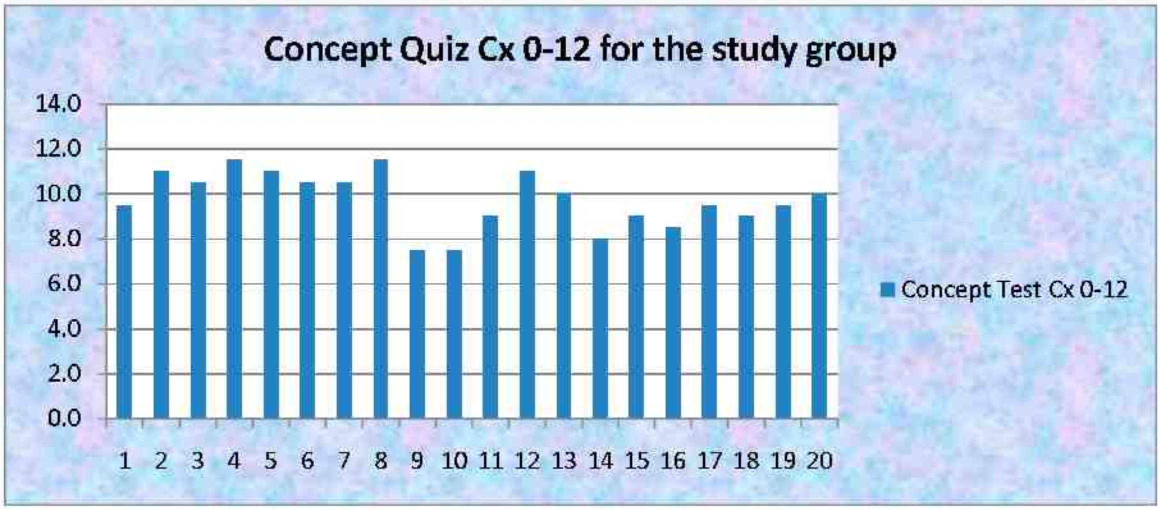


Figure 4.15 Results on Concept Quiz.

4.5 Evaluation of the performance for Study Group

To evaluate effectiveness of teaching the Ladder method for finding the LCD in the study group, we analyzed the mean of students' performance by Paired t test between their pre-test and post-test results:

- Test statistics $t = \dfrac{\bar{d}}{s_d / \sqrt{n}}$
- Ho: $\mu_x \geq \mu_X$
- Ha: $\mu_x < \mu_X$ Original Claim (left tail test)
- Paired t test (x , X)
- **P-value using Excel:** =TTEST(N7:N26,O7:O26,1,1)

  P-value = **0.00000525** < **0.05** (Alpha 0.05 is significance level)

Hence at a confidence level of 95% we reject the Null hypothesis (Ho) and we accept the alternative hypothesis (Ha). Therefore we conclude that the mean for post-test scores is significantly greater than the mean for pre-test scores. This means that the Ladder method was helpful to the participants in the Study Group in that their performance on fraction operations improved significantly immediately after they learned the Ladder method.

Comparing pre-test and post-test

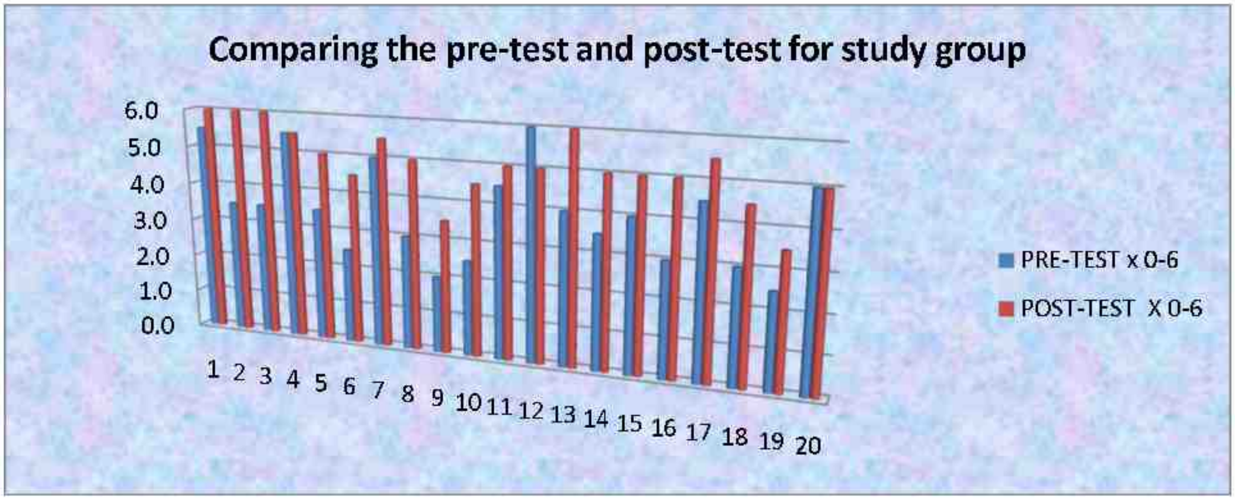


Figure 4.16 Performance on pre-test and post-test by students in the study group.

The bargraph in Figure 4.16 shows pre-test and post-test scores for individual participants. Note that all students performed at least as well on the post-test as on the pre-test. The majority of the low scoring students improved after the Ladder method activities, and some of them even obtained a perfect score. Hence we can conclude that the Ladder method of finding the LCD helped many of them in performing fraction operations.

5 Data Analysis for Control Group

5.1 Description of data

Now we present the collected data from the survey of the Control Group in the table below. Each question was graded using a predetermined suitable rubric with the scores described in the Range row.

	Control Group		n=20 students							
	Survey	Questions								
	Q1 - Visual Learner	Q2 - Gender	Q3 - Hrs Work	Q4 - Units Passed	Q5 - Play Game	Q6 - Study math	Q7 - math level	Q8 - Like math	Q9 - Op. Fracs.	Q10 - Math Methods
RANGE	0-1	F0-M1	0-30	0-50	0-30	0-20	0-4	1-10	1-10	0-3
	1	0	5	0	0	4	3	6	5	2
	1	0	0	17	0	4	3	0	6	2
	1	1	2	0	0	6	2	7	9	1
	1	1	0	14	0	3	1	2	8	2
	1	0	0	0	0	2	1	8	7	2
	1	0	0	0	0	2	3	5	3	2
	1	1	0	20	0	6	1	5	10	0
	1	1	12	0	0	6	1	7	6	0
	1	0	0	0	7	3	1	3	7	1
	1	0	10	0	0	5	2	7	10	2
	1	0	0	0	0	2	2	4	3	0
	1	0	0	5	0	2	3	6	1	2
	1	0	22	0	0	4	1	3	9	2
	1	0	17	0	0	5	2	7	5	2
	1	0	0	0	0	3	3	4	4	2
	1	0	0	0	0	0	1	7	10	1
	1	0	0	0	0	3	4	7	9	2
	1	0	0	0	0	3	3	3	8	2
	1	0	0	12	0	7	3	4	10	1
	1	0	24	0	0	1	2	3	10	2
Mean	1	0.2	4.6	3.4	0.35	3.55	2.1	4.9	7	1.5
Mode	1	0	0	0	0	3	3	7	10	2
Median	1	0	0	0	0	3	2	5	7.5	2
Stdev	0.00	0.41	7.94	6.58	1.57	1.85	0.97	2.15	2.77	0.76

The mean of the math level of the students in the Control Group was 2 (College Algebra) and they too didn't like to play computer games. They used to study about 3.5

hours each week for their math class. Note that some of them worked over 20 hours per week, while no student in the Study Group worked that many hours. As a group, they did like mathematics as much as the participants from the other group, and their self-reported knowledge of fractions were on similar level. We have calculated correlations of the survey data with test scores in Section 5.2.

The table below displays pre-test and post-test data, as well as data collected on the concept quiz for the Control Group. The grading range for each rubric is displayed in the first numerical row. Students' codes are in the first column.

		Math95	
	PRE-TEST	POST-TEST	Concept Test
	y	Y	CY
	0-6	0-6	0-12
Gi.	4.0	4.0	7.5
Ale.	3.0	4.0	8.0
Joh.	3.0	2.5	10.5
Ke.	4.0	3.0	9.5
Jen.	2.5	3.0	7.0
Ali.	1.0	3.0	10.5
Bri.	3.5	1.0	10.0
Est.	3.5	5.0	8.0
Jer.	3.0	4.5	7.5
Sad.	6.0	6.0	11.5
Sha.	3.0	5.0	6.5
Eli.	3.0	3.5	8.5
Ver.	3.0	4.0	10.5
Ros.	6.0	4.0	9.5
Pap.	3.5	3.5	10.5
Ken.	4.0	5.0	11.5
Dai.	6.0	5.5	10.0
Mar.	4.0	3.5	9.5
Dan.	6.0	5.0	10.0
Brea.	1.5	2.5	9.0
Mean	3.68	3.88	9.28
Mode	3	4	10.5
Median	3.5	4	9.5
Stdev	1.42	1.20	1.46

Note that the mean of the students' scores in this group was increased from 3.68 to 3.88. Hence there was an improvement of their performance. We will test the significance of this improvement in Section 5.5.

The following tables show the parameters for the pre-test and the post-test for the control group.

Pre-Test			Post-Test	
Column1			*Column1*	
Mean	**3.68**		**Mean**	**3.88**
Standard Error	**0.32**		**Standard Error**	**0.27**
Median	**3.50**		**Median**	**4.00**
Mode	**3.00**		**Mode**	**4.00**
Standard Deviation	**1.42**		**Standard Deviation**	**1.20**
Sample Variance	**2.01**		**Sample Variance**	**1.44**
Kurtosis	**-0.07**		**Kurtosis**	**0.31**
Skewness	**0.36**		**Skewness**	**-0.38**
Range	**5.00**		**Range**	**5.00**
Minimum	**1.00**		**Minimum**	**1.00**
Maximum	**6.00**		**Maximum**	**6.00**
Sum	**73.50**		**Sum**	**77.50**
Count	**20**		**Count**	**20**

Note that the mean on the post-test is larger than on the pre-test and that the standard deviation decreased. This means that the weaker students improved their scores (as for many good students there was not much space to improve). We have observed a similar situation previously for the Study Group. We will test the statistical significance of this improvement later.

Next we searched the Control Group data for correlations between the test scores and answers on survey questions to evaluate certain variables outside of the study which might influence the collected data. The following table shows the results of our calculations.

Note that because in the case of the Study Group, we observed only very weak or, in most of the cases, zero correlations between survey questions and the pre-test or post-test results, we can conclude that learning in our study was not influenced by the variables evaluated by the survey questions. The results are consistent for both groups.

Therefore variables like visual learner, gender, number of work per week, number of units passed, playing computer games, number of hours studying math per week, math level, interest in math, and self-evaluations of fraction skills did not influence the students' performance in our study.

5.2 Results of Correlation Analysis for Control Group

The table below shows the results of our calculations.

Control group		
Pearson Values between survey questions and pre-test		
PEARSON(Q1, y)	undefined	**No correlation was calculated.**
PEARSON(Q2, y)	-0.063364635	**Almost no correlation between survey Q2 and pre-test.**
PEARSON(Q3, y)	-0.028528593	**Very weak negative correlation between survey Q3 and pre-test.**
PEARSON(Q4, y)	0.079596846	**Almost no correlation between survey Q4 and pre-test.**
PEARSON(Q5, y)	-0.112141371	**very weak negative correlation between survey Q5 and pre-test.**
PEARSON(Q6, y)	0.463634795	**Moderate positive correlation between survey Q6 and pre-test.**
PEARSON(Q7, y)	0.216851453	**Weak positive correlation between Q7 and pre-test.**
PEARSON(Q8, y)	0.2825494	**Weak positive correlation between Q8 and pre-test.**
PEARSON(Q9, y)	0.321631594	**Weak positive correlation between Q9 and pre-test.**
PEARSON(Q10, y)	0.03661751	**Almost no correlation between Q10 and pre-test.**
Control Group		
Pearson Values between survey questions and post-test		
PEARSON(Q1, Y)	undefined	**No correlation was calculated.**
PEARSON(Q2, Y)	-0.426886807	**Weak negative correlation between survey Q2 and post-test.**
PEARSON(Q3, Y)	0.024812292	**Almost no correlation between survey Q3 and post-test.**
PEARSON(Q4, Y)	-0.372707916	**Moderate negative correlation between survey Q4 and post-test.**
PEARSON(Q5, Y)	0.122418188	**Very Weak positive correlation between survey Q5 and post-test.**
PEARSON(Q6, Y)	0.0088832	**Almost no correlation between survey Q6 and post-test.**
PEARSON(Q7, Y)	0.192312122	**Weak positive correlation between Q7 and post-test.**
PEARSON(Q8, Y)	0.178274212	**Weak positive correlation between Q8 and post-test.**
PEARSON(Q9, Y)	0.007899895	**Almost no correlation between Q9 and post-test.**
PEARSON(Q10, Y)	-0.014390351	**Almost no correlation between Q10 and post-test.**
Pearson Value between pre-test and post-test		
PEARSON(y, Y)	0.562243738	**Moderate positive correlation between pre-test and post-test.**
Pearson Value between pre-test, post-test, and concept test		
PEARSON(Cy , y)	0.30539576	**Weak positive correlation between concept test and pre-test.**
PEARSON(Cy, Y)	0.013086051	**Almost no correlation between concept test and post-test.**

5.3 Analysis of raw data for the survey for the Control Group

The following chart displays raw data of students' answers to each survey question. Note that questions were graded on different scales (see Figure 5.1).

Results and Findings in Survey (Control Group)

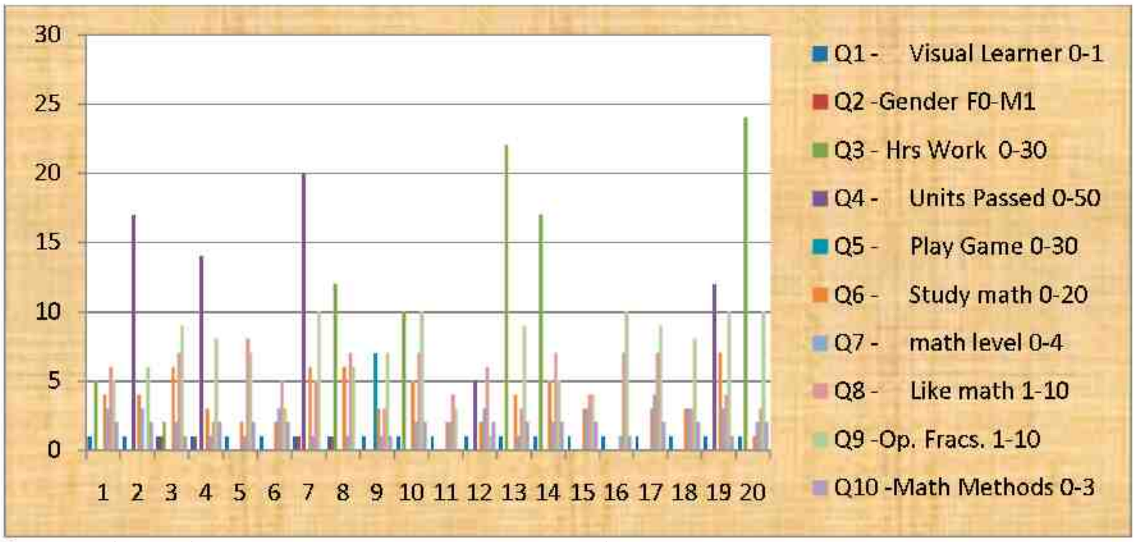


Figure 5.1 Results for Survey Questions for Control Group

Most of the students were at math level 2 (College Algebra), and most of the students didn't like to play computer games. They typically studied about 5 hours for their math class each week. They claimed that their knowledge of operations on fractions was above the average, but most of them didn't know which method to use while adding or subtracting fractions. Note that this group did not like doing mathematics as much as the Study Group and the students were employed more hours.

Question 1- Are you a visual learner (Yes=1 and No= 0)?

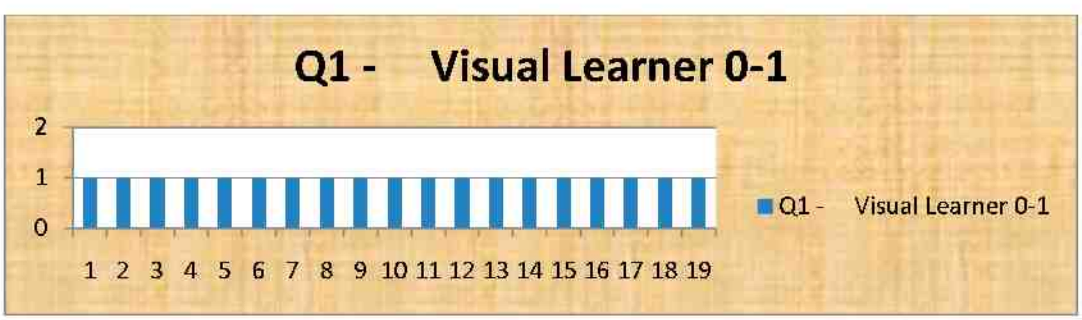


Figure 5.2 All of the students declared themselves as visual learners.

As we see in Figure 5.2, all students were aware that they are visual learners. They used prime factorization of denominators to find the LCD. Since the Ladder method uses visual display of factors in an organized way, it may appeal to this group of learners and they may perform better after learning it.

Question 2- What is your gender (Male=1 and Female= 0**)?**

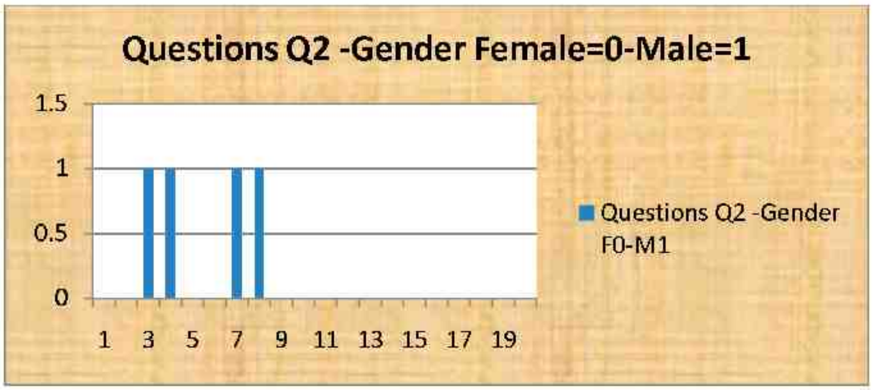


Figure 5.3 There were 4 males and 16 females' students in the control group.

Question 3- How many hours per week do you work?

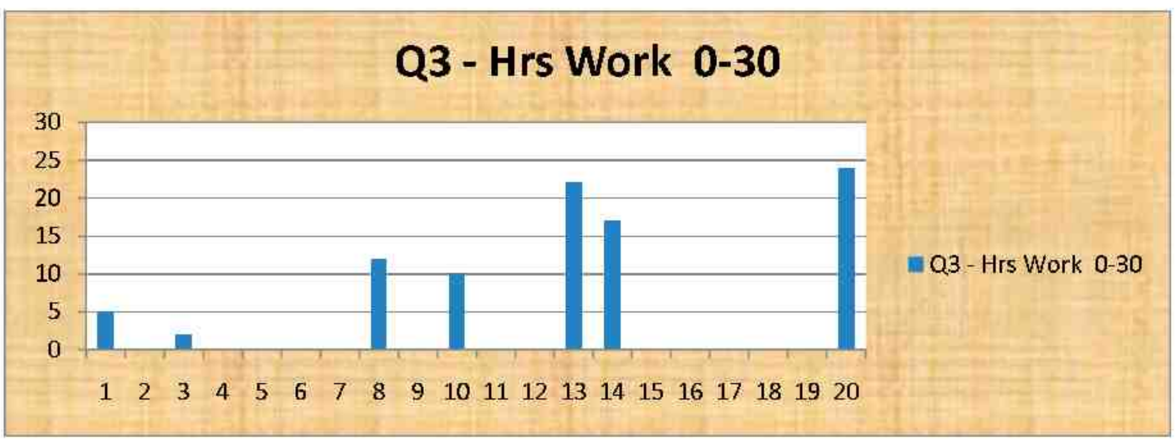


Figure 5.4 Seven out of 20 students were employed.

The above group shows that majority of students did not work. This is not surprising because all of them were freshman. Two students worked fewer than 5 hours. Only five students worked more than 5 hours; and of these three students worked 15 to 25 hours. Hence a working hours variable should not influence this group's performance.

Question 4- How many university units have you passed so far?

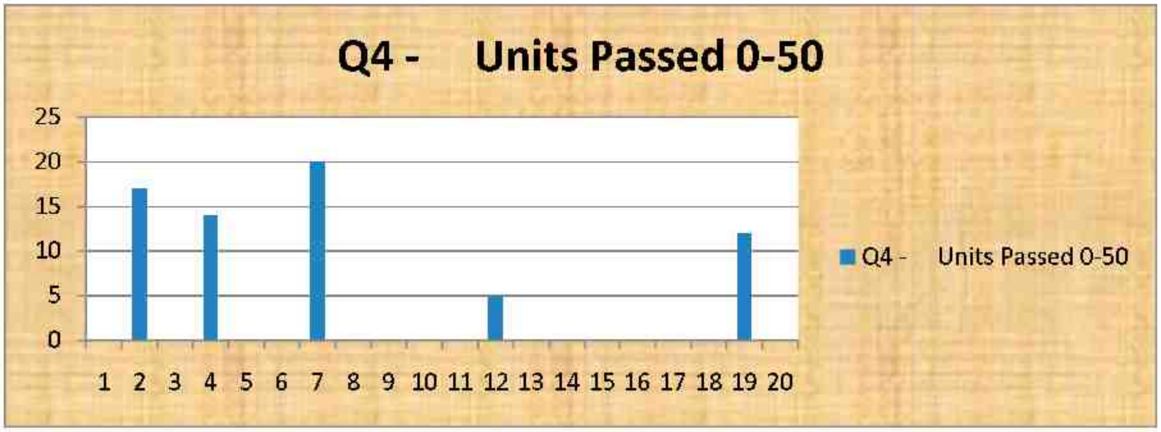


Figure 5.5 Four students had passed 10 to 20 units, and one student had passed 5 units.

While all students were freshmen, five of them had college experience already. Four of them had collected units equivalent to one semester of university course work.

Question 5- How many hours per week do you play computer games?

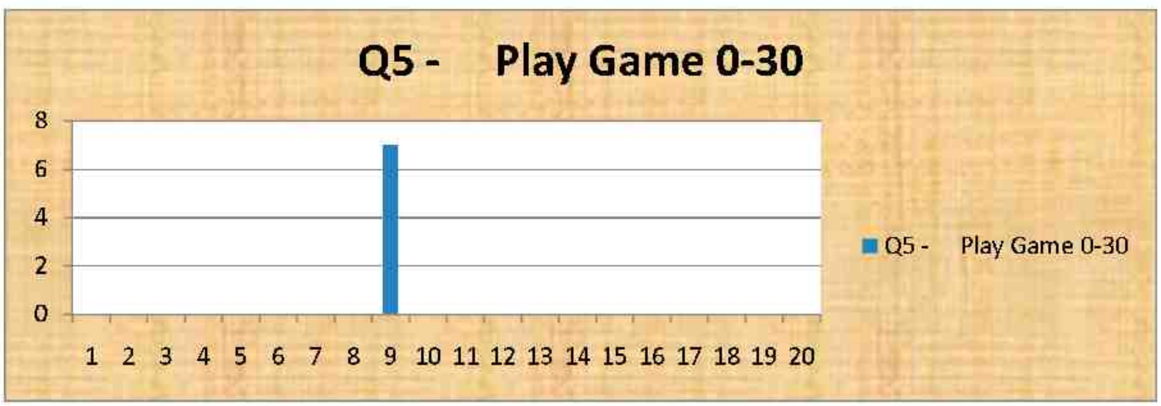


Figure 5.6 Most of the students didn't spend time playing computer games.

Except for one student, this group was not interested in computer games. This makes it similar to the Study Group.

The graph in Figure 5.7 displays self-reported time spent on mathematics homework.

Question 6- How many hours per week do you study for your math class?

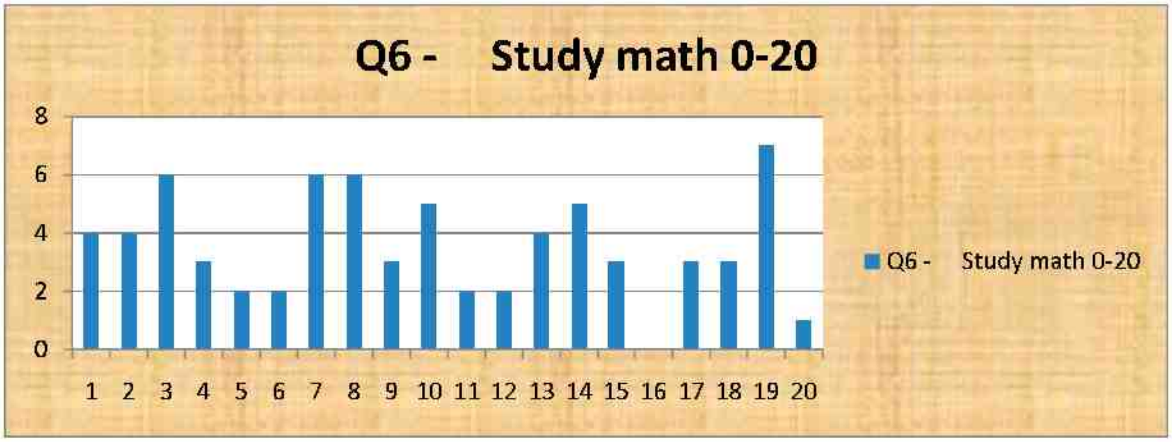


Figure 5.7 Most of the students typically studied about 4 hours per week for math.

The majority of the students in the Control Group reported spending at least two hours per week studying for their algebra class. This was an expected time to spend on homework. These results are similar to the ones noted for the Study Group.

Question 7- What is your highest math level class (0-4)?
Pre-algebra=0, Intermediate Algebra=1, College Algebra=2, Statistics=3, and Calculus=4

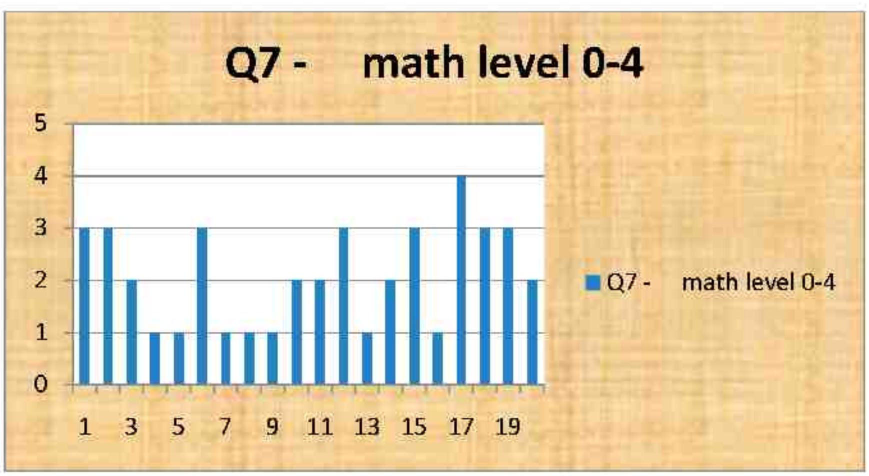


Figure 5.8 Most of the students' math level was College Algebra.

Figure 5.8 confirms our expectation that students were on the same level. One student had taken Calculus in high school while seven were in statistics courses.

However, these students were placed into Algebra courses by the university due to their weak math skills. This data is similar to the Study Group data.

Question 8- On the scale of 1-10 (10 being the highest**), how much do you like math?**

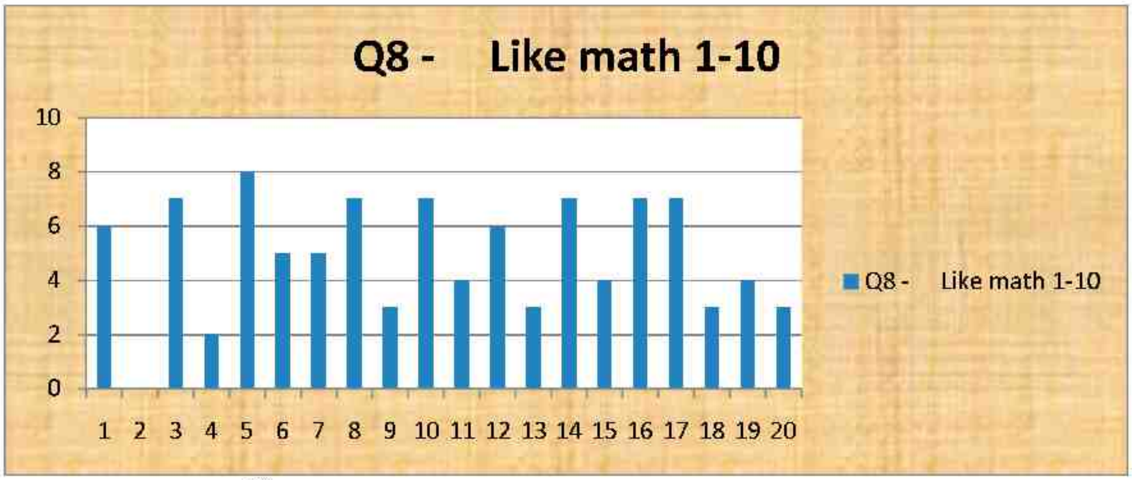


Figure 5.9 About 50% of the students liked math.

Note that the average for this group was slightly lower than the Study Group's average. However, more than half of them liked mathematics enough to assign a score of five or more to this question.

Question 9- On the scale of 1-10 (10 being the highest**), how good are you in adding and subtracting fractions?**

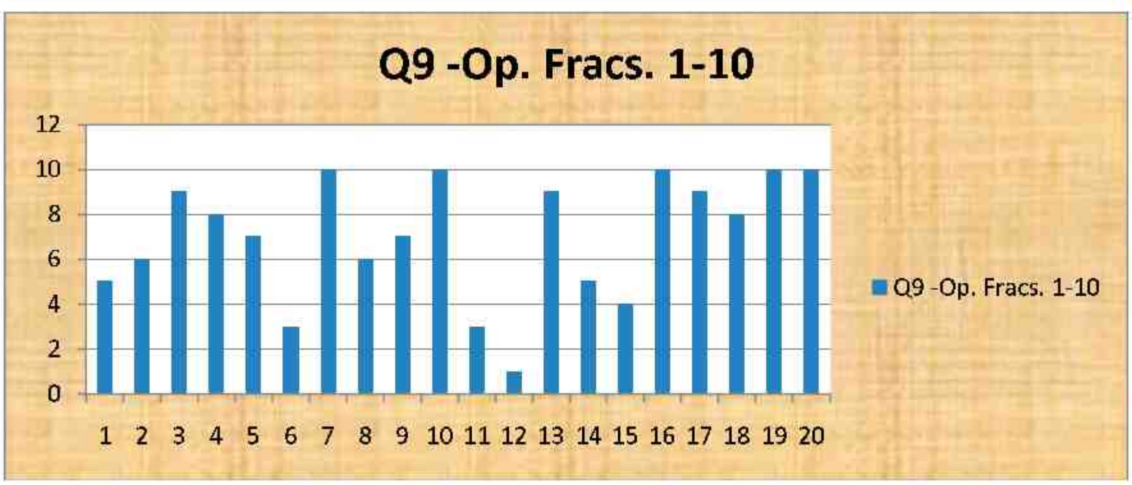


Figure 5.10 Most of the students knew operations on fractions.

The bar graph in Figure 5.10 shows that most of the students reported that they were familiar with operations on fractions. Only three students felt that they had somewhat weak skills in this area. In this aspect both groups were similar.

Question 10- What methods do you use when adding or subtracting fractions? List them and describe.

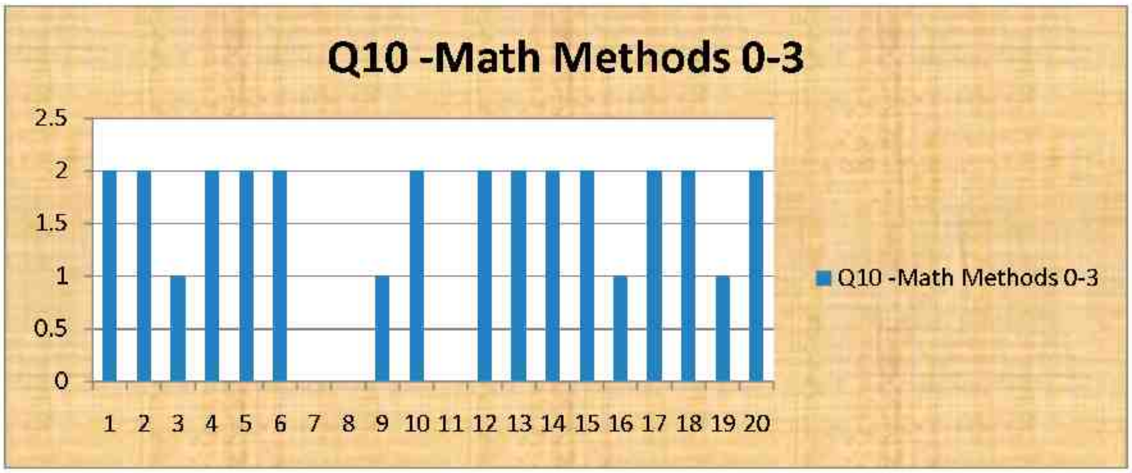


Figure 5.11 Self-reported knowledge of adding and subtracting fractions.

Three students in the Control Group could not describe any methods for performing operations on fractions. These results are similar to the results for the Study Group.

Note that the table in section 5.2 shows that variables included in the survey were not strongly correlated with students' performance on administered pre-tests and post-tests.

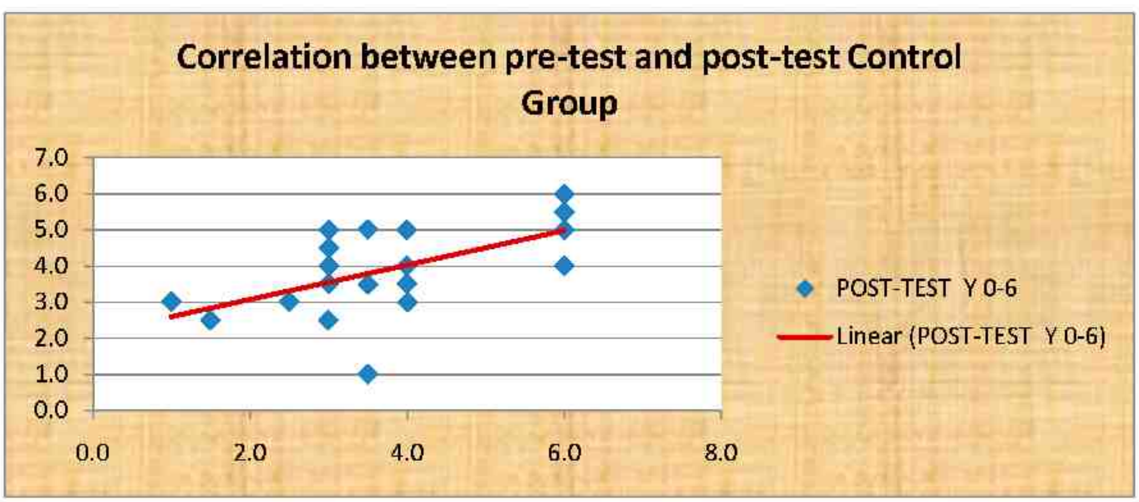


Figure 5.12 Correlation between pre-test and post-test Control Group.

Figure 5.12 shows that there was only a moderate positive correlation between pre-test and post-test scores in the Control Group. This is because most of the students that earned a perfect score on pre-test did well on post-test as well.

5.4 Concept Quiz for Control Group

As for the Study Group, in order to evaluate whether students in the Control Group had a conceptual understanding of fraction operations, we administered the same quiz with conceptual questions. In various contexts most of the students did well on the concept quiz (see Figure 5.13). This showed that they were familiar with fractions, but many still had difficulties with addition and subtraction when denominators were large.

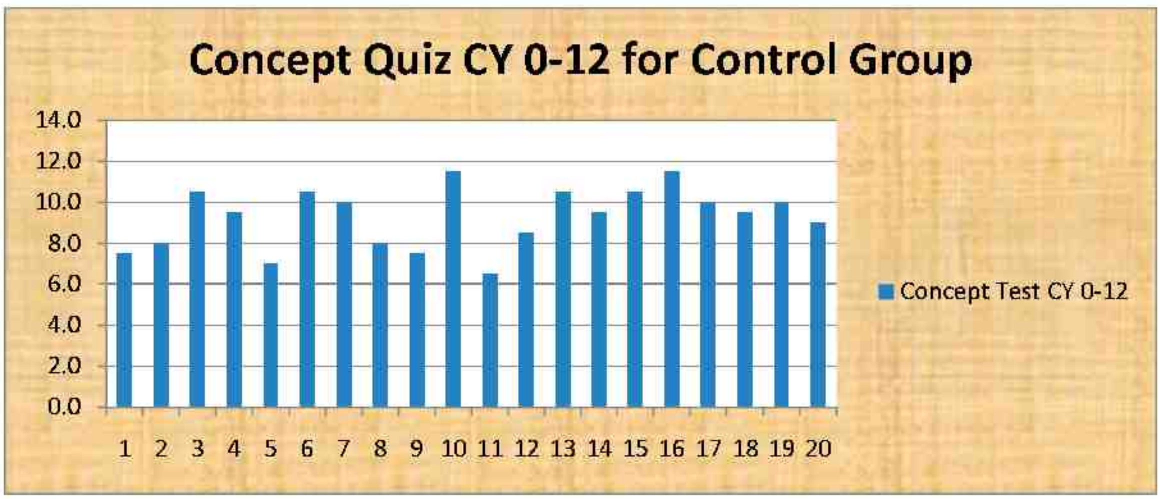


Figure 5.13 Results of the Concept Quiz.

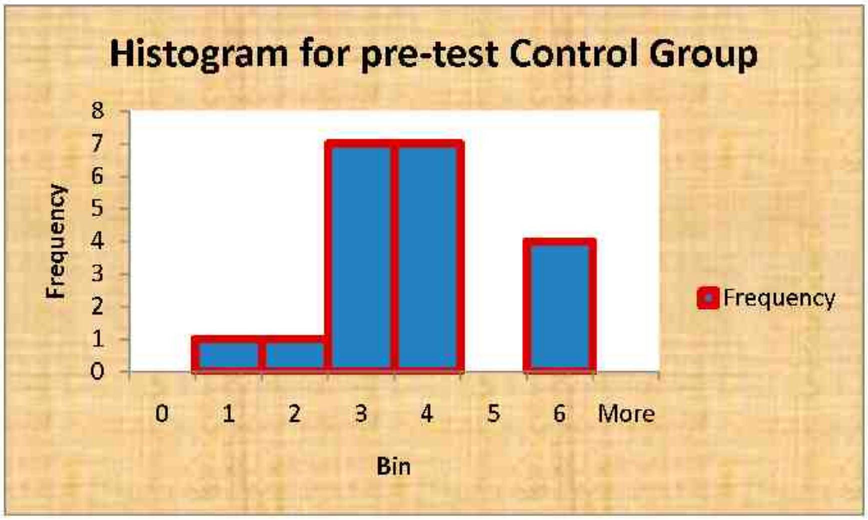
5.5 Evaluation of the performance for Control Group

To evaluate the effectiveness of teaching the prime factorization method for finding the LCD in the Control Group, we analyzed students' performance by applying paired *t*-test between their pre-test and post-test results.

- Test statistics $t = \dfrac{\bar{d}}{s_d / \sqrt{n}}$
- Ho: $\mu_y \geq \mu_Y$
- Ha: $\mu_y < \mu_Y$ Original Claim (left tail test)
- Paired t test(y , Y)
- **P-value using Excel:** =TTEST(N61:N80,O61:O80,1,1)
  P-value = **0.2396996** > **0.05** (Alpha 0.05 is significance level)

At a confidence level of 95% we failed to reject the null hypothesis (Ho), and we accepted the null hypothesis (Ho). Hence the mean for the post-test scores was not greater than the mean scores for the pre-test scores. This means that the standard method was not helpful to the students. Even though they were exposed to extra learning activities, they did not perform better. Note that the students were exposed to the Prime Factorization Method in their pre-college level mathematics courses, but they did not realize any significant improvement of their skills even after our additional activities that presented the method they were familiar with.

Comparing the results in Control Group



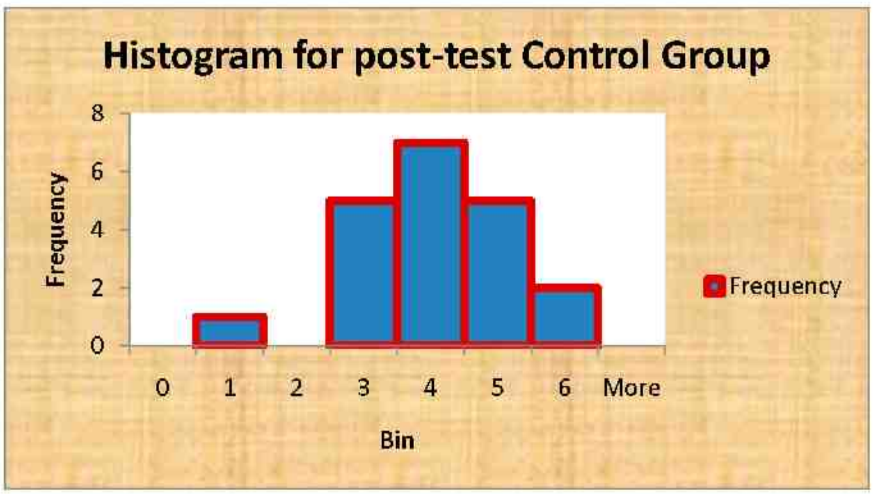


Figure 5.14 Comparing Histogram of pre-test and post-test for Control Group.

Most of the students' grades on the pre-test were between 3 and 4 out of 6 and for the post-test were between 3 and 5 out of 6; hence we observed some improvement. But there was no significant difference between the pre-test and post-test scores in the Control Group. Therefore, we conclude that extra instructions based on the Standard Method did not improve students' skills on the LCD.

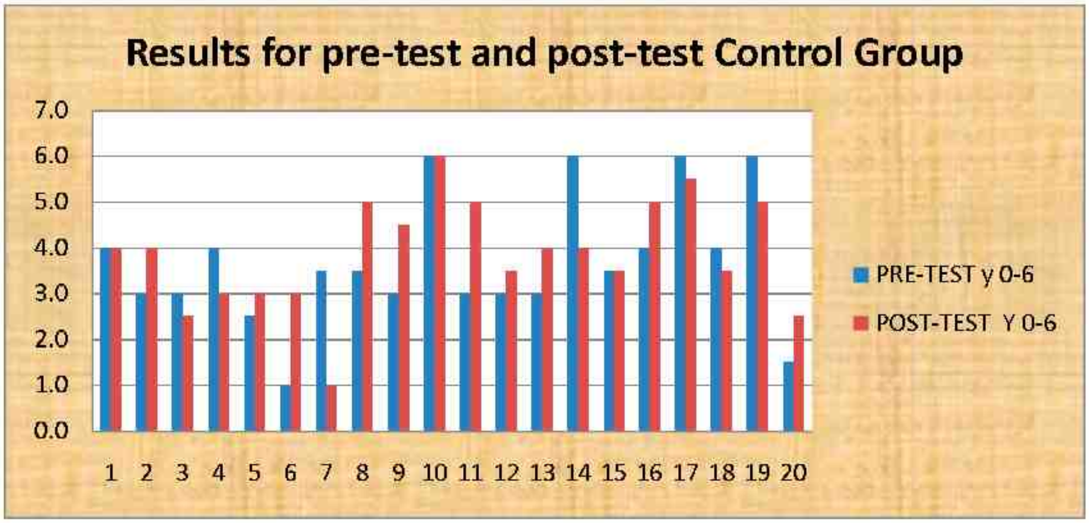
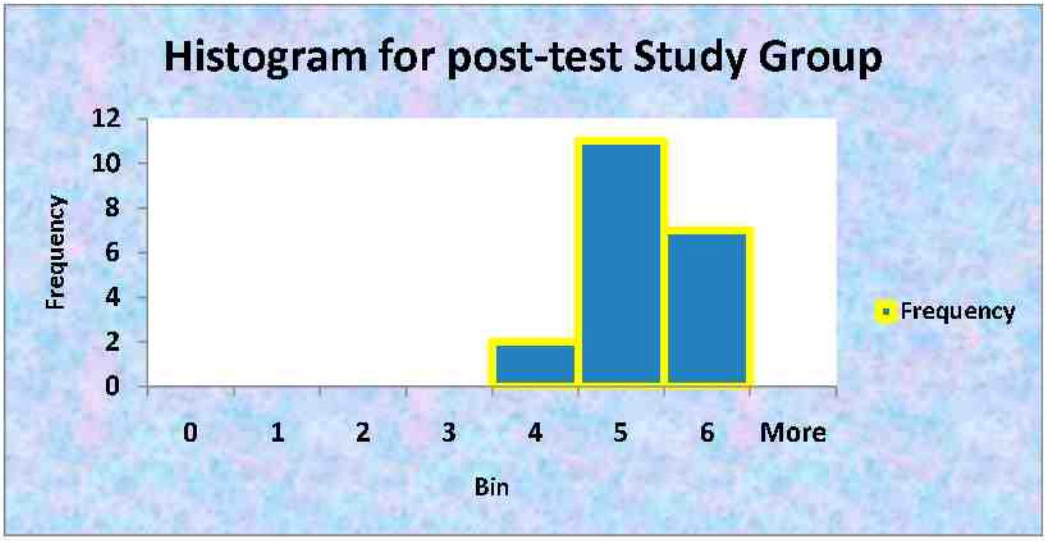


Figure 5.15 Results for pre-test and post-test for the Control Group.

We noticed that students in the Control Group showed mixed performance on both the pre-test and the post-test. The improvement was hard to predict in this group. Because the questions were comparable but different on each test, by using the same method for solving the problems some Control Group students did worse on the post-test.

6 Comparing Performance of the two groups.

We start comparison of the post-test scores for the Study Group and the Control Group by comparing the graphs below.



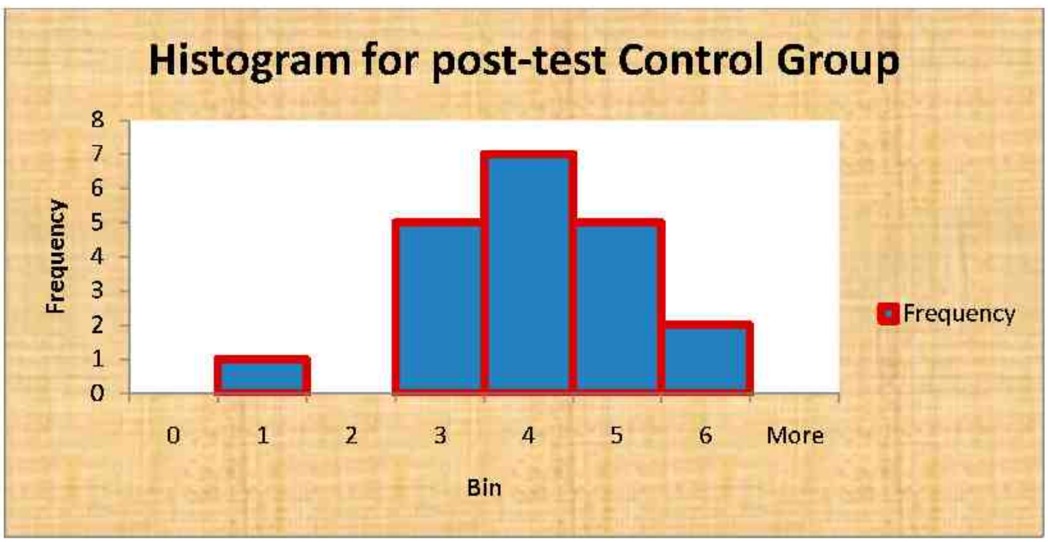


Figure 6.1 Histograms for the study group and control group.

A quick glance at the graphs in Figure 6.1 shows that scores on post-test were between 4 and 6 for students in the Study Group and between 3 and 4 for all but one student in the Control Group. The average for the Study Group was higher than the Control Group's average. Since the groups were comparable at the beginning of the study, we may conclude that the performance of participants' using the Special Method (the Ladder method) increased more than the performance of other students.

6.1 Comparing performance of the Study Group and the Control Group

To evaluate the effectiveness of the Ladder Method, we compared performance of the students in both groups by hypothesis testing. We performed an unpaired t-test for the **post-test** mean test scores (X , Y) to test the following hypothesis for the means:

- Ho: $\mu_X \leq \mu_Y$
- Ha: $\mu_X > \mu_Y$ O.C. (original claim, right tail test)

We obtained the following p-value:

P-value = TTEST(O7:O26,O61:O80,1,2) = **0.0002976, Significant**

Since **0.0002976** < **0.05** , we rejected Ho and we accepted Ha with a 95% confidence level**.** The scores in study group for the post-test were significantly better than the results in the control group. Therefore we can say that the Ladder method significantly improved students' skills. Note that our **p-value** is less than 0.001, **0.0002976** < **0.001**, hence we have extremely strong evidence that H_0 is not true with a 99% confidence level.

Therefore the new method was significantly better than the standard method. See Figure 6.2 for graphical representation of our hypothesis testing results.

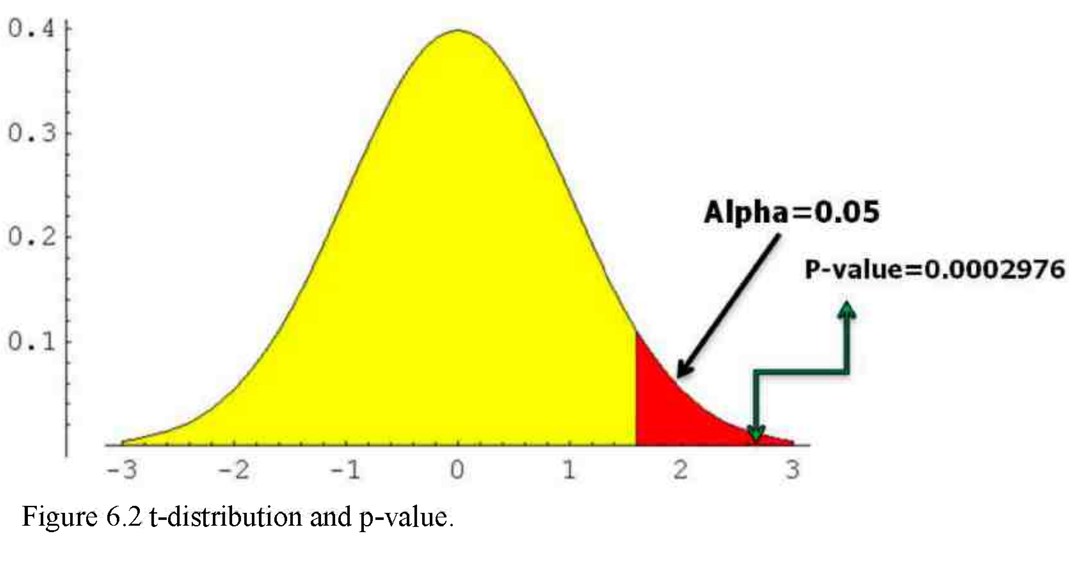


Figure 6.2 t-distribution and p-value.

6.2 Confirming the result for post-tests using the five steps method

In this section we present the calculations from section 6.1 using five easy steps.

Step 1: *Formulation of the null and alternative hypothesis.*

The Study Group performed better than the Control Group on the post test, meaning that the average score in the first group (denoted by X) was significantly better than the average score in the second group (denoted by Y). Hence the teaching activities used in the Study Group improved students' performance significantly.
We tested the following hypothesis.

Ho: $\mu_Y \leq \mu_Y$

Ha: $\mu_X > \mu_Y$ O.C. (original claim, right tail test)

Step 2: *Finding the rejection region for the confidence level α .*

Since both samples were small (n < 30), we used *t*-test. We calculated the degree of freedom as follows: df = $(n_1 + n_2 - 2)$ = **20 + 20 - 2 = 38**

Hence *t*-critical value = 2.03 from the *t*-table for df = 38 and α = 0.05 for one tail test.

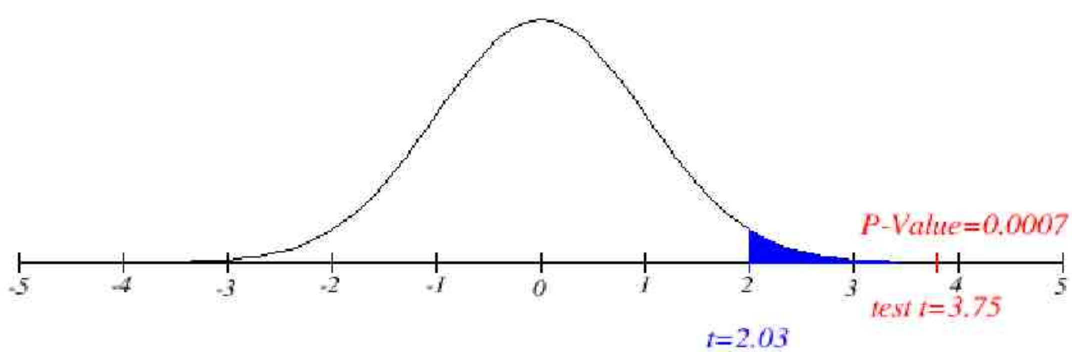


Figure 6.3 t-distribution and test statistics

Step 3: *Calculating the test statistics*

$$S_p^{\,2} = \frac{(n_1-1)S_1^{\,2}+(n_2-1)S_2^{\,2}}{n_1+n_2-2} = \frac{(20-1)(0.724)^2+(20-1)(1.202)^2}{20+20-2} = 0.9844$$

$$t = \frac{(\overline{X}_1-\overline{X}_2)}{\sqrt{S_p^{\,2}(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{5.05-3.875}{\sqrt{0.9844(\frac{1}{20}+\frac{1}{20})}} = 3.75$$

Step 4: *Accepting or rejecting the hypothesis.*

Since the test statistic value t = **3.75 > 2.03,** it falls in the rejection region; hence we reject the H_0 (see Figure 6.3).

Step 5: *Conclusion.*

At the 95% confidence level our hypothesis Ha is accepted, meaning that the new method of finding the LCD significantly improves students' performance related to fraction operations.

We repeated our testing once again using MegaStat software. The calculations are displayed below.

0-6	0-6	
5.050	3.875	mean
0.724	1.202	std. dev.
20	20	n
	38	df
	1.1750	difference (0-6 - 0-6)
	0.9839	pooled variance
	0.9919	pooled std. dev.
	0.3137	standard error of difference
	0	hypothesized difference
	3.75	t
	0.00029764	p-value (one-tailed, upper)

Once again our p-value 0.00029764 is less than α =0.05; hence we reject the null hypothesis and accept the alternative hypothesis Ha. Therefore, we conclude that the Ladder Method improves the students' performance related to operations on fractions.

7 Using Markov Chains to model student learning

Markov Chains are mathematical systems with a finite or countable number of possible states that undergo transitions from one state to another with assigned probabilities. It is useful to present a Markov Chain as a matrix to model and to predict the behavior of a system from one state to another state as in [Ga]. Note that the consecutive state of the system depends on the current state only therefore we have the condition fulfilled for the chain to be Markov. In our study of students' learning we have assumed that next test result depends only on the current test result. Hence we were able to construct the Markov Chain process representing changes in the students' performance.

We defined the possible states of our learning system as the letter grade results on consecutive tests obtained by the participants in the study, starting with pre-test and post-test grades assigned to students in the Study Group. We used the following rubric for translating scores into grades (i.e. the states of our system).

A for scores between 5 and 6
B for scores between 4 and 4.9
C for scores between 3 and 3.9
D for scores between 2 and 2.9
F for scores below 2

To model our learning, we assumed that the next state of the group test scores depends only on the current test students are taking as in [Ga]. We used the following matrix as a transition matrix for our learning system representing the probabilities of transition from each state to any other state. The matrix P, based on the Study Group participants' grades (see table on page 14), represents the actual probabilities of the changes between states (grades) as defined as follows:

$P =$

	A	B	C	D	F
A	0.98	0.02	0	0	0
B	0.91	0.08	0.01	0	0
C	0.75	0.21	0.03	0.01	0
D	0.17	0.45	0.35	0.03	0
F	0.01	0.36	0.53	0.09	0.01

For example, the probability that a student with an A grade on pre-test will get a B on the post-test is 2%, and the probability that someone with a B grade on the pre-test will get an A on the post-test is 91%. Note that the students had a learning session between each test.

To model the learning process after *n*-steps, we can calculate the P^n that will be representing the state of the learning system (i.e. test grades of this particular group of students after *n* learning sessions). Here are the calculations for the first repetitions.

$$P^2 = P \times P = \begin{bmatrix} 0.9786 & 0.0212 & 0.0002 & 0 & 0 \\ 0.9721 & 0.0267 & 0.0011 & 0.0001 & 0 \\ 0.9503 & 0.0426 & 0.0065 & 0.0006 & 0 \\ 0.8437 & 0.1264 & 0.0255 & 0.0044 & 0 \\ 0.7503 & 0.1844 & 0.0563 & 0.0089 & 0.0001 \end{bmatrix}$$

$$P^3 = P^2 \times P = \begin{bmatrix} 0.97847 & 0.02131 & 0.00022 & 0.00000 & 0.00000 \\ 0.97780 & 0.02185 & 0.00034 & 0.00001 & 0.00000 \\ 0.97504 & 0.02405 & 0.00083 & 0.00008 & 0.00000 \\ 0.96172 & 0.03432 & 0.00357 & 0.00039 & 0.00000 \\ 0.94684 & 0.04562 & 0.00670 & 0.00084 & 0.00000 \end{bmatrix}$$

$$P^4 = P^2 \times P^2 = \begin{bmatrix} 0.97846 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97838 & 0.02138 & 0.00023 & 0.00000 & 0.00000 \\ 0.97806 & 0.02164 & 0.00029 & 0.00001 & 0.00000 \\ 0.97646 & 0.02290 & 0.00059 & 0.00005 & 0.00000 \\ 0.97458 & 0.02437 & 0.00095 & 0.00009 & 0.00000 \end{bmatrix}$$

Note here that if your original test grade was B, after 4 steps your grade will be A with the probability of 0.97838.

$$P^5 = P^4 \times P = \begin{bmatrix} 0.97846 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02133 & 0.00022 & 0.00000 & 0.00000 \\ 0.97841 & 0.02136 & 0.00023 & 0.00000 & 0.00000 \\ 0.97822 & 0.02151 & 0.00026 & 0.00001 & 0.00000 \\ 0.97800 & 0.02168 & 0.00030 & 0.00001 & 0.00000 \end{bmatrix}$$

$$P^6 = P^5 \times P = \begin{bmatrix} 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02133 & 0.00022 & 0.00000 & 0.00000 \\ 0.97843 & 0.02134 & 0.00023 & 0.00000 & 0.00000 \\ 0.97840 & 0.02136 & 0.00023 & 0.00000 & 0.00000 \end{bmatrix}$$

$$P^7 = P^6 \times P = \begin{bmatrix} 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02133 & 0.00022 & 0.00000 & 0.00000 \end{bmatrix}$$

Note that due to the round-off procedure implemented by the computer, if we multiply the P^7 by P we get the same result. Hence with the given accuracy we can assume that $P^7 = P^n$ for n > 6. This will give us a way to calculate expected grades of the students after *n* study sessions on the *n*-th tests using the initial distribution vector of the grades.

Let vector π_0 represent the Initial Distributions of grades A, B, C, D, and F on the pre-test (the first test). This means that the probability of someone getting A on the first post-test is 23%, and the probability of someone getting B in the first post-test is 25%, and so on. Note that this data was obtained from samples in our study (see table on page 14),

$\pi_0 = [0.23 \quad 0.25 \quad 0.32 \quad 0.20 \quad 0]$

Now we can calculate the $\pi_n = \pi_0 P^n$ (the final grade distribution) after *n*-repetions of the learning activity.

$$\pi = [0.23 \quad 0.25 \quad 0.32 \quad 0.20 \quad 0] \begin{bmatrix} 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \\ 0.97845 & 0.02133 & 0.00022 & 0.00000 & 0.00000 \end{bmatrix}$$

By multiplying, we obtain:

$$\begin{array}{ccccccc} & & \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \\ \pi = \pi_n = & [& 0.97845 & 0.02132 & 0.00022 & 0.00000 & 0.00000 \;] \end{array}$$

π is called the Steady State Distribution. This means that if we continue with the study sessions followed by testing beyond seven tests about 97.845% of students will get an A, about 2.132% of the students will get a B; and only about 0.022% will get a C. Hence all students will pass. This result shows that repeating learning activities will improve students' performances to an at least satisfactory level.

8 Results and findings

We compared two groups of students improving their skills involving operations on fractions. The groups were similar, on the same level of mathematical development with no significant statistical differences. The Study Group, involved in learning common denominators using the visual and organized Ladder method, performed significantly better on the post-test than the Control Group that used the prime factorization method. Therefore we conclude that the Ladder method is more effective and should be adopted as a teaching tool. We agree with the following statement made by Lind, Marchal, Mason, and Cooper in [LMMC] "Good research generates dependable data, being derived by practices that are conducted professionally and that can be used reliably for managerial decision making" (p 9). Data from our study showed significant improvement of student performance in the Study Group. Therefore we can advocate for using the Ladder Method for the teaching of finding least common multiples. Even though for our experiment we tried to select our survey questions to account for various outside variables influencing students' learning, after we analyzed statistical calculations, we found that outside variables proved to be irrelevant to the study. Hence our main results are based solely on the effectiveness of our new method rather than on other variables that may have been relevant to students' learning.

9 Future Plans

I really enjoyed working with real life data and educational settings. I would like to do more research for other special topics that are difficult for students. Since the Ladder method was successful in this study, we may want to do a long term follow-up with a study on knowledge retention. Testing the special method (the Ladder method) with larger groups of students at various levels should follow up this study as well.

For teaching effectively and successfully in today's rapidly changing world and economy, we need to ensure that we continue to utilize teaching techniques that are tested for effectiveness. The teaching must also be involving and accurate in order to improve students' skills and understandings in an efficient way. I hope to apply my statistical knowledge and various research methods to all my educational endeavors in the future.

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Software used: MS Excel and MegaStat

Appendix A

Activity: How to find LCM (or LCD for fractions**)?**

The **Ladder Method** can be used to find the **GCF** and **LCM** (LCD for fractions).

Example 1, let's use the Ladder Method to find the GCF and LCM for two numbers **72** and **84**.

1- We write the numbers on the top of a line as follows:

$$\begin{array}{c|cc} & \mathbf{72} & \mathbf{84} \\ \hline & & \end{array}$$

2- We need to find a number that is a factor of both **72** and **84**. For example, let's use **4** as the factor. We write **4** outside in the left column, and we write the corresponding quotients on the same row as follows:

$$\begin{array}{c|cc} & \mathbf{72} & \mathbf{84} \\ \hline \mathbf{4} & \mathbf{18} & \mathbf{21} \end{array}$$

3- In the next step we need to identify a factor of **18** and **21**. It is clear that **3** is a good choice. Hence we write **3** in the left column and the quotients on the same row as follows:

$$\begin{array}{c|cc} & \mathbf{72} & \mathbf{84} \\ \hline \mathbf{4} & \mathbf{18} & \mathbf{21} \\ \hline \mathbf{3} & \mathbf{6} & \mathbf{7} \end{array}$$

4- Now we check if there is a common factor for **6** and 7. Since there is none (other than 1), we have reached our result. The GCF can be found by multiplying the numbers in the left column because there is no other common factor, as follows:

$$\begin{array}{c|cc} & \mathbf{72} & \mathbf{84} \\ \hline \mathbf{4} & \mathbf{18} & \mathbf{21} \\ \hline \mathbf{3} & \mathbf{6} & \mathbf{7} \end{array}$$

GCF = 4 x 3 =12

5- We find **GCF of 72** and **84** is **12**= **4** x **3**. This method works regardless of the factors chosen at each step. We could have started with a 2 as a common factor rather than 4, and we will end up with the same result.

To find the **LCM (**or LCD for fractions**)** we multiply the numbers in the left column and the last row (inside the circled L). We get **LCM** = **LCD** = **4** x **3** x **6** x **7** = **504** and **GCF**= **4** x **3** = **12**

	72	**84**
4	**18**	**21**
3	**6**	**7**

→ **LCM** = **LCD** = **4** x **3** x **6** x **7** = **504**

Example 2, let's use the Ladder Method to find the GCF and LCM for three numbers **12, 18,** and **24** or to find the LCD (LCM) for the following fractions:

$$\frac{13}{24}+\frac{7}{18}-\frac{1}{12}$$

	12	**18**	**24**
2	**6**	**9**	**12**
3	**2**	**3**	**4**

GCF = 2 x 3 = 6

Considering there is no common factor for 2, 3, and 4 (or nothing goes in 2, 3, and 4 except 1), the **GCF** is 2 x 3 = 6 but there is a common factor for 2 and 4 which is 2. We write 2 in the left column under 3, then divide only 2 and 4 by 2 and bring down 3.

	12	**18**	**24**
2	**6**	**9**	**12**
3	**2**	**3**	**4**
2	**1**	**3**	**2**

→ **LCM** = **LCD**= **2** x **3** x **2** x **1** x **3** x **2** = **72**

We just need to multiply the circled numbers in the L shape to get the LCM or LCD.

Appendix B

Finding the LCM (or LCD for fractions) by prime factorization.

We start by finding prime factorization for each number and writing each number as a product of its factors.

Then we circle factors in common and multiply the first number by all not circled factors in the second number.

Example: finding LCM (or LCD in fractions) of 18 and 60.

We find the prime factorization for each number.

60 = 2 x 2 x 3 x 5

18 = 2 x 3 x 3

Hence one 2 and one 3 are common factors.

We multiply 60 by 3 (the only not common factor in 18) to obtain 180 = LCM (LCD).

LCM = LCD = $2^2 \times 3^2 \times 5 = 180$

Appendix C

Survey Questions:

Please be honest with your responses. You will get extra credit no matter what your answers are.

1- Are you a visual learner (Yes=1 and No= 0)?

2- What is your gender (Male=1 and Female= 0)?

3- How many hours per week do you work?

4- How many university units have you passed so far?

5- How many hours per week do you play computer games?

6- How many hours per week do you study for your math class?

7- What is your highest math level class (0-4)?
(Pre-algebra=0, Intermediate Algebra=1, College Algebra=2, Statistics=3, and Calculus=4)

8- On the scale of 1-10 (10 being the highest), how much do you like math?

9- On the scale of 1-10 (10 being the highest), how good are you in adding and subtracting fractions?

10- What methods do you use when adding or subtracting fractions? List them and describe.

Appendix D

Pre-test questions: Find the LCD for the following fractions. Then solve the problems.

1- $\frac{3}{8}+\frac{5}{12}$ LCD=

2- $\frac{2}{5}-\frac{3}{7}$ LCD=

3- $\frac{3}{15}+\frac{5}{35}$ LCD=

4- $\frac{3}{28}+\frac{5}{14}-\frac{2}{21}$ LCD=

Post-test questions: Find the LCD for the following fractions. Then solve the problems.

1- $\frac{7}{24}-\frac{5}{36}$ LCD=

2- $\frac{7}{9}+\frac{2}{5}$ LCD=

3- $\frac{9}{20}-\frac{7}{25}$ LCD=

4- $\frac{13}{24}+\frac{7}{18}-\frac{1}{12}$ LCD=

Appendix E

Fraction Concepts Quiz

Name: ____________________________.

Please answer the following questions (extra credit, each question 1 point):

1- In the fraction $\frac{5}{8}$

a- What does the 5 stand for? Please explain.

b- What does the 8 stand for? Please explain.

c- Draw a picture representing $\frac{5}{8}$

2- Please answer the following questions for $3\frac{2}{5}$

a- What do we call this type of fraction?

b- Convert to improper fraction.

c- Draw a picture representing $3\frac{2}{5}$

3- Convert this fraction to a mixed number $\frac{23}{4}$

4- Explain why do we need the Least Common Denominator (LCD) to add or subtract fractions?

5- Please add the following fractions.

a- $\frac{2}{3}+\frac{5}{7}$

b- $\frac{a}{2}+\frac{b}{3}$

c- $\frac{1}{a}+\frac{2}{b}$

d- $\frac{a}{b}+\frac{c}{d}$

6- Explain if you can, why most of the students don't like using the Least Common denominator (LCD)?